Quadtree-adaptive global atmospheric modelling on parallel systems

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Quadtree-adaptive global atmospheric modelling *and* parallel systems

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Why adaptivity? Scaling of solution cost

Number of degrees of freedom scales like

$$C\Delta^{-4}$$

(4 $=$ 3 spatial dimensions $+$ time)

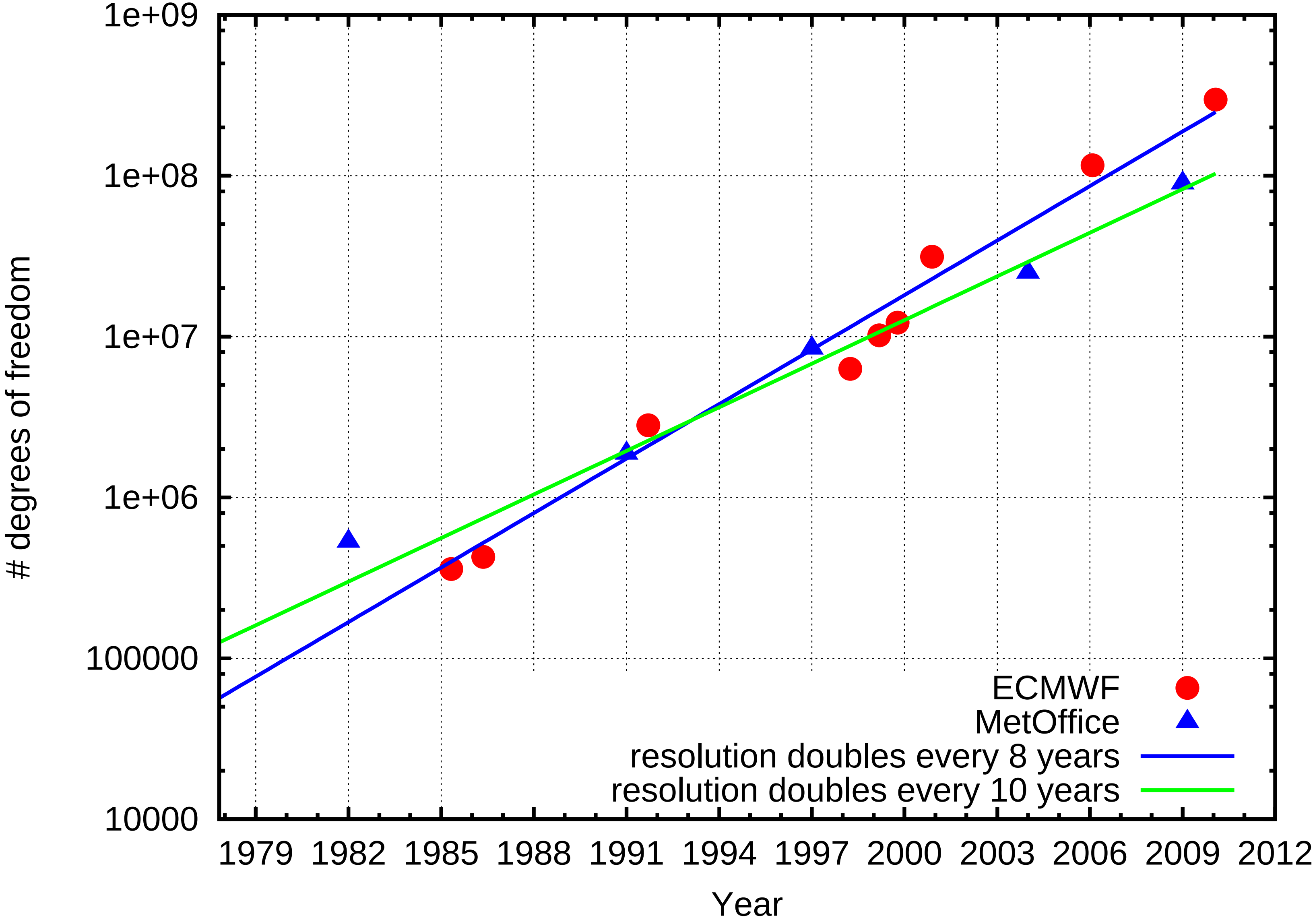
Moore's law

Computing power doubles every two years

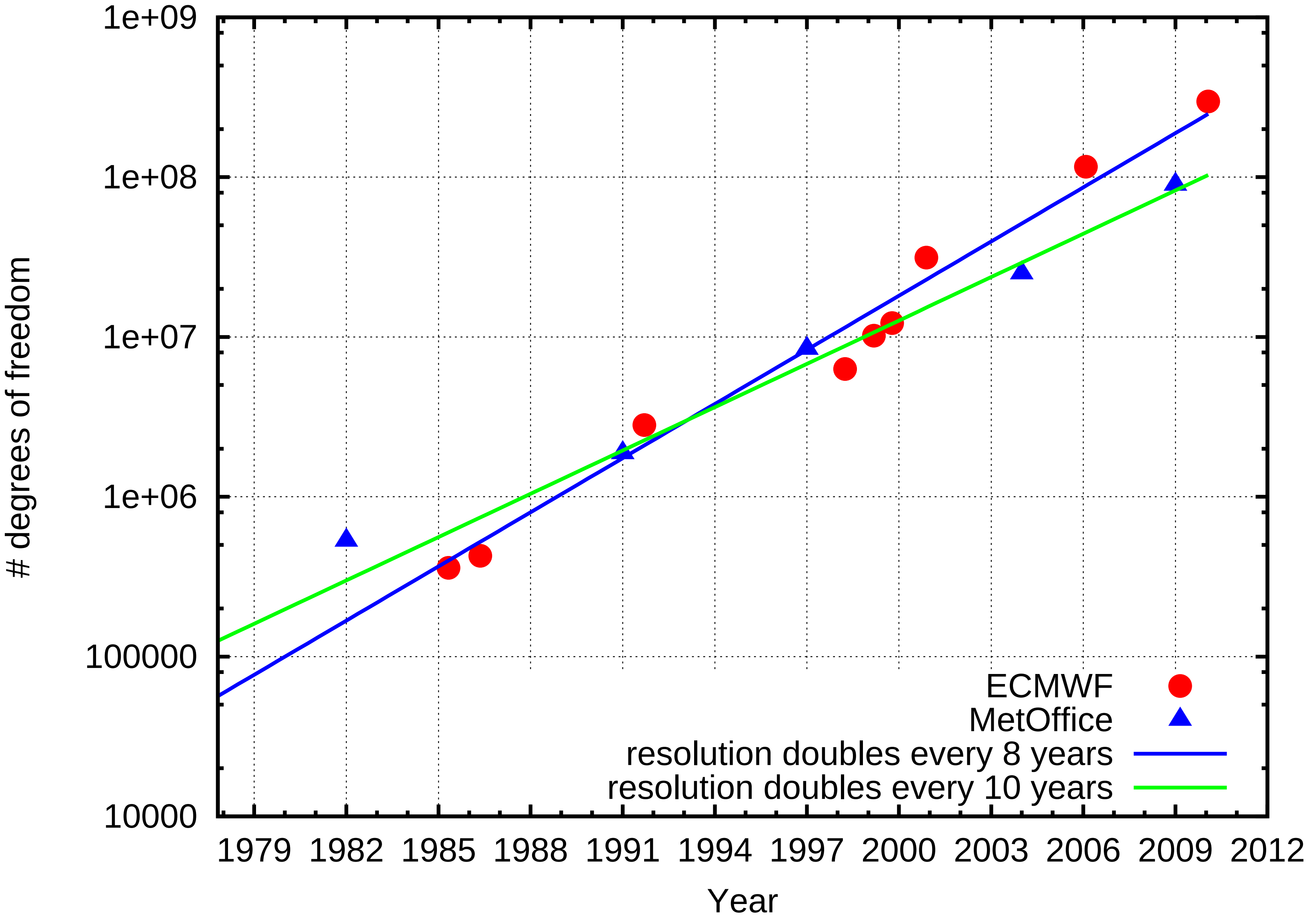
combined with the above scaling gives

Spatial resolution of climate models doubles every eight years

Is this a good prediction?

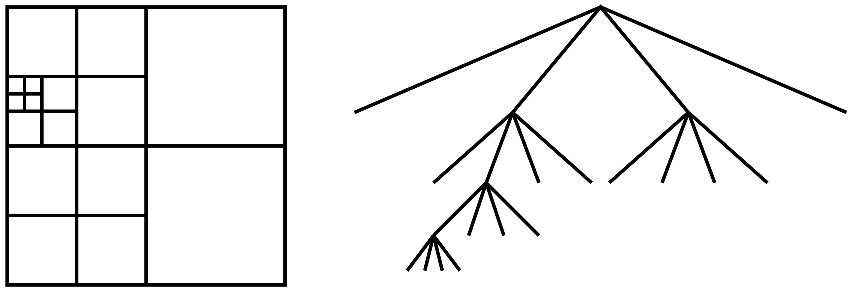


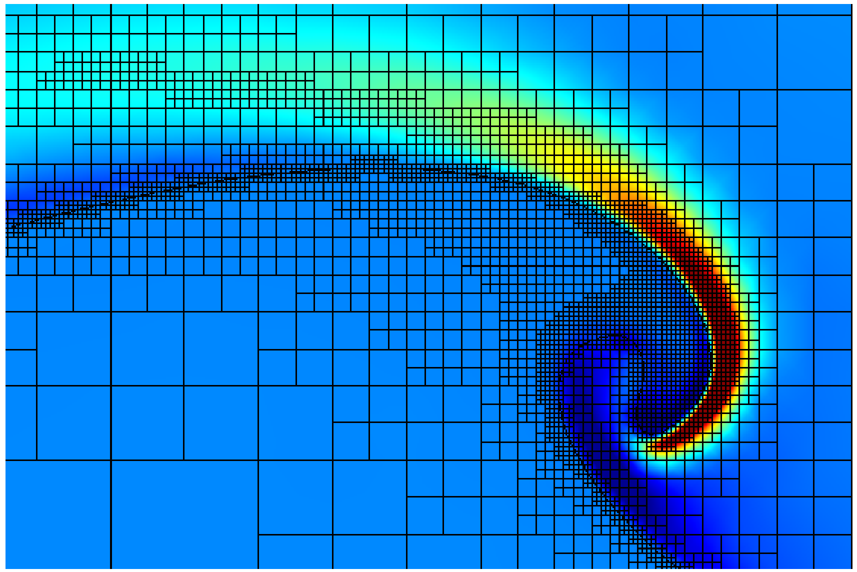
Can we do better with adaptivity?



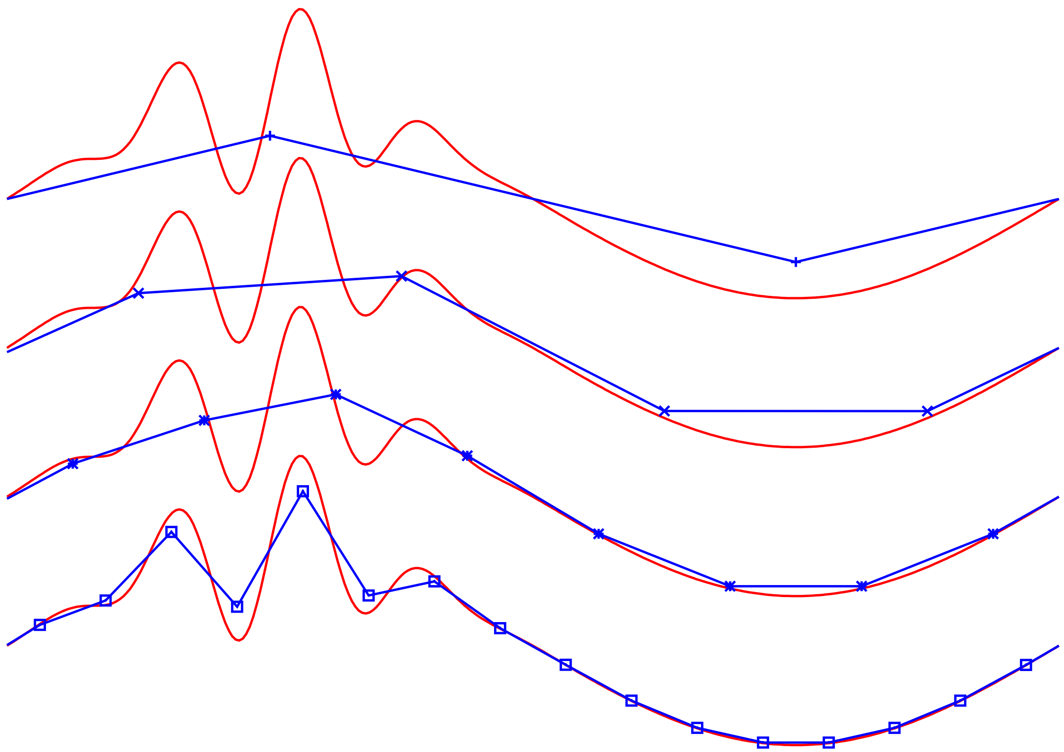
Adaptive framework for Partial Differential Equations

- Gerris Flow Solver `gfs.sf.net`
- Poisson/Helmholtz, Navier–Stokes, Euler, Saint-Venant etc...
- Adaptive quad/octree discretisation
- Colocated A-grid + approximate projection stabilisation
- Free Software (GPL) since its first version in 2001
- Popinet (2003, 2009), *JCP*, Popinet & Rickard (2007), *Ocean Modelling*





A natural multi-scale/frequency representation

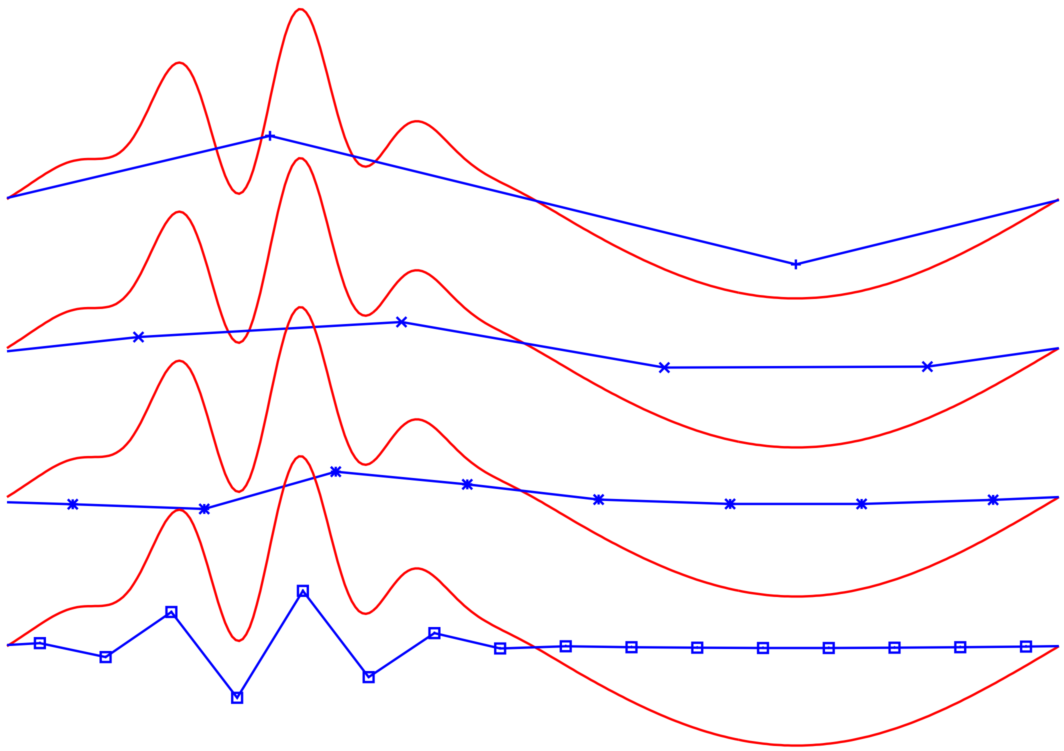


⇒ Efficient multigrid solvers for linear/nonlinear systems

⇒ A large collection of other efficient “divide-and-conquer” algorithms: spatial indexing, compression etc...

⇒ Formally linked to wavelets / “multiresolution analysis”

A natural multi-scale/frequency representation



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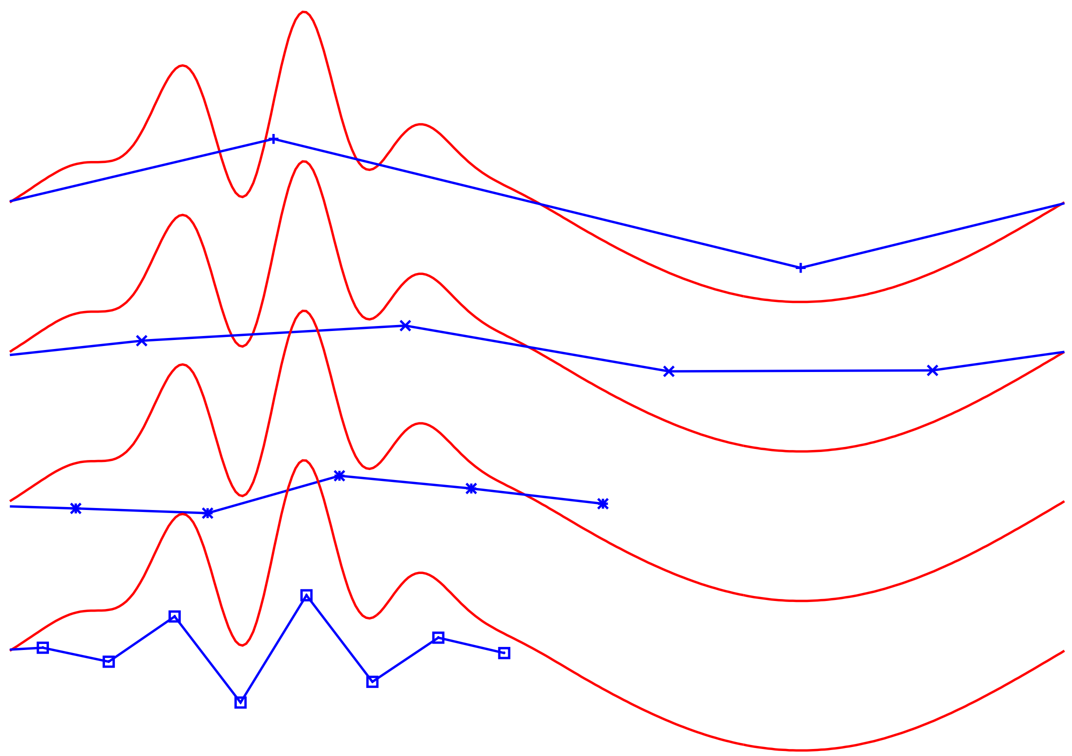

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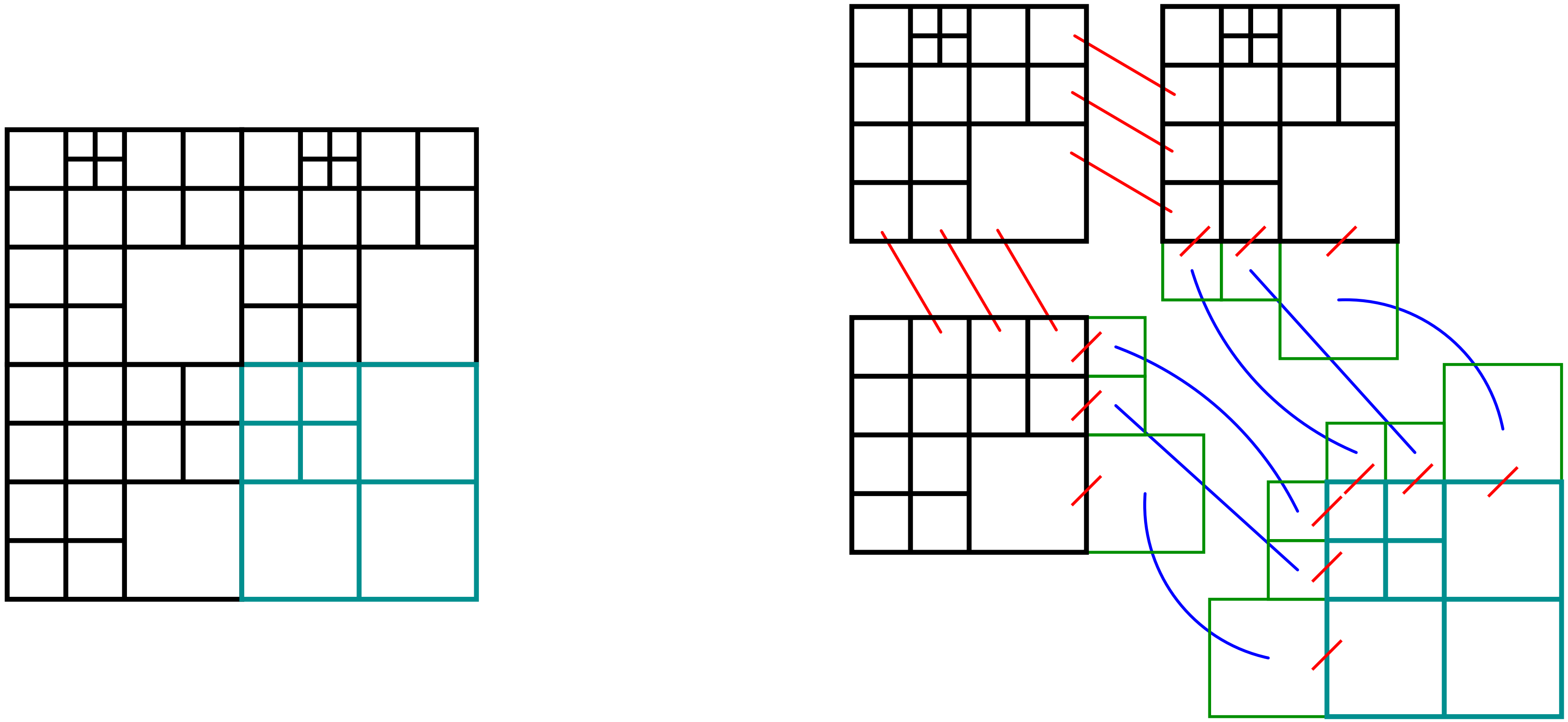
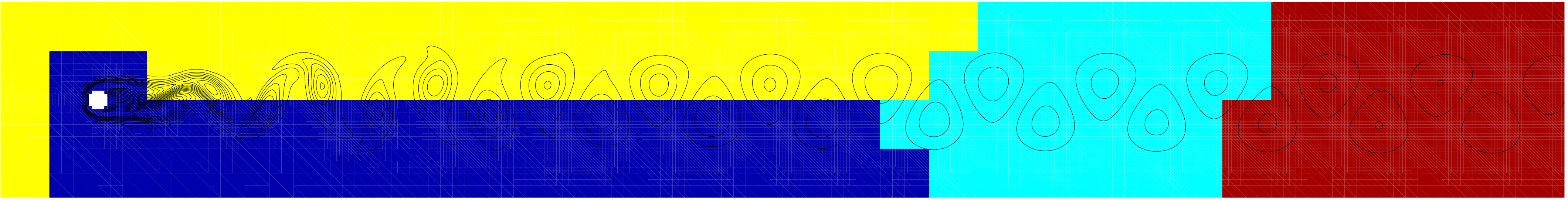
Parallel domain decomposition



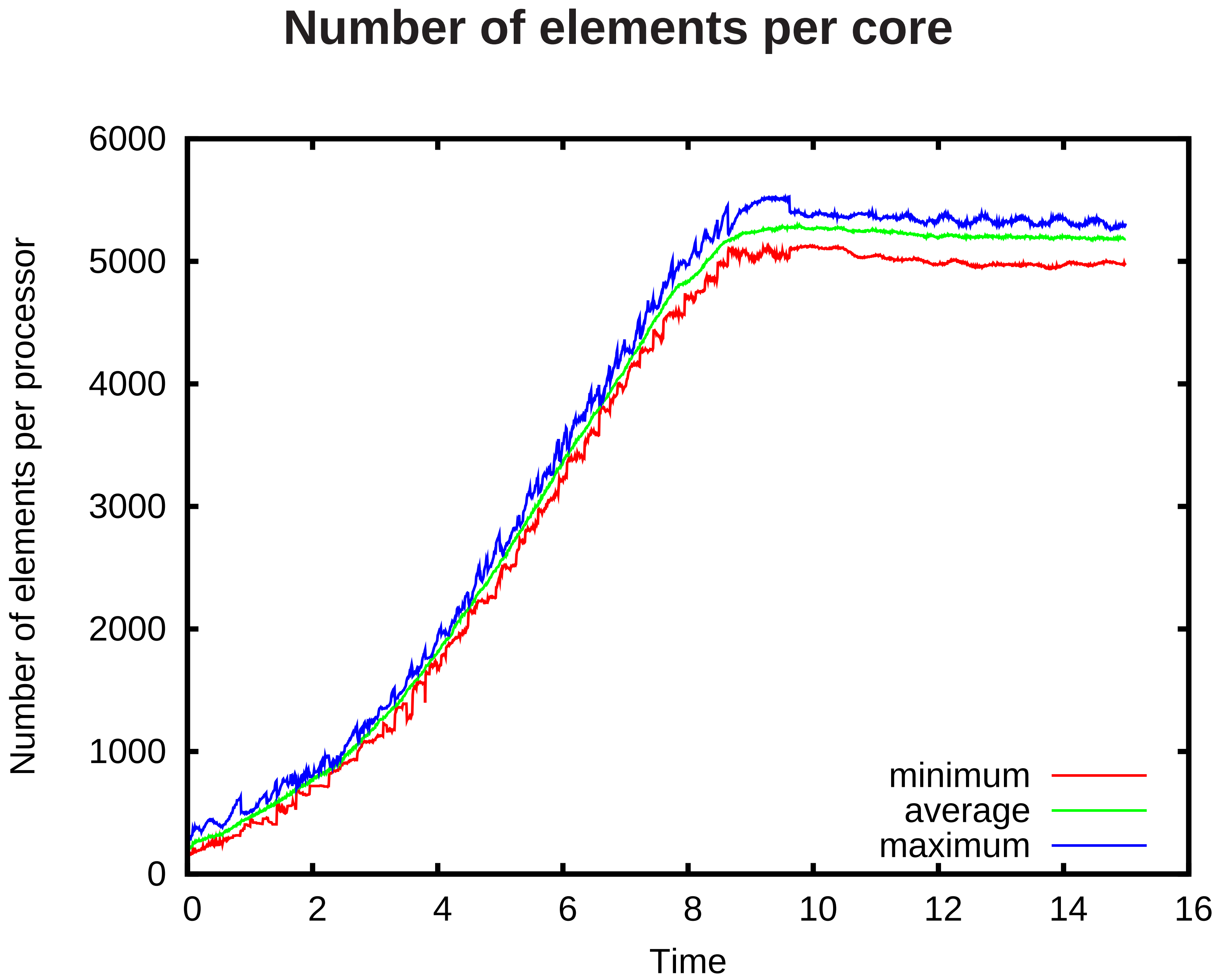
coarse-grained parallelism

preserves multigrid hierarchy

Example of dynamic load-balancing

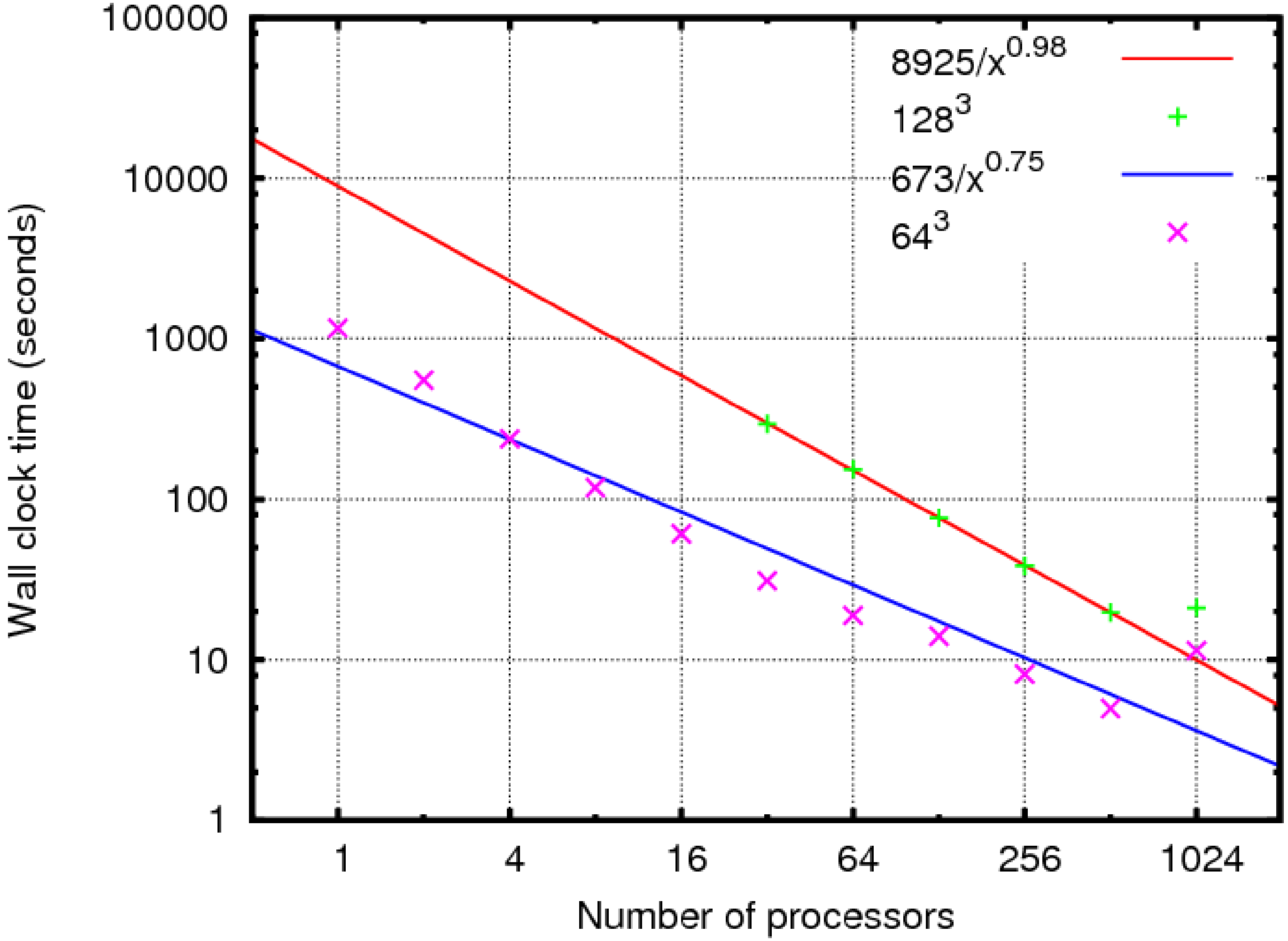


- Balancing fluxes computed in parallel as a “Poisson problem on a graph”
- Heuristics for choosing which quad/octrees to shift between processes
- *Parallel simulation of multiphase flows using octree adaptivity and the volume-of-fluid method*, Agbaglah et al, Cr. Aca. Sci., 339, 2011.



Number of elements per core
Number of elements per processor
Time
0
1000
2000
3000
4000
5000
6000
0
2
4
6
8
10
12
14
16
minimum
average
maximum

Parallel scaling on “large” systems



3D turbulent lid-driven cavity problem

IBM Power 575, 1024 cores

Global atmospheric model

Hydrostatic, compressible atmosphere

$$\begin{aligned}
\frac{D}{Dt}\boldsymbol{v}_h + f\boldsymbol{k} \times \boldsymbol{v}_h + \boldsymbol{\nabla}_p \Phi &= \mathcal{F} \\
\frac{\partial \Phi}{\partial p} + \alpha &= 0 \\
\boldsymbol{\nabla}_p \cdot \boldsymbol{v}_h + \frac{\partial \omega}{\partial p} &= 0 \\
\alpha &= \alpha(\theta, p) = \frac{\partial \Pi}{\partial p}\theta \\
\frac{D}{Dt}\theta &= \frac{\mathcal{Q}_\theta}{\Pi}
\end{aligned}$$

Global atmospheric model: numerical ingredients

- **Advection**: $\frac{D}{Dt}\theta$, $\frac{D}{Dt}\boldsymbol{v}_h$

  Second/third order Bell-Colella-Glaz monotonic scheme

- **Coriolis**: $f\boldsymbol{k} \times \boldsymbol{v}_h$

  Time-implicit (trivial on a colocated grid)

- **Poisson–Helmholtz**: $\boldsymbol{\nabla}_p \cdot \boldsymbol{v}_h + \frac{\partial \omega}{\partial p} = 0$

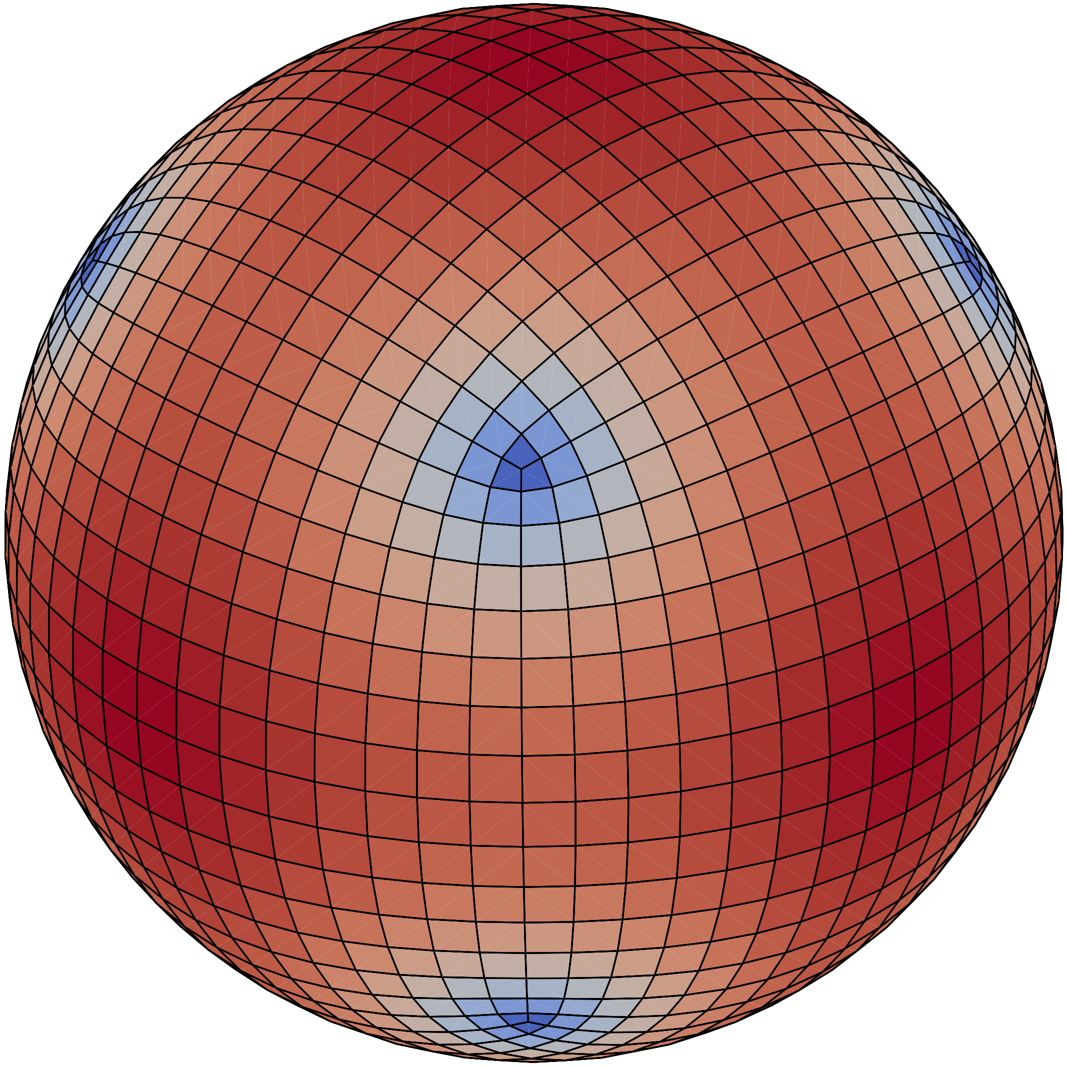
  Implicit free-surface, quadtree multigrid solver, approximate projection

  implicit A-grid mode-filtering

- **Source term parameterisations**: $\mathcal{F}$, $\mathcal{Q}_\theta$

- **Spherical geometry**

Conformal cubed sphere



Conformal = Orthogonal + Isotropic

Rancic et al, 1996

Rossby–Haurwitz wave

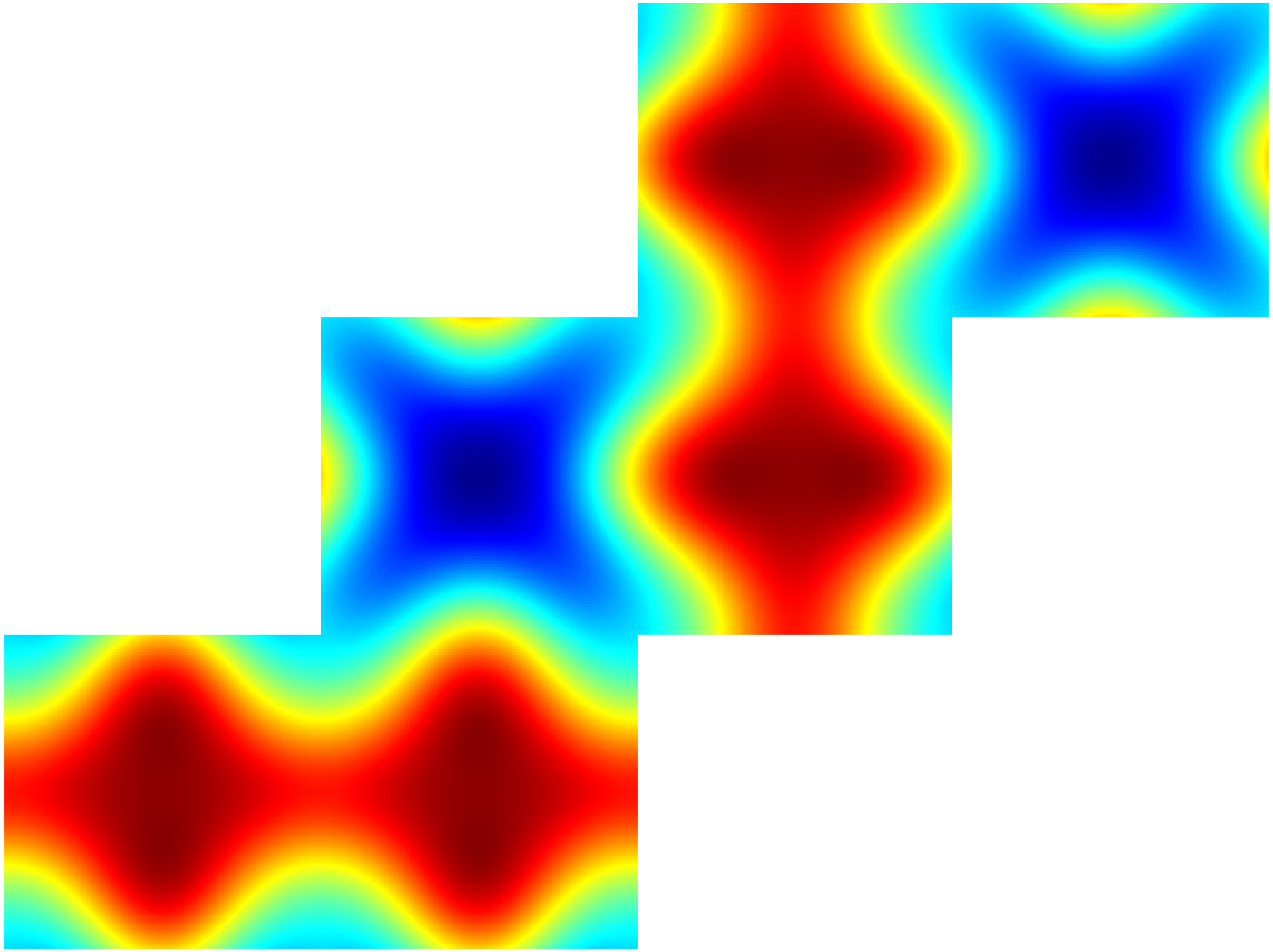
Analytical solution of the 2D incompressible Euler equations on the rotating sphere

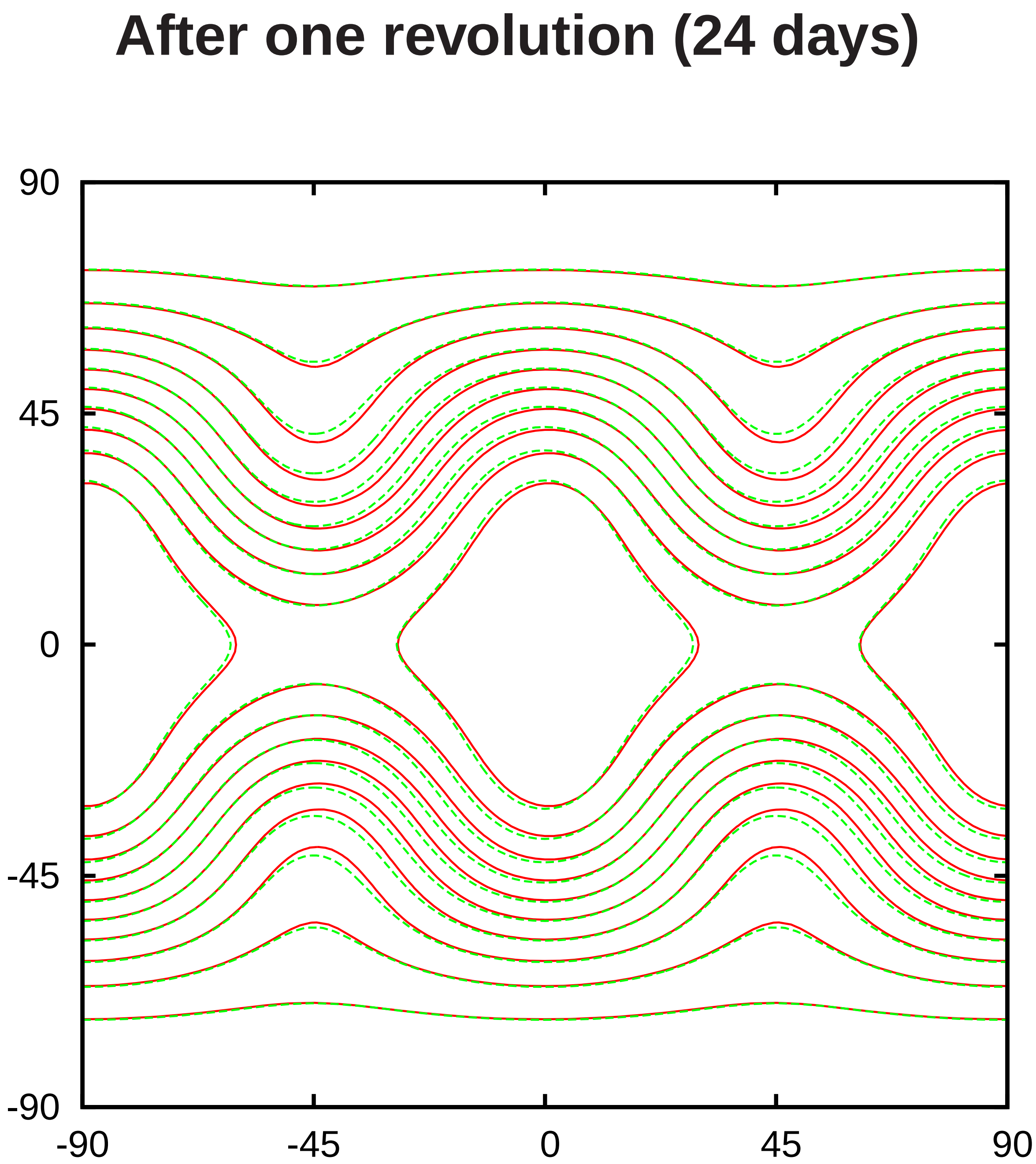
$$\psi(\lambda, \theta) = -a^2 \omega \sin\theta + a^2 K \cos^R \theta \sin\theta \cos R\lambda$$

A classical test case for global atmospheric models

(Phillips 1957, test case #6 Williamson et al, 1992).

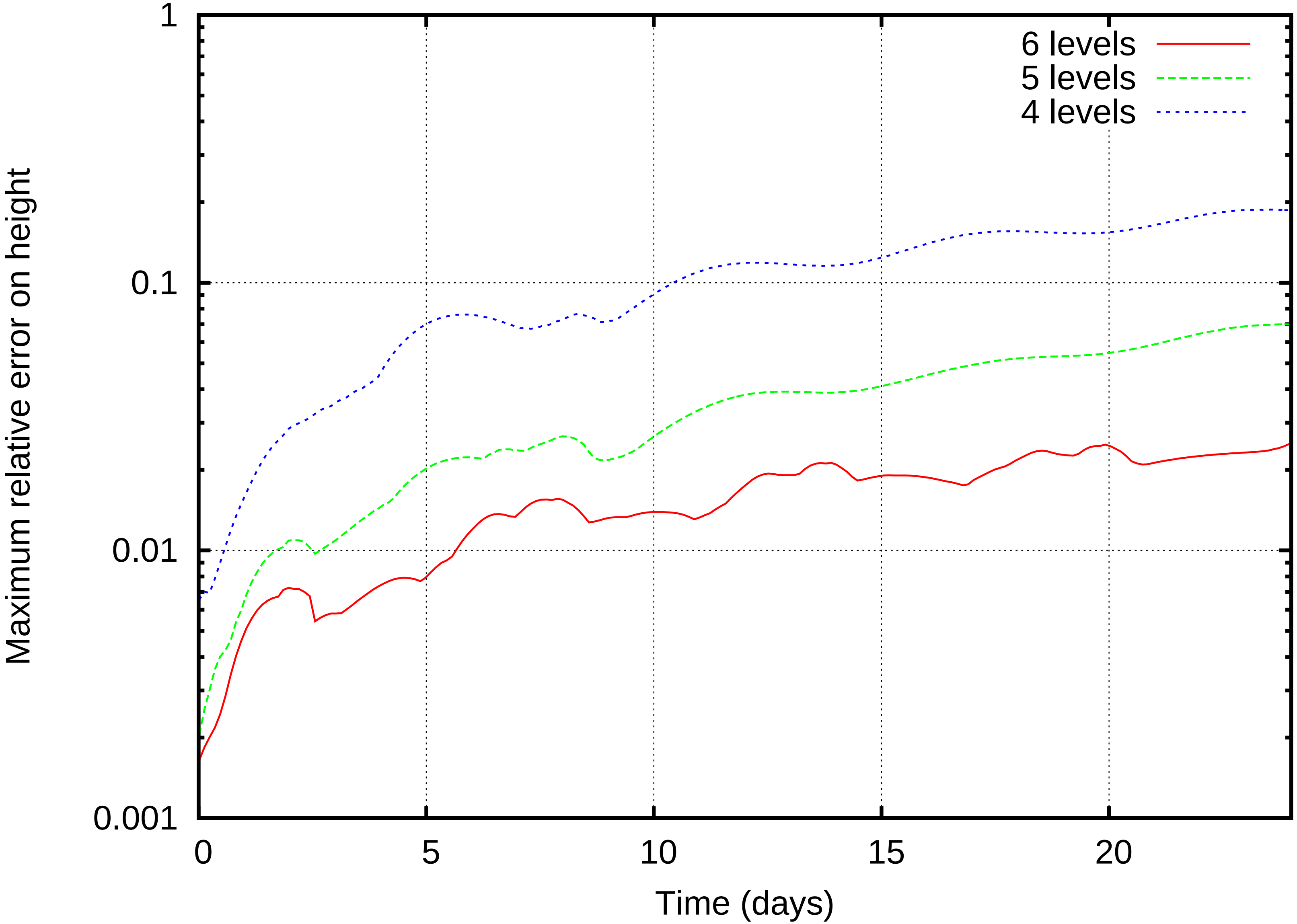
Rossby–Haurwitz wave





After one revolution (24 days)
90
45
0
-45
-90
-90
-45
0
45
90

Evolution of relative error



Order of convergence ≈ 1.5

The Held–Suarez climate model

Simple physical parameterisations

Energy input: “Radiative forcing”

$$\partial_t T = ... - k_T(\varphi, \sigma)\,[T - T_{eq}(\varphi, \sigma)]$$

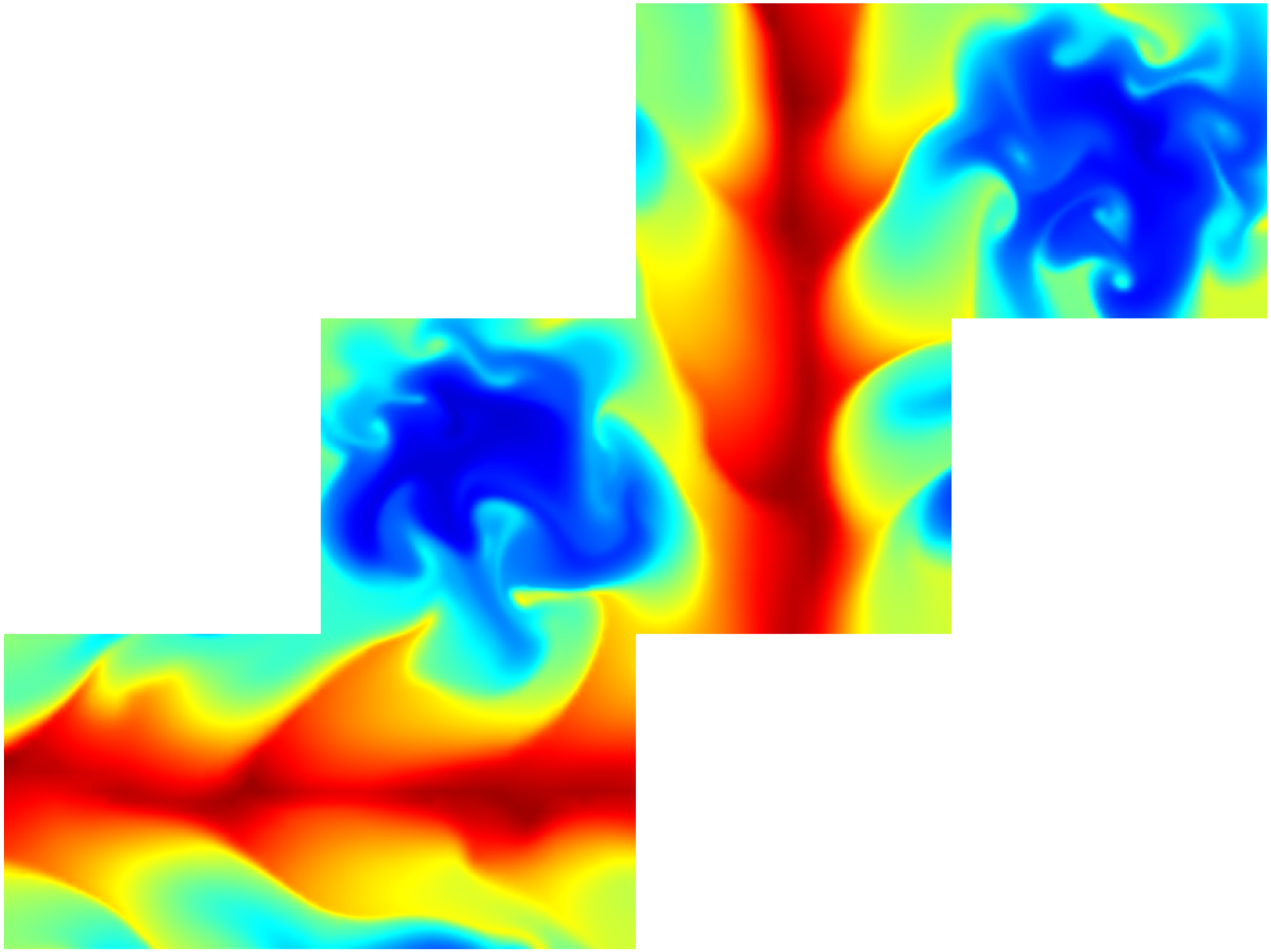
Energy dissipation: Boundary-layer friction

$$\partial_t \mathbf{v}_h = ... - k_v(\sigma)\mathbf{v}_h$$

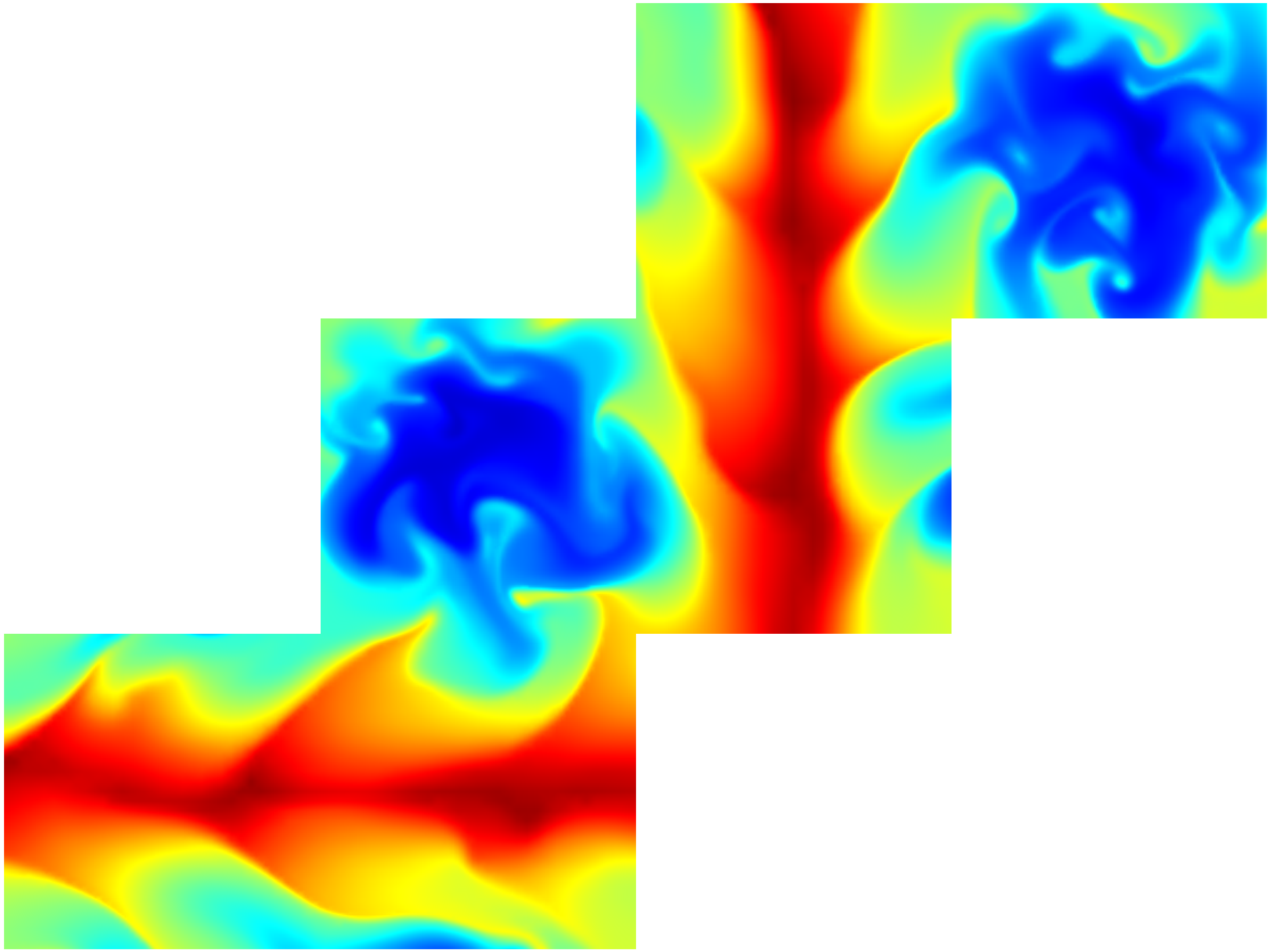
Vertical coordinate

20 equidistant pressure levels

Evolution of surface potential temperature with time

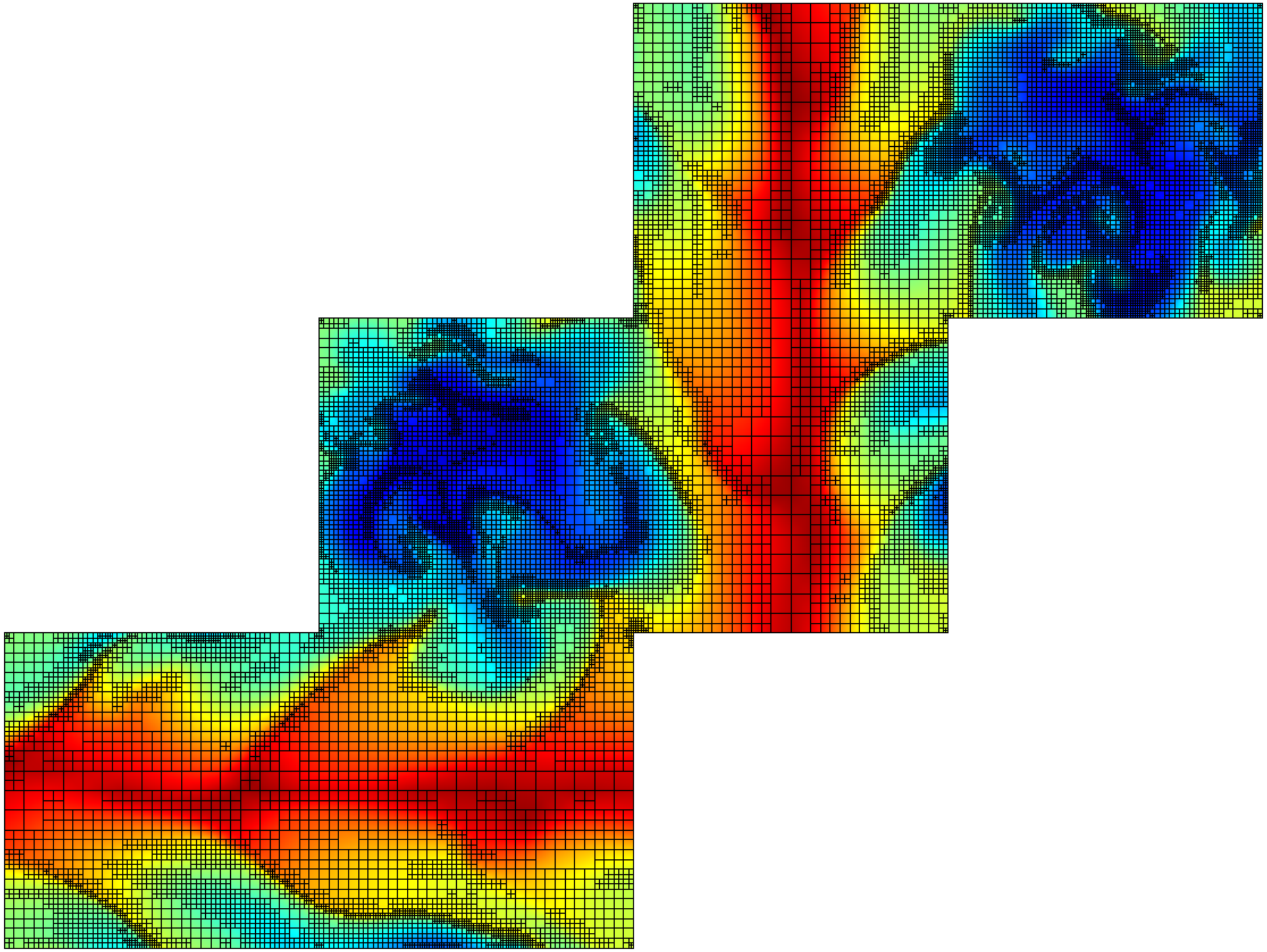


Adaptive mesh



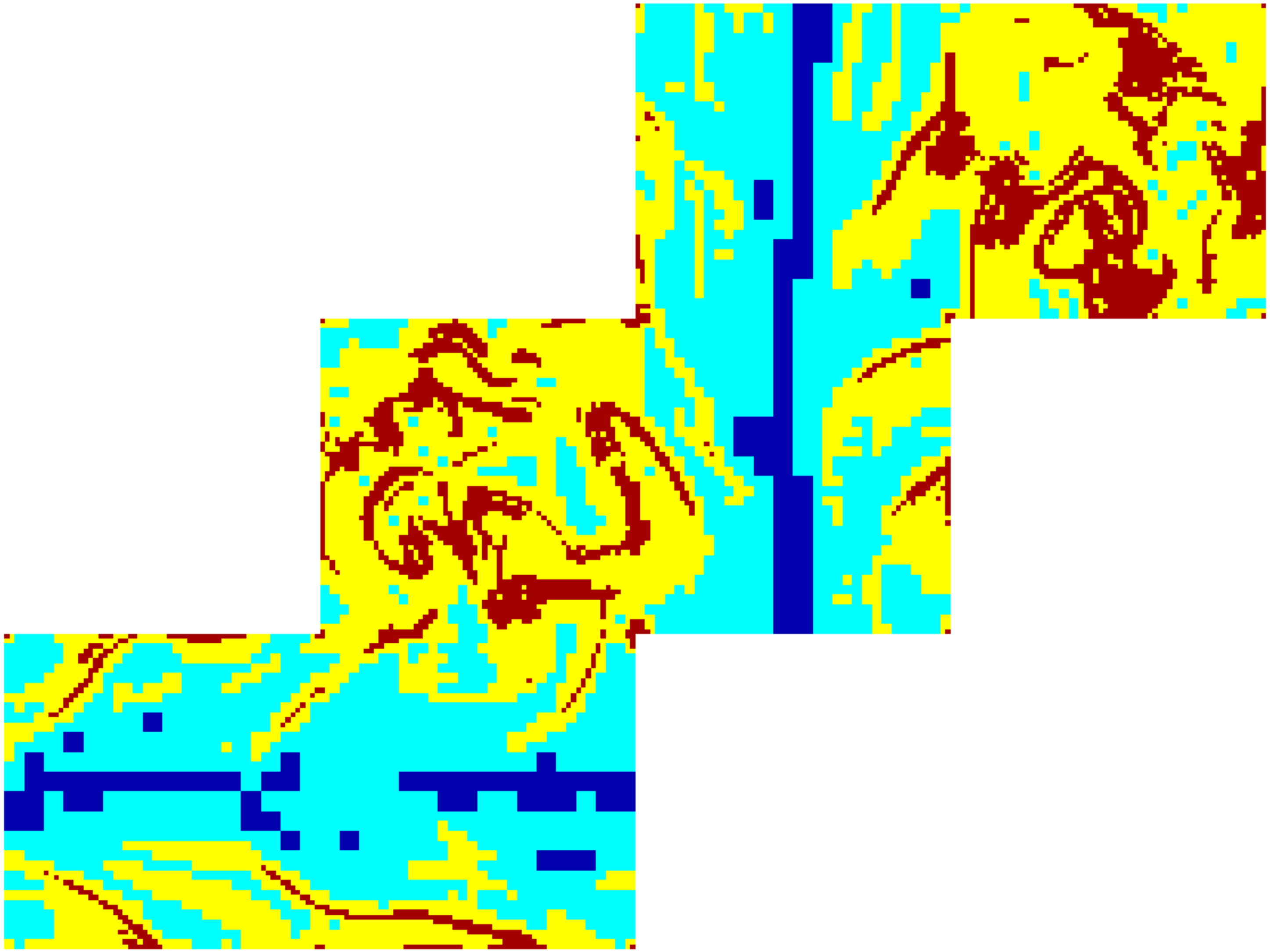
Error on potential temperature $< 0.05^\circ$

Adaptive mesh



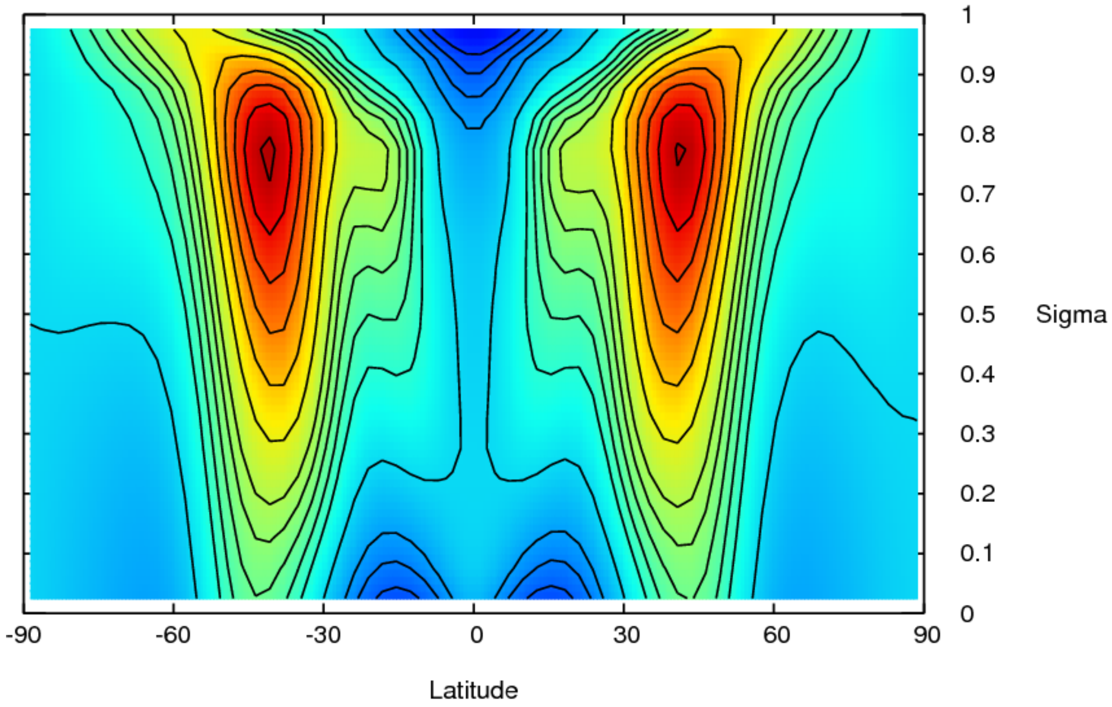
Error on potential temperature $< 0.05^{\circ}$

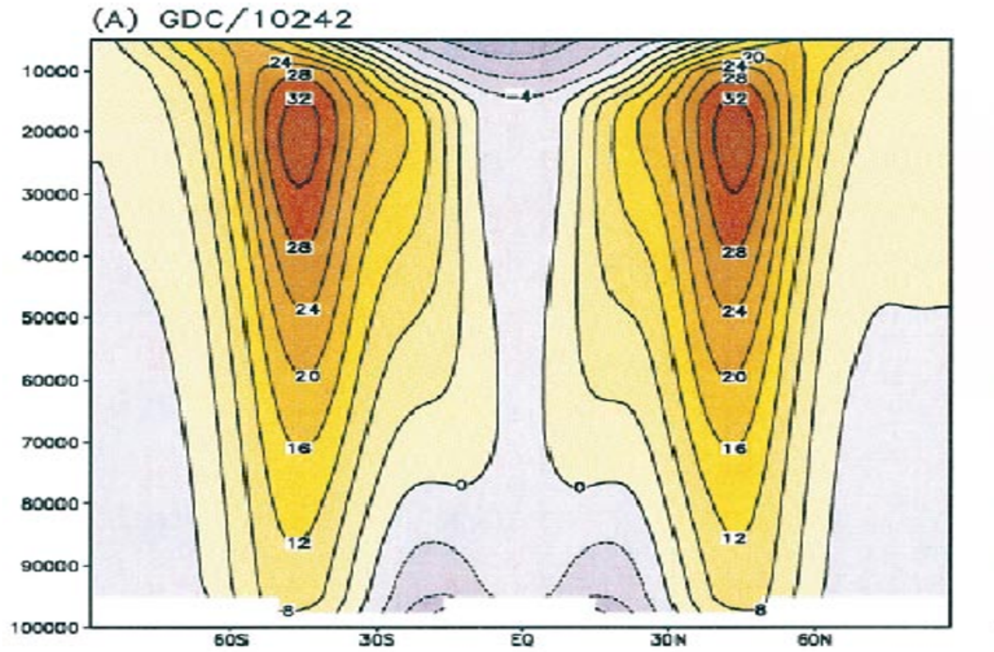
Adaptive mesh



$0.7^\circ \leq$ Spatial resolution $\leq 5.6^\circ$

900-day zonal average



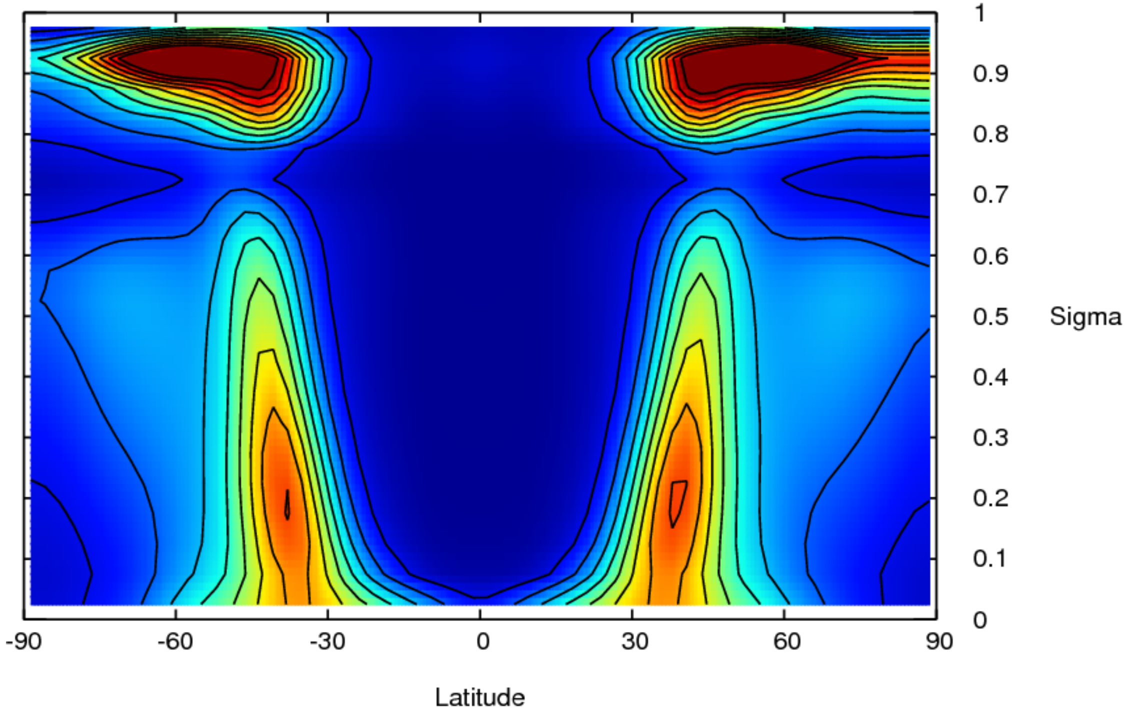


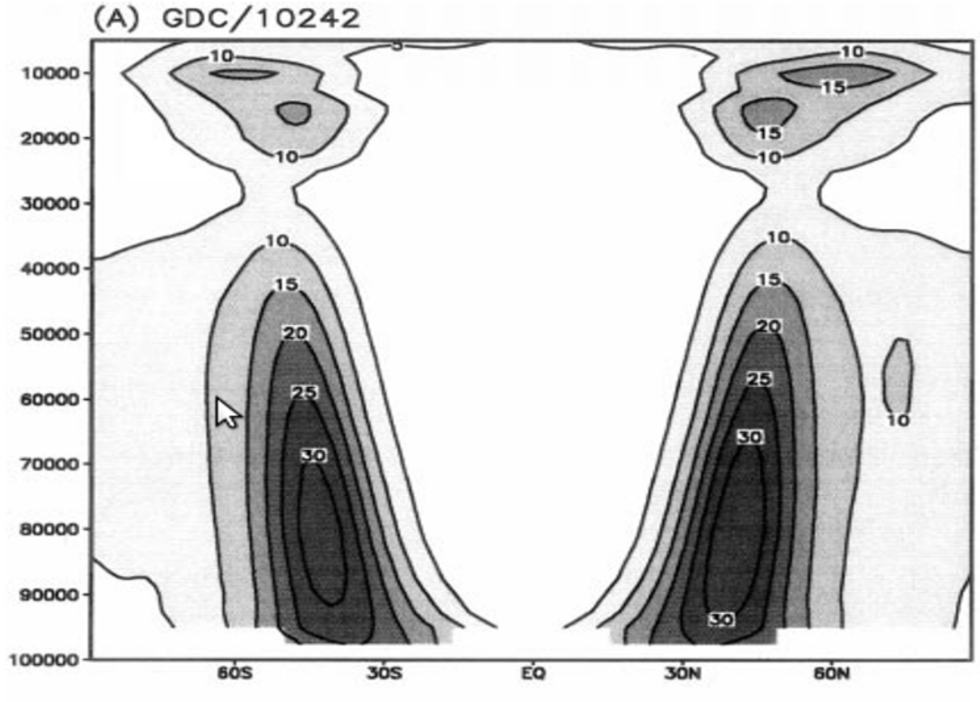
Ringler et al, MWR, 128, 2000

Zonal velocity

Maximum spatial resolution $\approx 3^\circ$

900-day zonal average



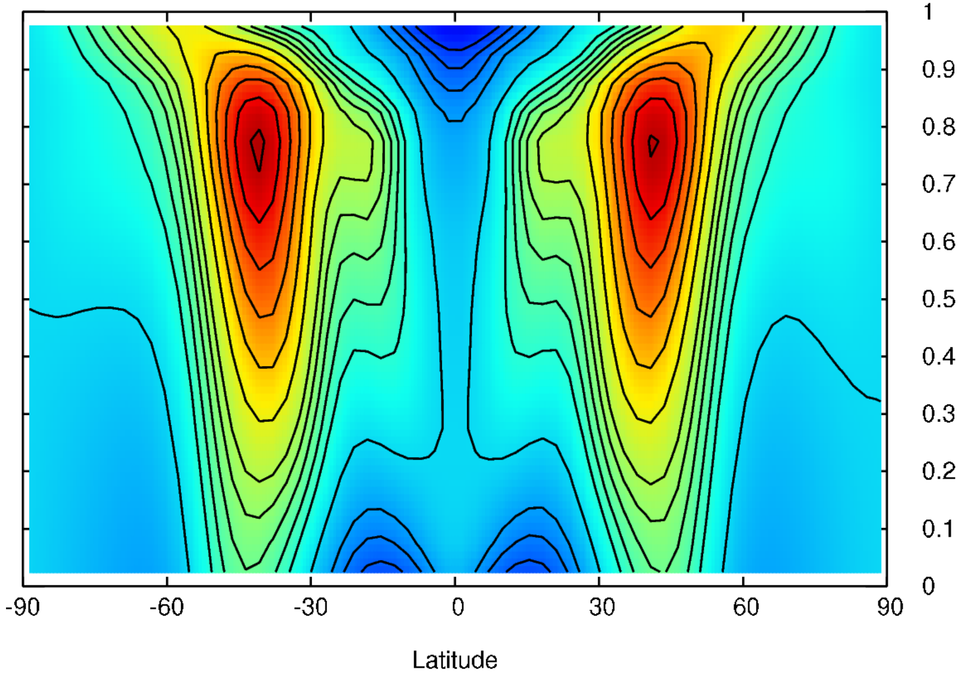


Ringler et al, MWR, 128, 2000

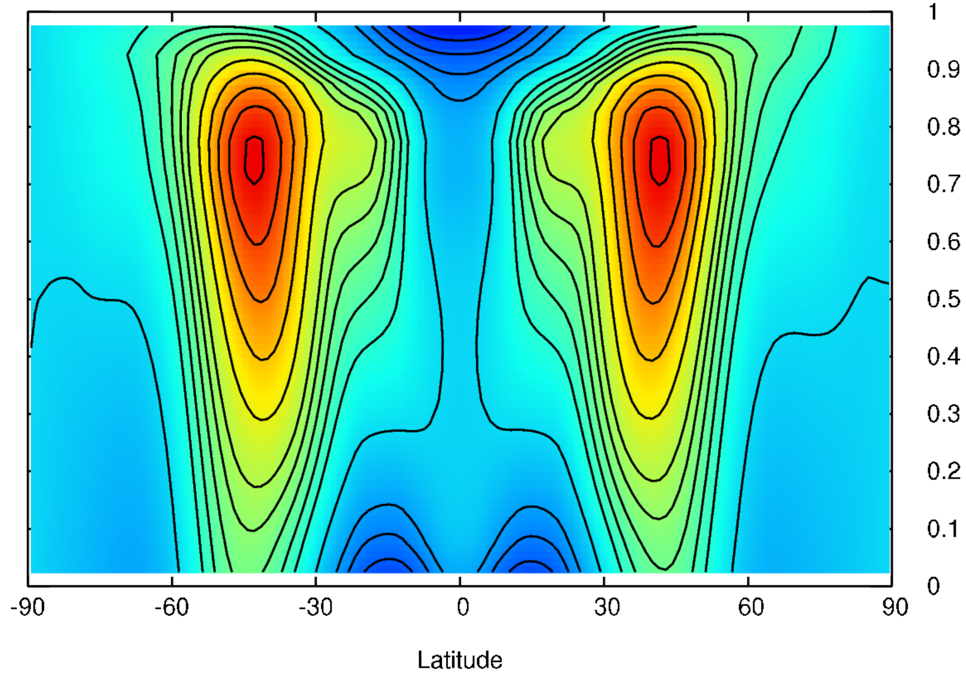
Variance of potential temperature

Maximum spatial resolution $\approx 3^\circ$

Dependence on spatial resolution



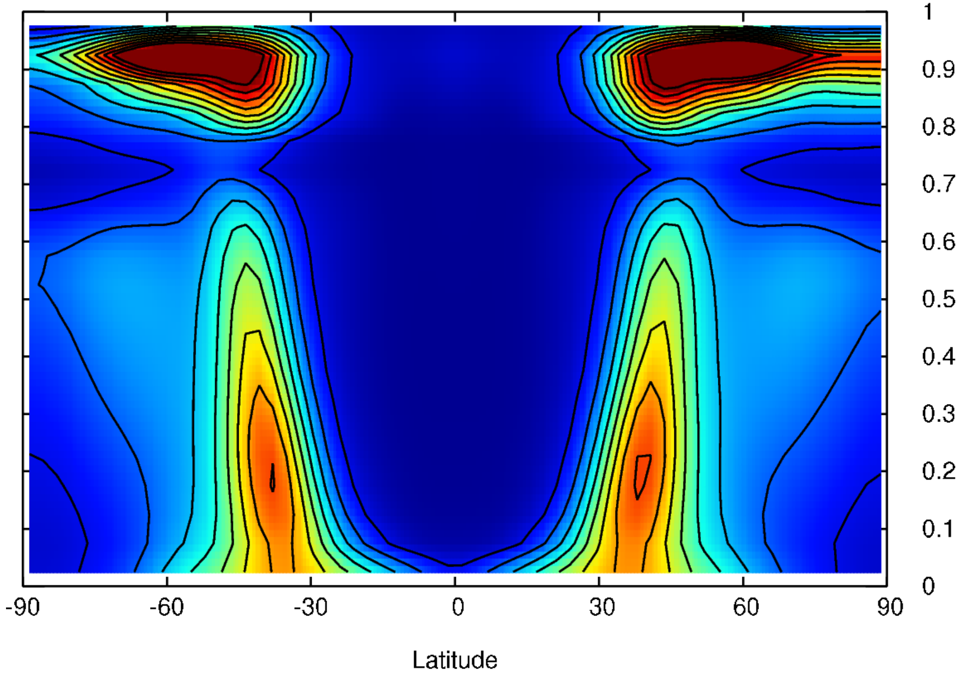
Maximum spatial resolution $\approx 3^\circ$

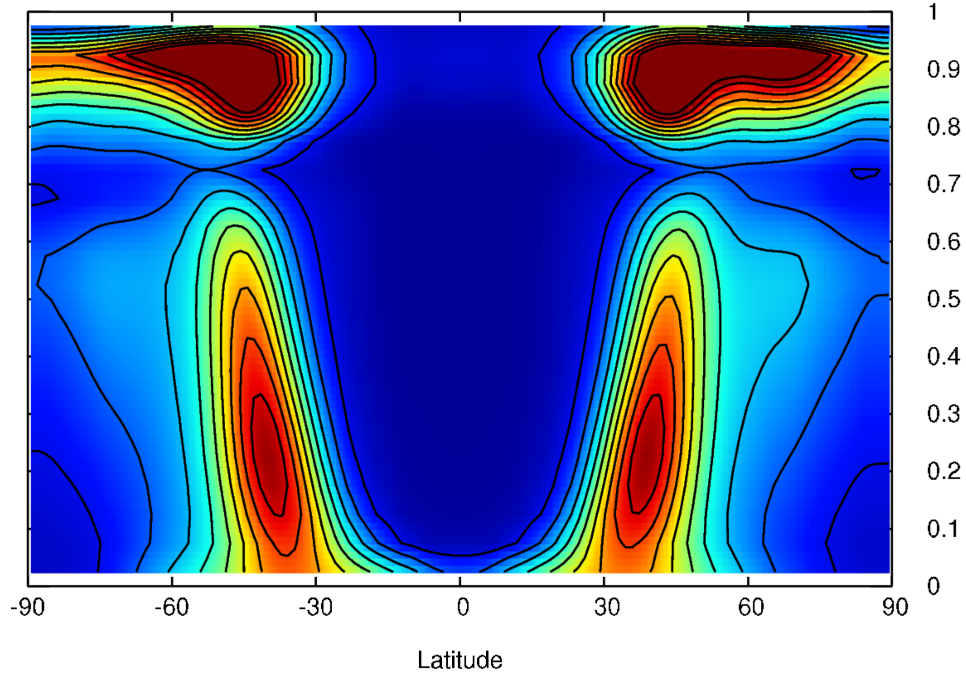


Maximum spatial resolution $\approx 1.5^\circ$

Zonal velocity

Dependence on spatial resolution

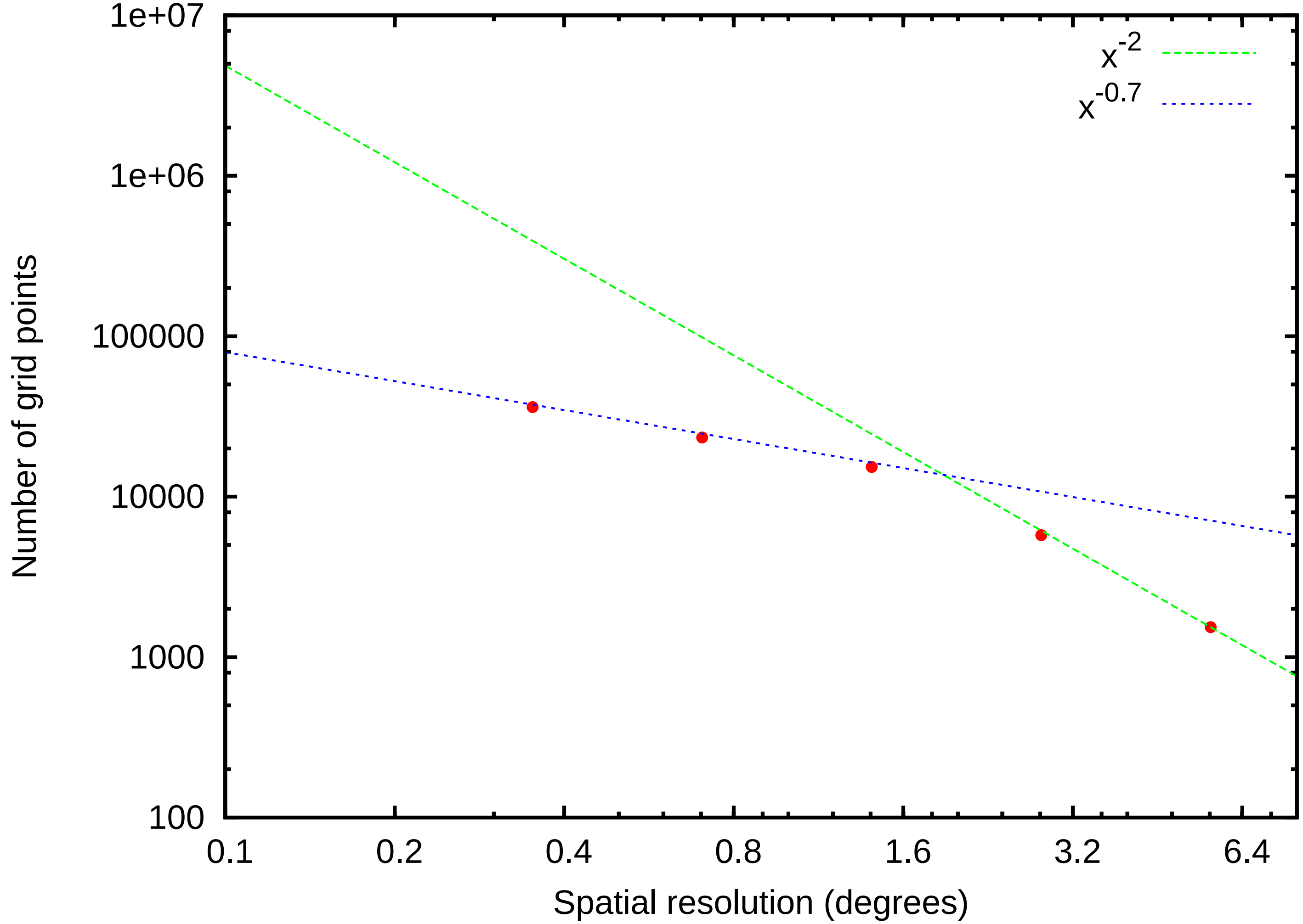




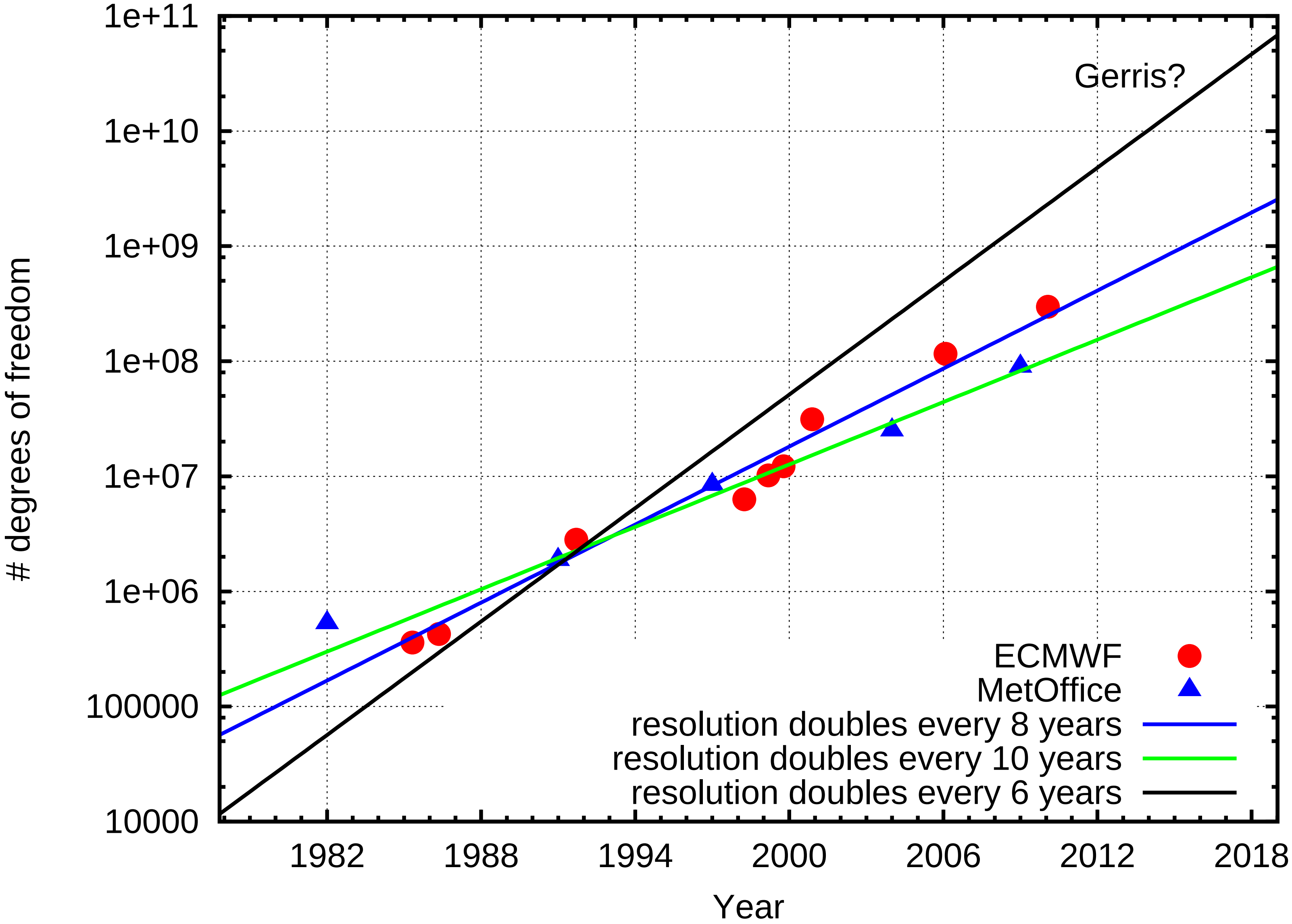
Maximum spatial resolution $\approx 3^\circ$ Maximum spatial resolution $\approx 1.5^\circ$

Variance of potential temperature

Scaling of computational cost



The future?



1e+11
1e+10
1e+09
1e+08
1e+07
1e+06
100000
10000
degrees of freedom
1982
1988
1994
2000
2006
2012
2018
Year
Gerris?
ECMWF
MetOffice
resolution doubles every 8 years
resolution doubles every 10 years
resolution doubles every 6 years

Conclusions

- "Conformal cubed sphere" + quadtrees + A-grid and approximate projection is a viable combination (at least for Held–Suarez)
- Computational cost scaling for the Held–Suarez atmosphere using adaptive refinement is encouraging
- These scaling properties can be explored using outputs from existing (constant-resolution) high-resolution models
- Needs testing on "real" parallel systems at higher resolutions
- Aquaplanet + tropical convection: scaling?
- How to push this prototype to a "full" climate model? Resources!