Gravity Flows, towards Tsunamis generation with the multi-layer solver

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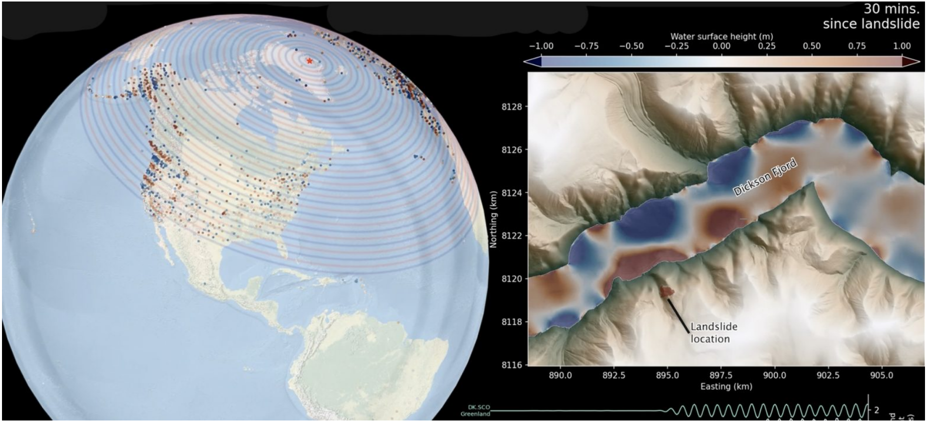
Gravity flows tsunamis

Massive landslide in Groenland - september 2023

25 millions m^3 of rocks and ice mixture collapse into the Dickson Fjord $\Rightarrow$ **200 m** wave



“Un énorme glissement de terrain déclenché par le changement climatique a fait vibrer la Terre pendant 9 jours” (*IPGP - Institut de Physique du Globe de Paris*)

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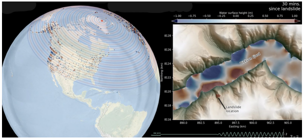

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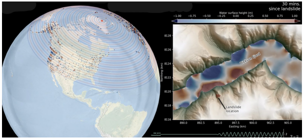

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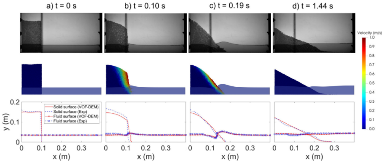
From field to laboratory: geophysical flow approximations

- Shallow water approximation ($L \gg h$)
- 2D continuous model and /or discreet, often local

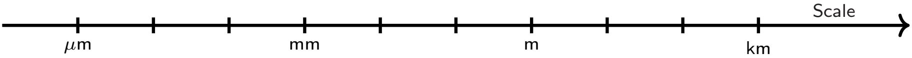


Granular collapse in water, expériment - top (*Cabrera et al, JGR, 2020*), numerical simulation - bottom (*Nguyen, PoF, 2022*)

What mechanism drives the formation of a wave when a **large amount of material** * enters the water

* Granular medium: glacier, rocks, volcano...

Gravity flow tsunamis – a multiscale and multiphase problem

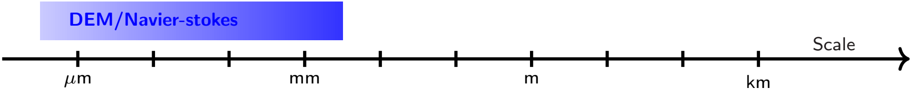


Landslide in the Dorset, UK (left), (*James Loveridge Photography*), wave breaking (right), (*google image*)

The problem description

1. Granular material with complex rheology
2. Wave formation and breaking
3. Imbibition, phase mixture, triple line front evolution...

Gravity flow tsunamis – a multiscale and multiphase problem



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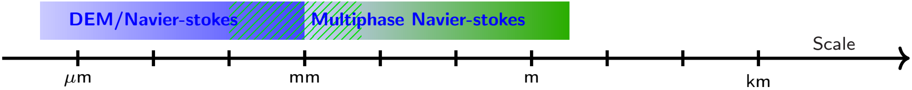

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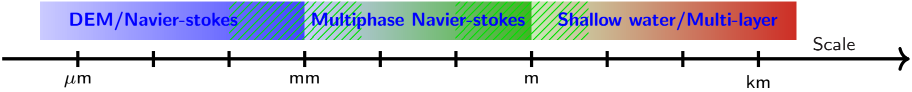

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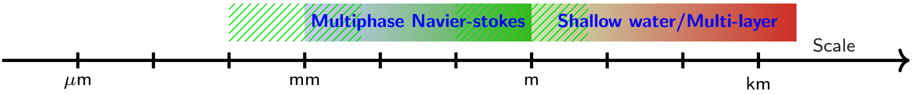


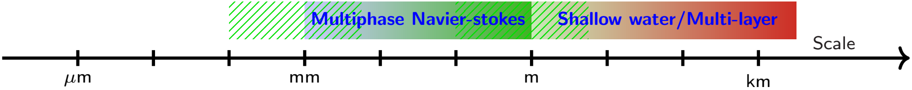
Wave breaking

Multiphase Navier-Stokes

- × Grid and refinement
- × Complex numerical schemes
- × High computational cost

Gravity flows

Tsunamis



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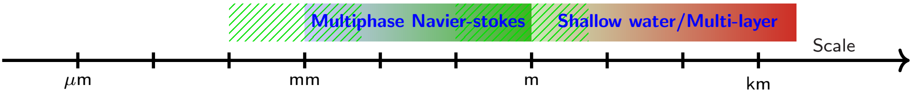

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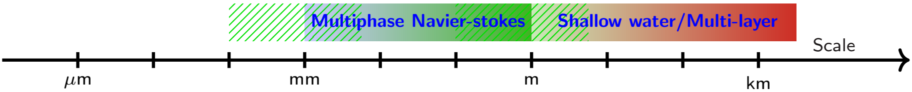
$\Rightarrow$

Multi-layer model

- ✓ Fixed grid (almost)
- ✓ Simple numerical schemes
- ✓ Lower computational cost

Gravity flows — Tsunamis



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⇒

Multi-layer model

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- ✓ Simple numerical schemes
- ✓ Lower computational cost

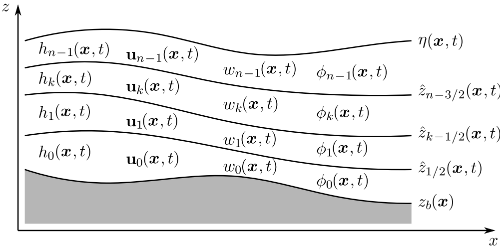
Proposition

Multi-layer solver for gravity flows and/or tsunamis formation

Multi-layer model

http://basilisk.fr/src/layered/hydro.h

The great wave off Kanagawa *(Wikipedia)*



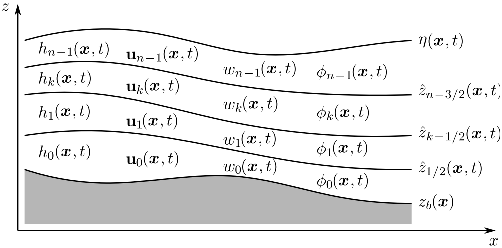
Definition of N layers *(credit: http://basilisk.fr/)*

$$\begin{aligned}
\partial_t h_k + \bar{\nabla}\cdot(h\bar{u})_k &= 0\\
\partial_t (h\bar{u})_k + \bar{\nabla}\cdot(h\bar{u}\bar{u})_k &= -g h_k \bar{\nabla}(\eta) - \bar{\nabla}(h\Phi)_k + [\Phi\bar{\nabla}]_k\\
\partial_t (hw)_k + \bar{\nabla}\cdot(hw\bar{u})_k &= -[\Phi]_k\\
\bar{\nabla}\cdot(h\bar{u})_k + [w - \bar{u}\cdot\bar{\nabla}\hat{z}]_k &= 0
\end{aligned}$$

With k the index of the layer, h_k its thickness, u_k, w_k, the horizontal and vertical velocity, g the gravity and Φ_k the non-hydrostatic pressure,

$$\eta \equiv z_b + \sum_k h_k \text{ and } \hat{z}_{k+1/2} \equiv z_b + \sum_{l=0}^{k} h_l$$

Landslide in Dorset, UK



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\end{aligned}
$$

With k the index of the layer, h_k its thickness, u_k, w_k, the horizontal and vertical velocity, g the gravity and Φ_k the non-hydrostatic pressure, ν the kinematic viscosity

$$\eta \equiv z_b + \sum_k h_k \text{ and } \hat{z}_{k+1/2} \equiv z_b + \sum_{l=0}^{k} h_l$$

Multi-layer model

http://basilisk.fr/src/layered/diffusion.h

Implicit viscous friction term:

$$\frac{(hu_l)^{n+1} - (hu_l)^{\star}}{\Delta t} = \nu \left(\frac{u_{l+1} - u_l}{h_{l+1/2}} - \frac{u_l - u_{l-1}}{h_{l-1/2}} \right)^{n+1}$$

Problem

Complex rheology for granular flow $\Rightarrow$ viscosity $\nu \neq$ cste

Neumann boundary conditions on the free surface

Navier slip on the bottom

$$\partial_z u|_t = \dot{u}_t$$

Default free slip $\dot{u}_t = 0$

$$u|_b = u_b + \lambda_b \partial_z u|_b$$

and no slip $u_b, \ \lambda_b = 0$

These equations can be expressed as the linear system

$$\mathbf{M}\mathbf{s}^{n+1} = \mathrm{rhs}$$

where **M** is a tridiagonal matrix.

Multi-layer model

http://basilisk.fr/sandbox/fanshuo/diffusionMT.h

Implicit viscous friction term:

$$\frac{(hs_l)^{n+1} - (hs_l)^\star}{\Delta t} = \left({\nu_{eq}}_{l+1/2} \frac{s_{l+1} - s_l}{h_{l+1/2}} - {\nu_{eq}}_{l-1/2} \frac{s_l - s_{l-1}}{h_{l-1/2}} \right)^{n+1}$$

Development

Complex rheology for granular flow $\Rightarrow$ variable viscosity ν_{eq}

Neumann boundary conditions on the free surface

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Gravity flow rheology

http://basilisk.fr/sandbox/fanshuo/mui.h

$\mu(I)$ rheology *(GRD midi, Lagrée et al, JFM, 2011)* $\Rightarrow$ tangential and normal stress are linked through:
$$\tau = \mu(I)p$$

with the friction law

$$\mu = \mu_s + \frac{\Delta\mu}{1 + I_0/I} \quad \text{where} \quad I = d_g \frac{\partial u/\partial z}{\sqrt{p/\rho}}$$

I is the inertial number and d_g the grain diameter;
Coefficients μ_0, $\Delta\mu$ and I_0 depend on the nature of the granular media

Equivalent rheology in the multi-layer model

The solver is tuned to consider non newtonian viscosity as a function of the shear

$$\nu_{eq\,k} = \nu + \frac{\tau_z/\rho}{\left|\frac{\partial u}{\partial z}\right|_k}$$

Typically, $\nu_{eq\,k}$ and $\left|\frac{\partial u}{\partial z}\right|_k$ also depends on the index layer k;

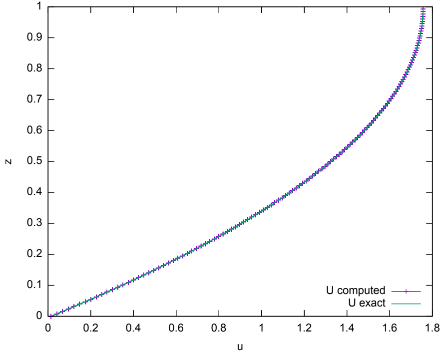
A shear regularisation is required to avoid infinite viscosity at rest

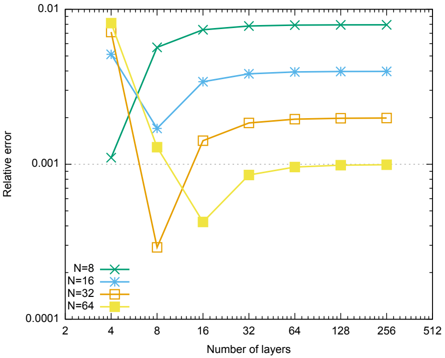
Periodic free surface with Newtonian flow

Poiseuille

A theoretical solution for the velocity:

$$u = \frac{G \sin\theta}{2\nu}(2 - z)z$$





Comparison between analytical solution and numerical solution (l); order of convergence (r) - $N_x = 16$, $nl = 128$, **Poiseuille**

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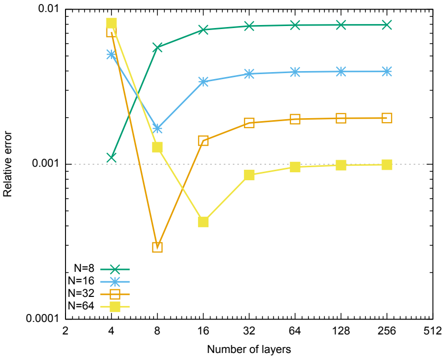
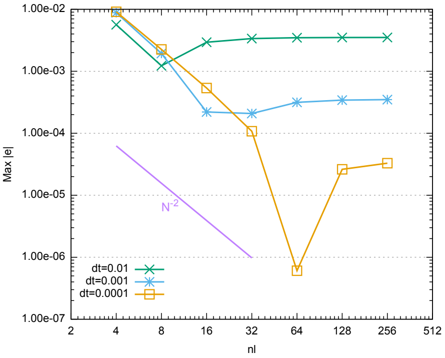




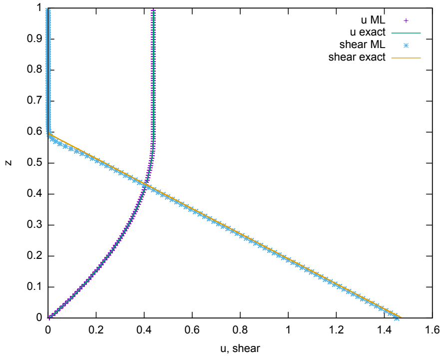
Order of convergence, influence of *dt* - **Poiseuille**

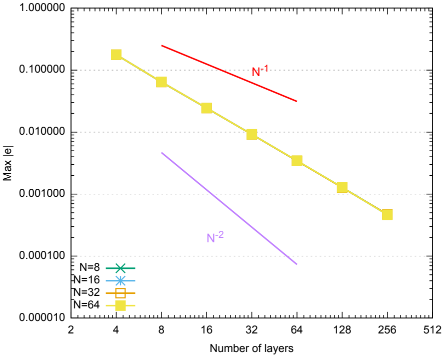
Periodic free surface with non Newtonian flow

Bingham

A theoretical solution for the velocity:

$$u = U\left(1 - \left(\frac{z - Z}{Z}\right)^2\right) \quad \text{with} \quad U = \frac{(\tau_z - h(-G\sin\theta))^2}{2\mu(G\sin\theta)} \quad \text{and} \quad Z = h - \tau_z/(-G\sin\theta)$$





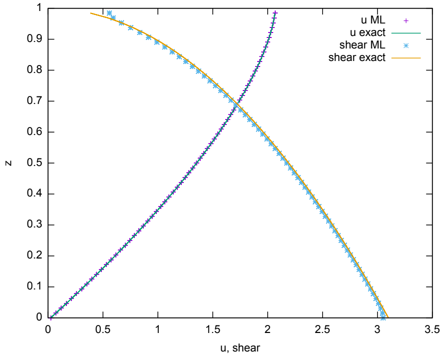
Comparison between analytical solution and numerical solution (l); order of convergence (r) - $N_x = 16$, $nl = 128$, **Bingham**

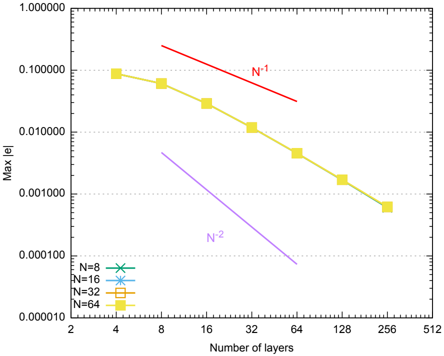
Periodic free surface with non Newtonian flow

Bagnold

A theoretical solution for the velocity:

$$u = \frac{2}{3}\frac{I_\theta}{d_g}\sqrt{gh^3 cos(\theta)}(1 - \left(1 - \frac{z}{h}\right)^{3/2} \text{ with } I_\theta = \frac{I_0(\tan\theta - \mu_0)}{\mu_0 + \Delta\mu - \tan\theta};$$

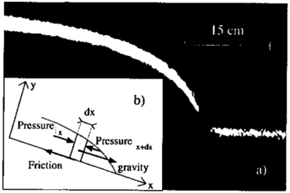




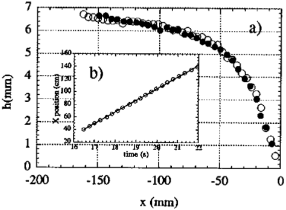
Comparison between analytical solution and numerical solution (l); order of convergence (r) - $N_x = 16$, $nl = 128$, **Bagnold**

Shape of a granular front down a rough inclined plane

An avalanche flowing along a mild slope



(a) Front illuminated by the laser sheet $\theta = 21^{\circ}$, $h_{\infty} = 9.5mm$, (b) Forces on a elementary material slice



(a) Front profiles 50cm from the outlet ○ and 150cm down the slope ●, (b) Position of the front with time showing the constant propagation velocity.

A theoretical solution for the front shape (*G. Saingier et al 2016*)

$$X = H - \frac{\frac{2}{3}\log(1-\sqrt{H}) - \frac{1}{3}\log(H+\sqrt{H}+1) + \frac{2}{\sqrt{3}}\arctan\left(\frac{2\sqrt{H}+1}{\sqrt{3}}\right) - (\alpha-1)Fr^2\ln\left(\frac{H^d}{(1-H^{3/2})^{2/3}}\right)}{d-1}$$

With $Fr = \frac{u_0}{\sqrt{gh_0\cos\theta}}$, u_0 the mean velocity; $H = h/h_0$; $X = x(\tan\theta - \mu_0)/h_0$;

$$d = (\tan\theta - \mu_0)/\Delta\mu.$$

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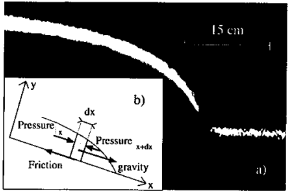

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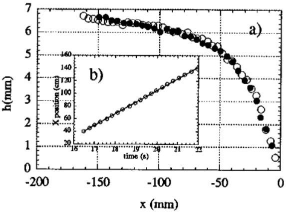

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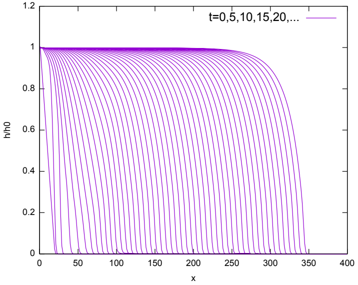

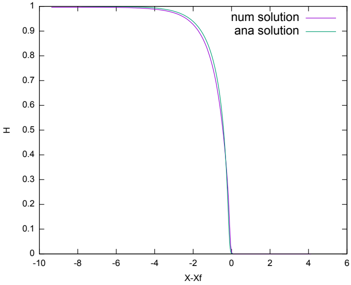

Shape of the front as a function of time (l), comparison between analytical solution and numerical solution (r), $N_X = 256$, $nl = 128$

Quick overview on multi-layer model for Gravity Flows

Ongoing works

- Variable viscosities in multi-layer solver
- Different existing rhelogies
- Validation with several test cases and examples

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- Variable viscosities in multi-layer solver
- Different existing rhelogies
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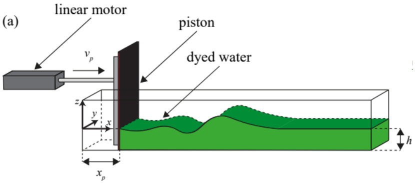
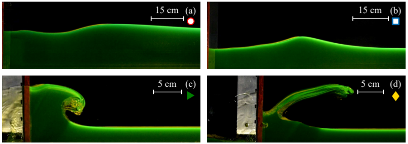
To do

- Time convergence of diffusion
- Extend Neumann to Navier BC on the free surface
- Two-phase problem (Poiseuille/Bingham) with variable viscosities in the Boussinesq approximation

Waves impulsed by granular collapse

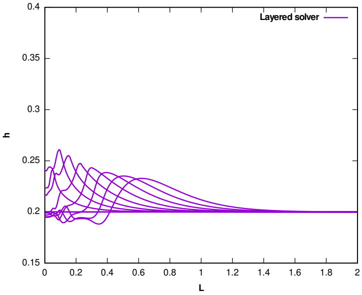


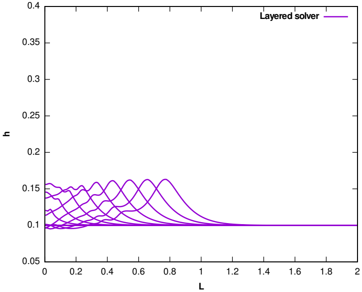
Experimental set-up, (*Sarlin et al, JFM, 2025*)



(a) A dispersive wave, (b) A solitary-like wave (*Sarlin et al, JFM, 2025*)

$$(a)\ L = 7cm,\ U = 0,42m.s,\ h = 20cm,\ Fr_p = 0,3$$
$$(b)\ L = 15cm,\ U = 1,09m.s,\ h = 10cm,\ Fr_p = 0,47$$

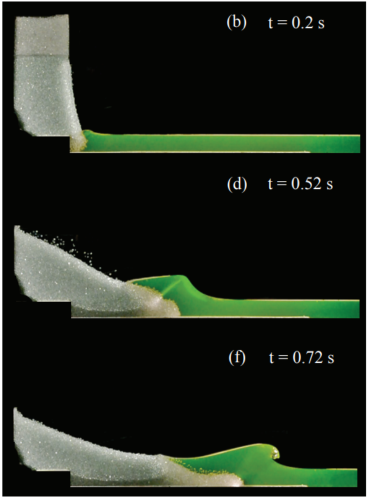




Gravity flows tsunamis

Physical challenge

Numerical features



Wave generation by granular collapse, $h_0 = 5cm$, (*Robbe-saule et al, JFM, 2021*)

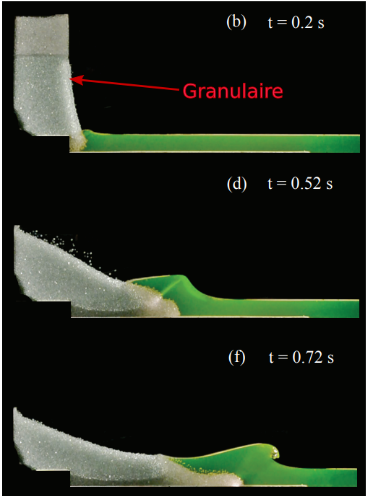
Gravity flows tsunamis

Physical challenge

1. Granular rheology

Numerical features

- Multi-layer and variable viscosity



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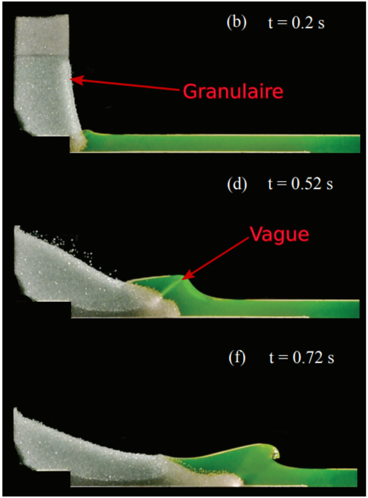
Gravity flows tsunamis

Physical challenge

1. Granular rheology
2. Wave formation

Numerical features

- Multi-layer and variable viscosity
- Multi-layer and variable density



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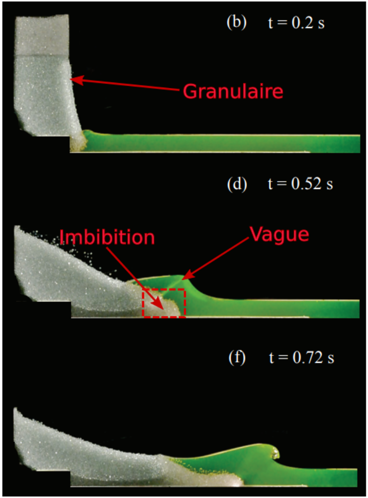
Gravity flows tsunamis

Physical challenge

1. Granular rheology
2. Wave formation
3. Imbibition/porous medium

Numerical features

- Multi-layer and variable viscosity
- Multi-layer and variable density
- Triple line modeling



Génération de vague par effondrement d'une colonne granulaire, $h_0 = 5cm$, (*Robbe-saule et al, JFM, 2021*)

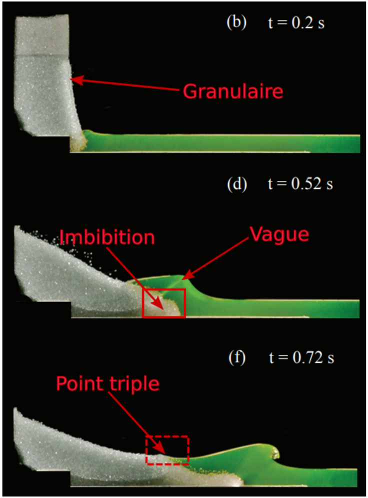
Gravity flows tsunamis

Physical challenge

1. Granular rheology
2. Wave formation
3. Imbibition/porous medium
4. Triple line

Numerical features

- Multi-layer and variable viscosity
- Multi-layer and variable density
- Triple line modeling



Génération de vague par effondrement d'une colonne granulaire, $h_0 = 5cm$, (*Robbe-saule et al, JFM, 2021*)

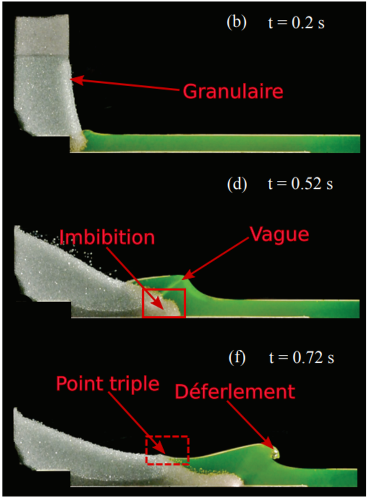
Gravity flows tsunamis

Physical challenge

1. Granular rheology
2. Wave formation
3. Imbibition/porous medium
4. Triple line
5. Breaking wave

Numerical features

- Multi-layer and variable viscosity
- Multi-layer and variable density
- Triple line modeling
- Multi-layer and Navier-Stokes/Lagrangian particles



Génération de vague par effondrement d'une colonne granulaire, $h_0 = 5cm$, (*Robbe-saule et al, JFM, 2021*)

Thank you for your attention

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