Contact

Can polymeric flows be the `Drosophila' of unsteady continuum mechanics?

Vatsal Sanjay

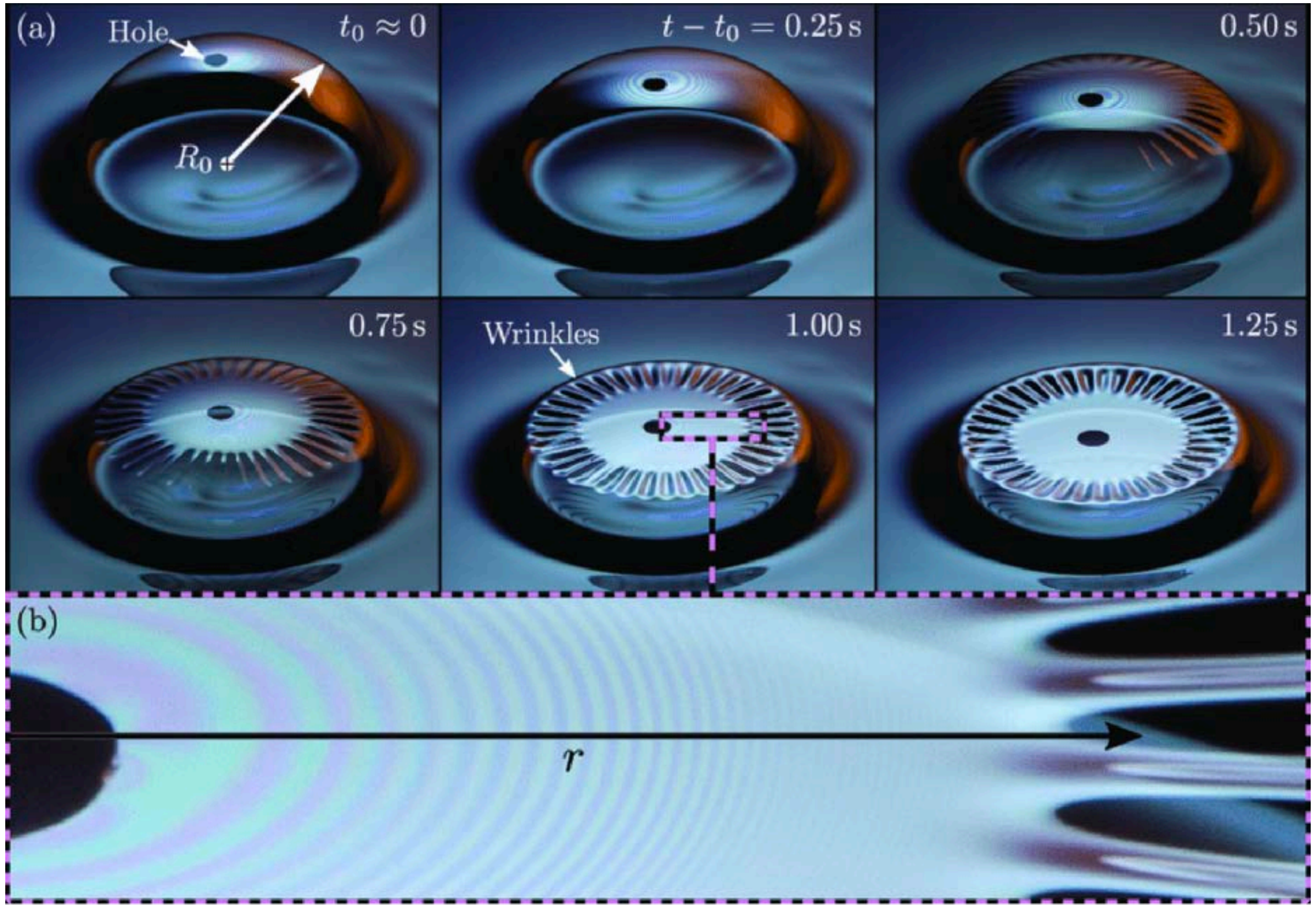
Durham University

CoMPhy Lab

Physics of Fluids

Different faces on continuum mechanics

Wrinkling of viscous surface bubble



A. Oratis, J. Bush, H. Stone, & J. Bird, Science, 369:6504, 685-688 (2020)

Wrinkling of elastic surfaces

D. Vella, A. Ajdari, A. Vaziri & A. Boudaoud, Phys. Rev. Lett., 107:17, 174301 (2011)

Why do we care about polymeric flows?

B. Scharfman, A. Techet, J. Bush & L. Bourouiba, Exp. Fluids, 57:2, 1-9 (2016)

Journey so far

Physics of Fluids

- **Detlef Lohse**
- Ayush Dixit*
- Aman Bhargava*
- Jnandeep Talukdar*
- **Saumili Jana**

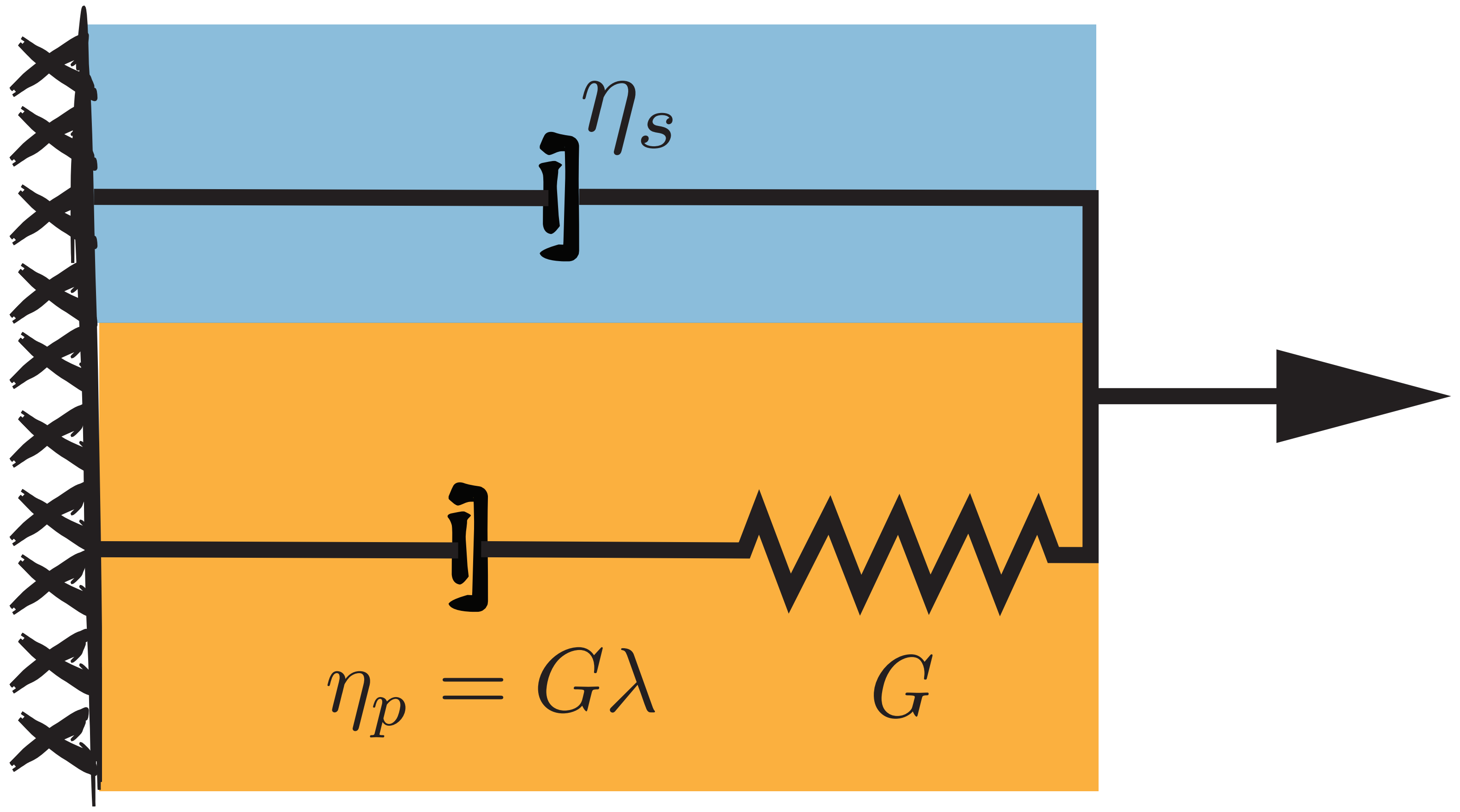
- **Jacco Snoeijer**
- **Alex Oratis**
- **Vincent Bertin**
- Alvaro Marin
- Tommie Verouden
- ...

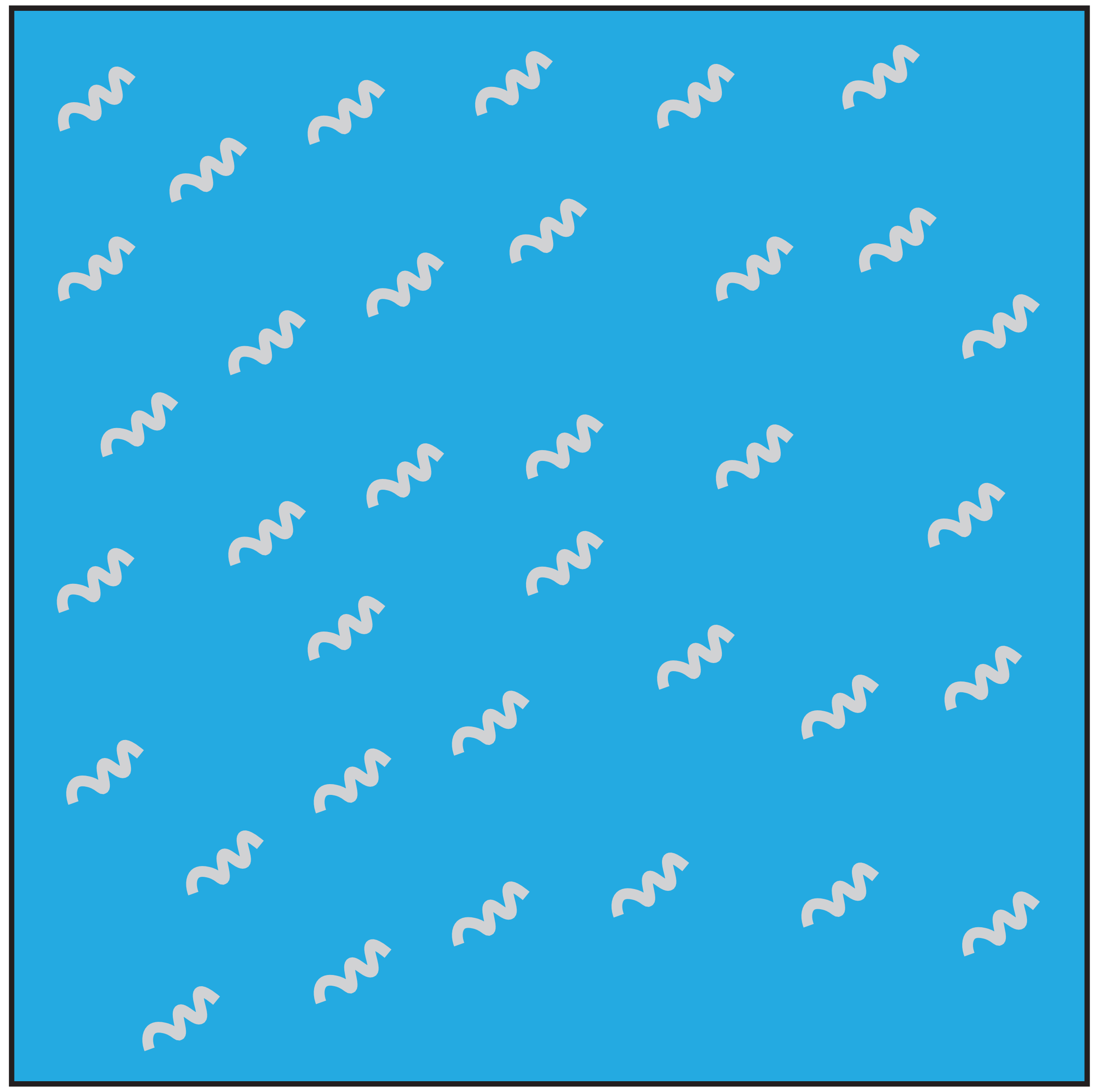
- Udo Sen (Wageningen Univ. & Research)
- Pierre Chantelot (Institut Langevin, ESPCI)
- Gareth McKinley (MIT, USA)
- Jie Feng (UIUC, USA)
- **John Kolinski (EPFL, Lausanne)**
- Mazi Jalaal (UvA, Amsterdam)
- Ari. Balasubramanian (KTH Sweden)
- Outi Tammisola (KTH Sweden)
- Konstantinos Zinelis (MIT, USA)
- Ricardo Constante Amores (UIUC, USA)
- ...

How do we model polymeric flows?

How to model polymeric flows?

Cauchy momentum equation: $$\frac{\partial(\rho \boldsymbol{u})}{\partial t} + \boldsymbol{\nabla}\cdot(\rho \boldsymbol{u}\boldsymbol{u}) = \boldsymbol{\nabla}\cdot\boldsymbol{\sigma}$$





$$\boldsymbol{\sigma} = -p\boldsymbol{\mathcal{I}} + 2\eta_s\boldsymbol{\mathcal{D}} + \boldsymbol{\sigma}_p(\boldsymbol{\mathcal{A}})$$

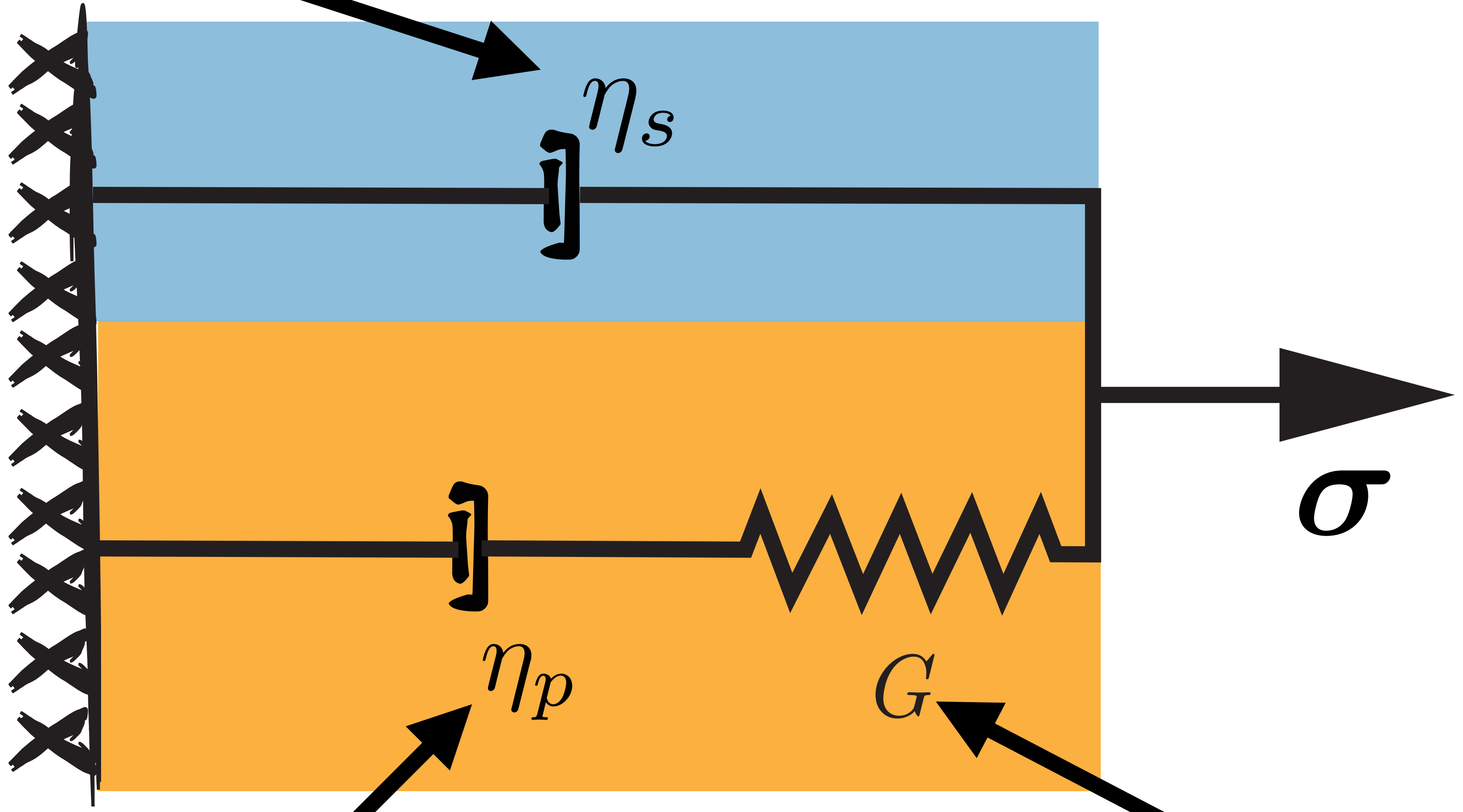
$$\boldsymbol{\mathcal{D}} = \frac{\boldsymbol{\nabla u} + (\boldsymbol{\nabla u})^T}{2}$$

Order parameter

for "polymer stretch"

Oldroyd-B family of models

Solvent viscosity



'Polymeric' viscosity

Elastic modulus

(Relaxation time)

$$\lambda = \frac{\eta_p}{G}$$

(Retardation time)

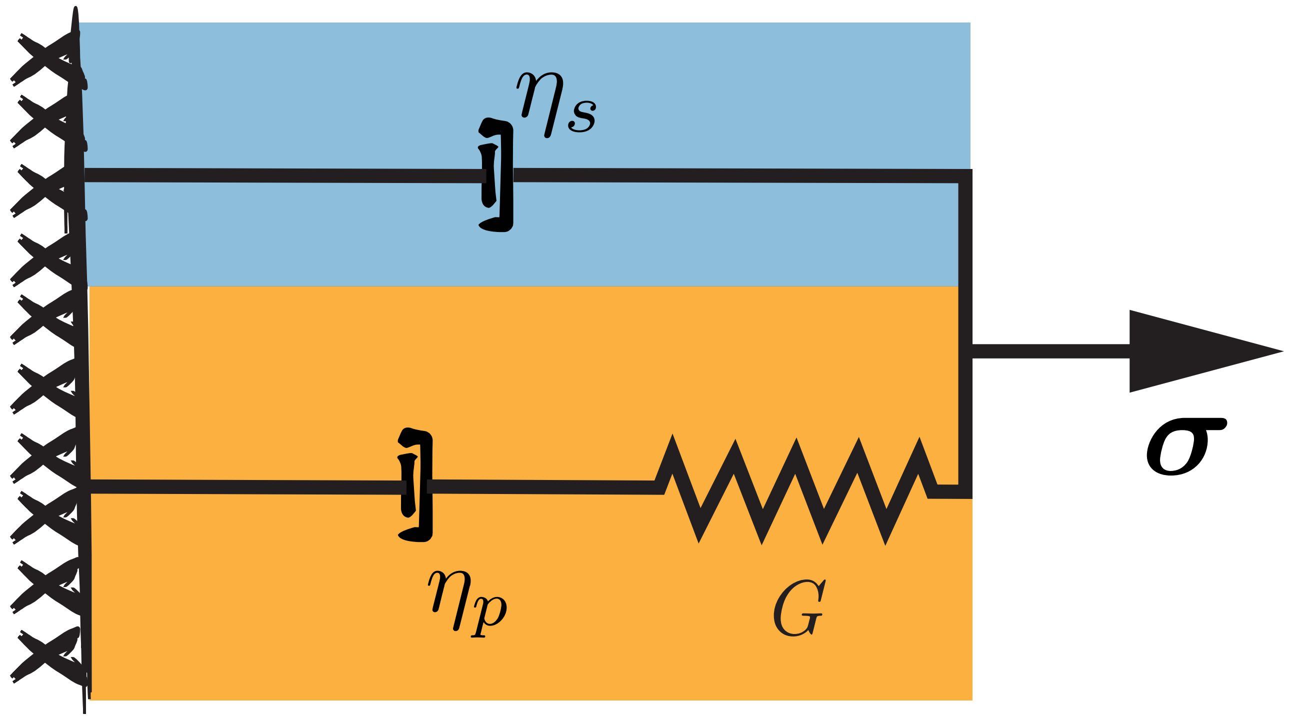
$$\lambda' = \frac{\eta_s}{G}$$

$$\boldsymbol{\sigma} = 2\eta_s \boldsymbol{\mathcal{D}} + G\left(\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{I}}\right)$$

$$\overset{\nabla}{\boldsymbol{\mathcal{A}}} = -\frac{1}{\lambda}\left(\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{I}}\right)$$

$$\frac{\partial \boldsymbol{\mathcal{A}}}{\partial t} + \left(\boldsymbol{u} \cdot \boldsymbol{\nabla}\right)\boldsymbol{\mathcal{A}} - 2\mathrm{Sym}\left(\boldsymbol{\mathcal{A}} \cdot \left(\boldsymbol{\nabla} \boldsymbol{u}\right)\right)$$

Oldroyd-B family



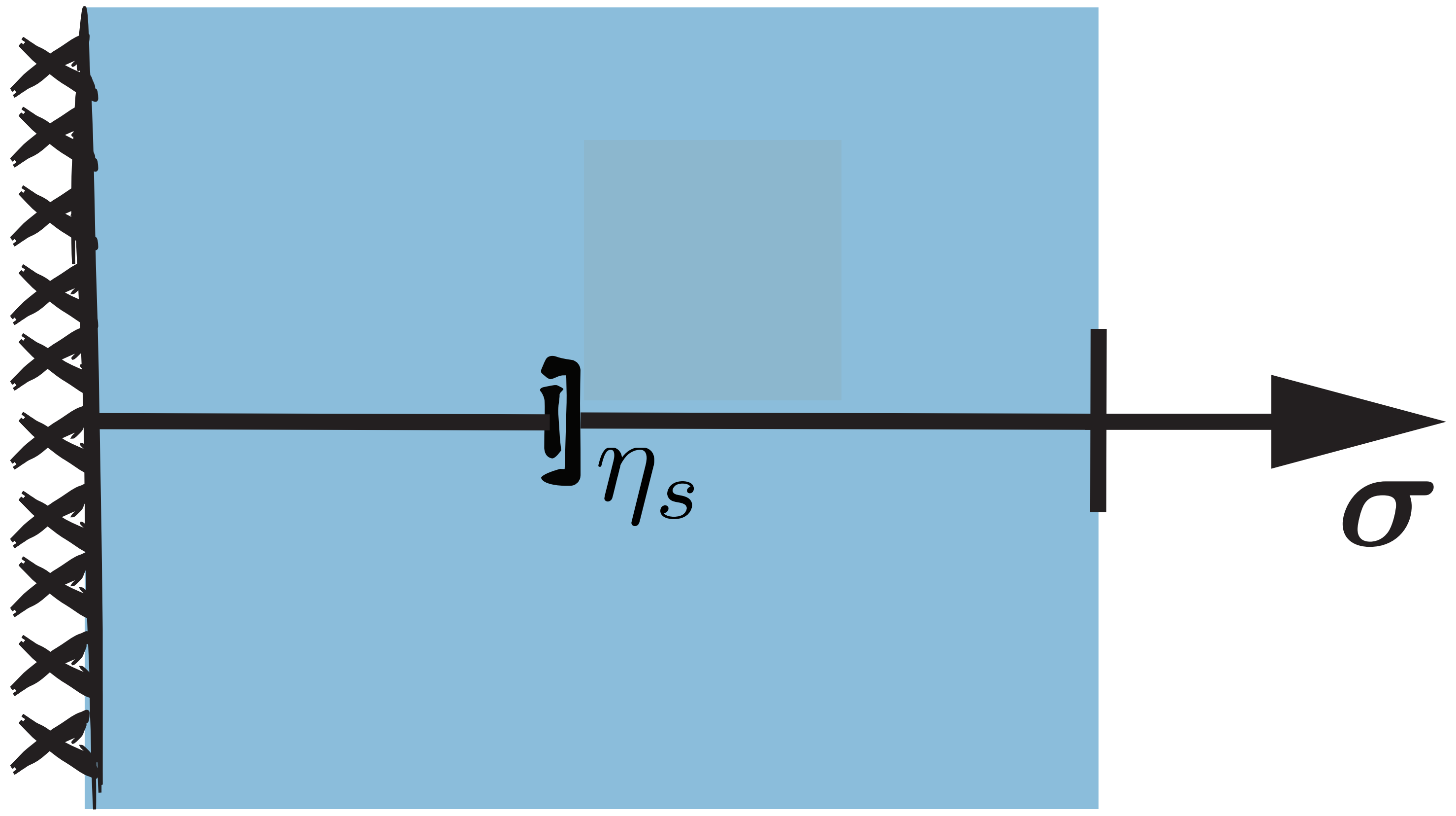
(Relaxation time)

$$\lambda = \frac{\eta_p}{G}$$

(Retardation time)

$$\lambda' = \frac{\eta_s}{G}$$

$$\lambda = 0$$

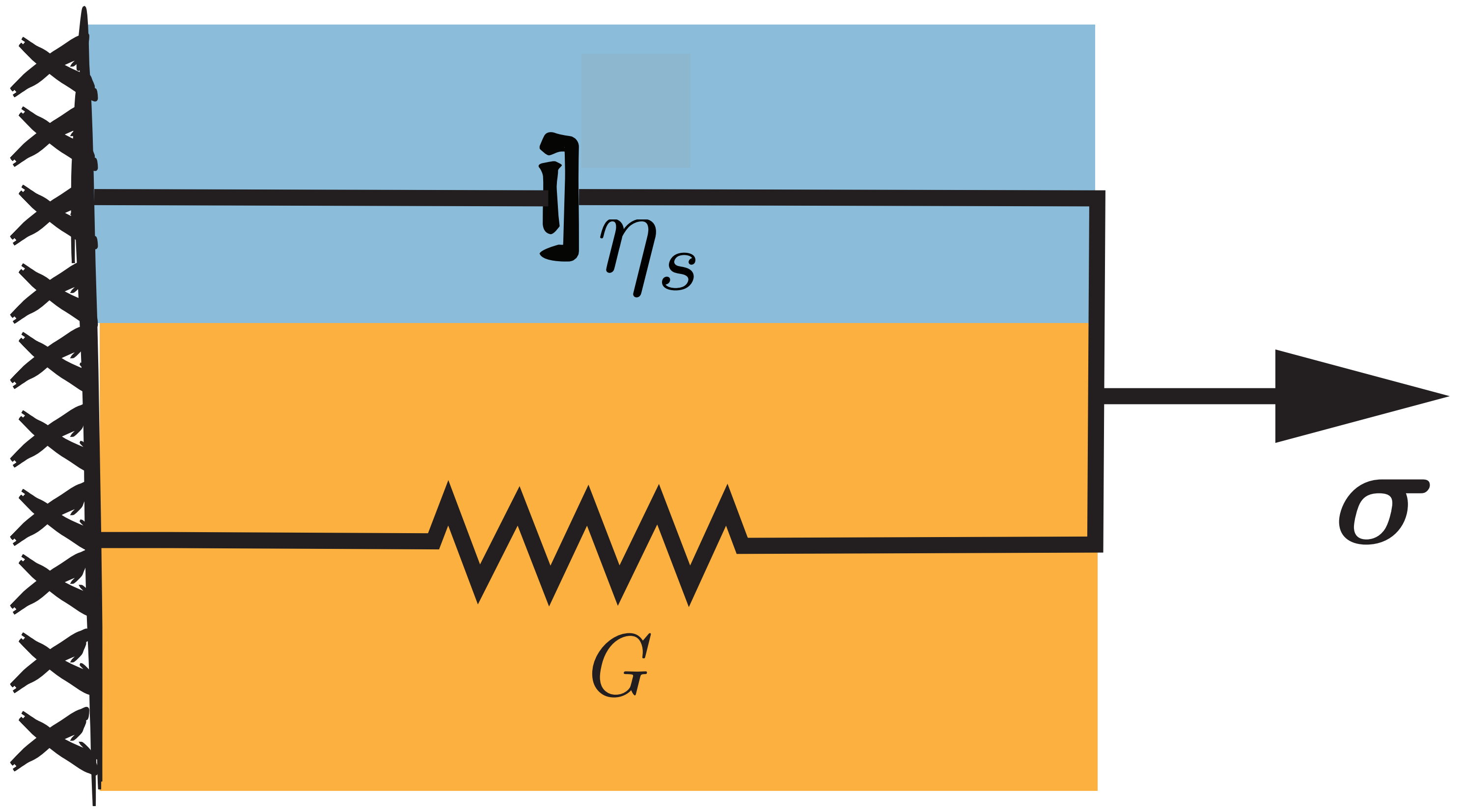


$$\boldsymbol{\sigma} = 2\eta_s \boldsymbol{\mathcal{D}}$$

$$\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{I}}$$

Viscous liquid

$$\lambda \to \infty$$



$$\boldsymbol{\sigma} = 2\eta_s \boldsymbol{\mathcal{D}} + G\left(\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{I}}\right)$$

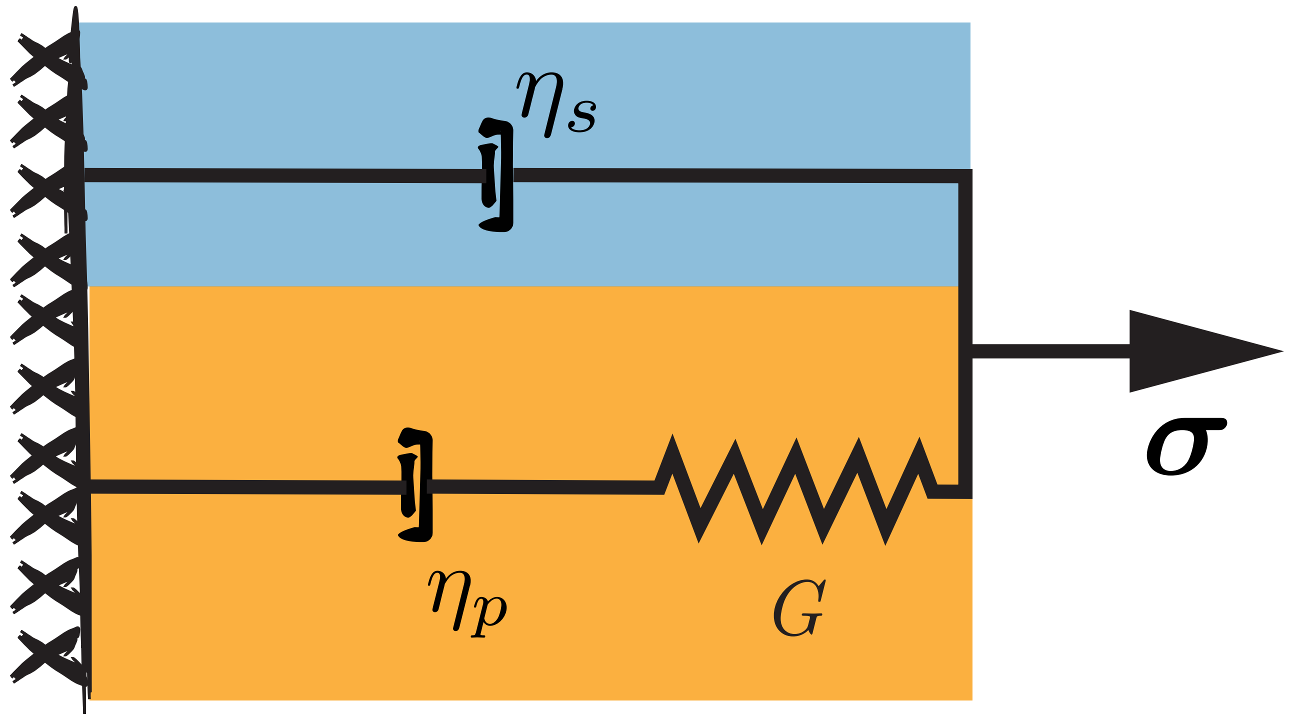
$$\overset{\nabla}{\boldsymbol{\mathcal{A}}} = 0$$

Kelvin–Voigt solid

$$\lambda' = 0$$

Elastic solid

$$\boldsymbol{\sigma} = G\left(\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{I}}\right)$$



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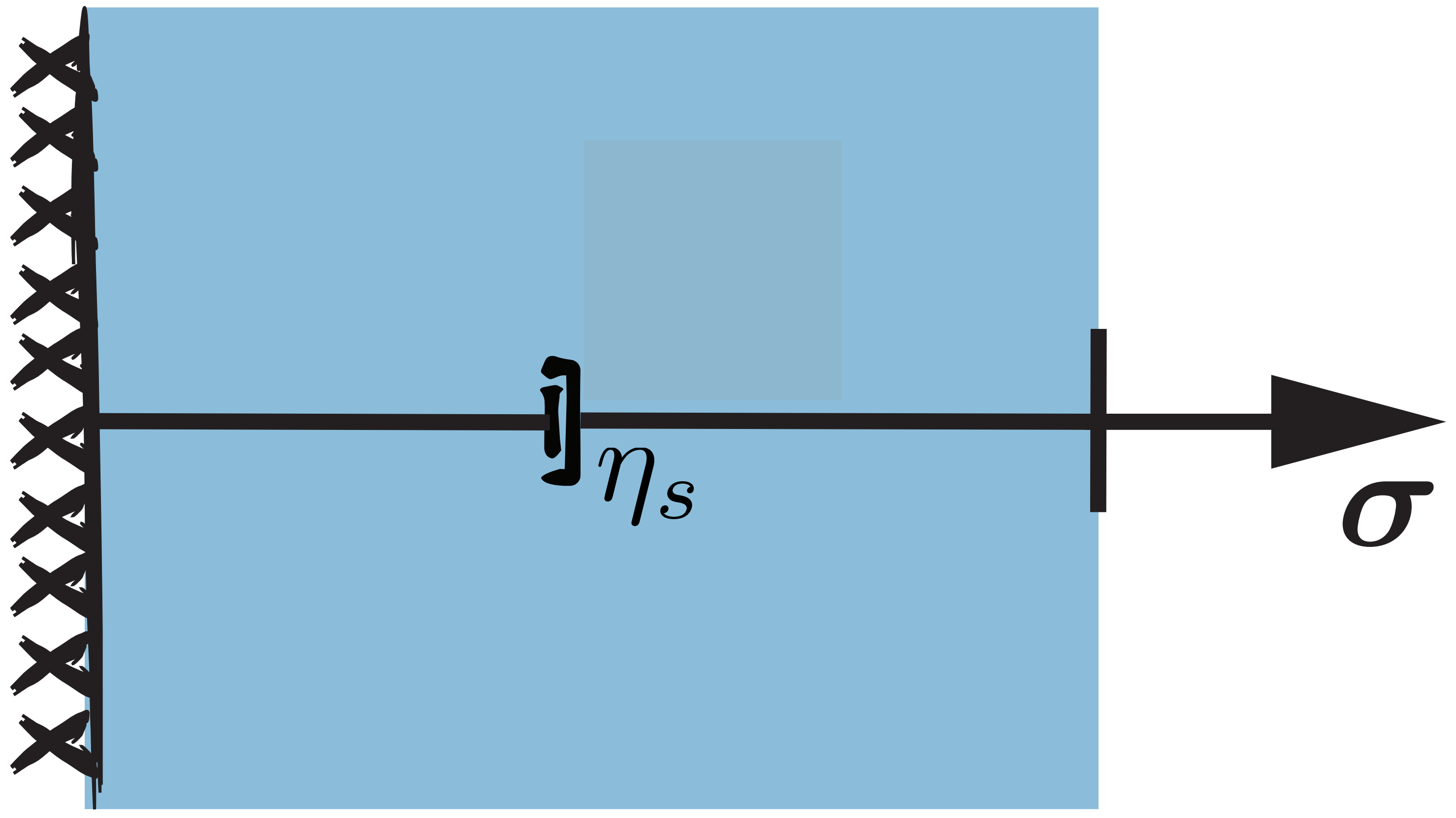

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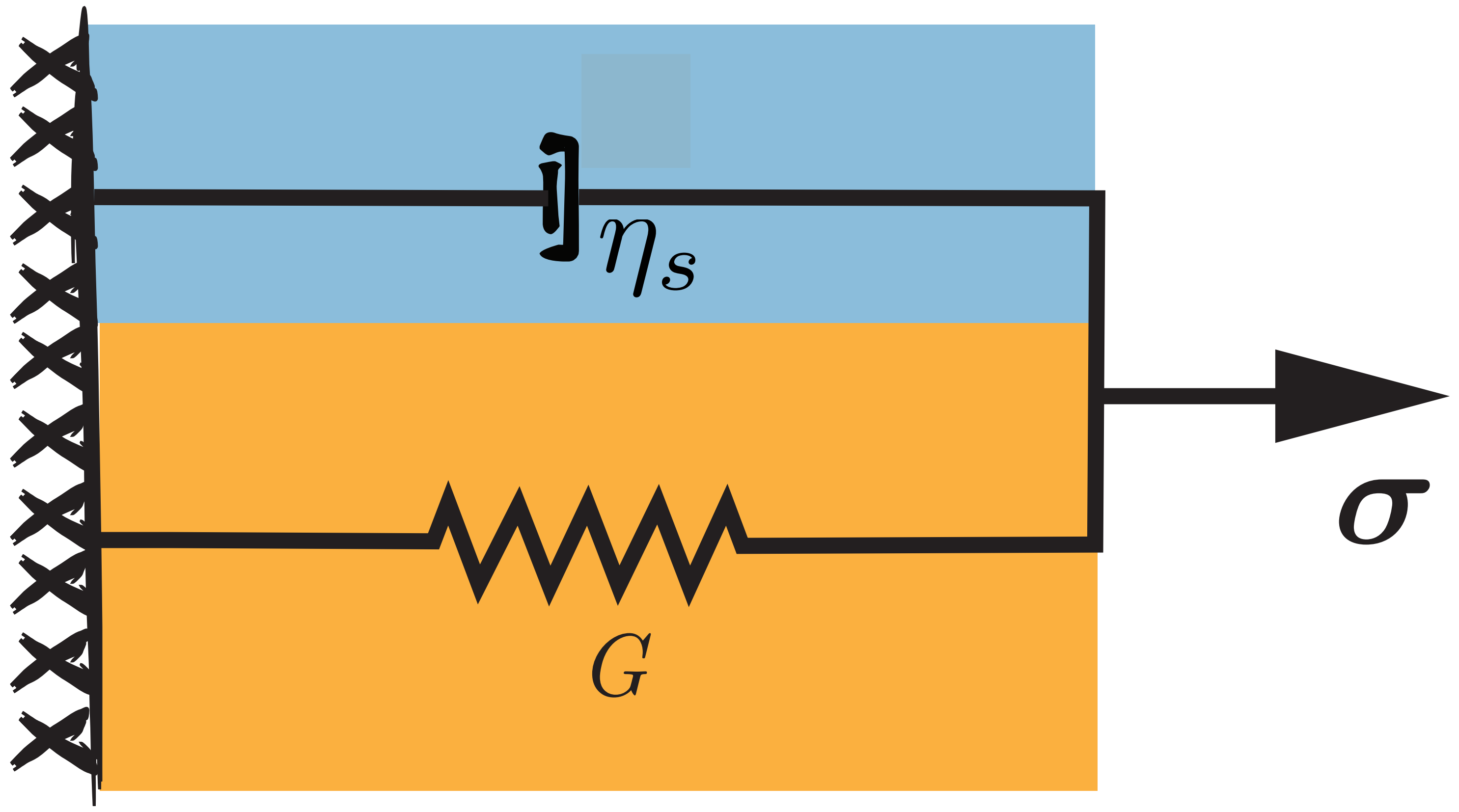
Viscous liquid

$$\lambda \to \infty$$



$$\boldsymbol{\sigma} = 2\eta_s \boldsymbol{\mathcal{D}} + G\left(\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{I}}\right)$$

$$\overset{\nabla}{\boldsymbol{\mathcal{A}}} = 0$$

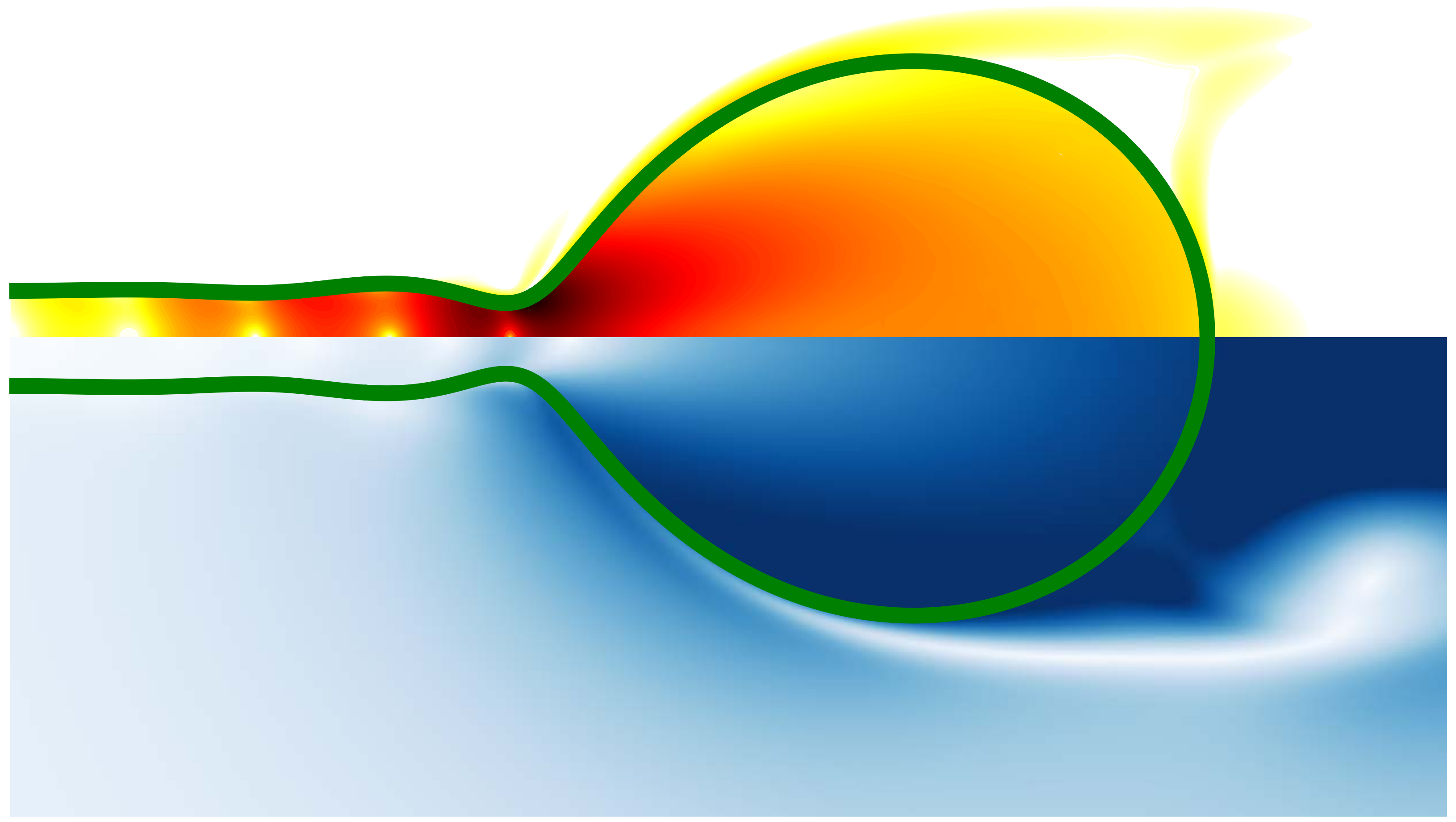
Kelvin–Voigt solid

$$\lambda' = 0$$

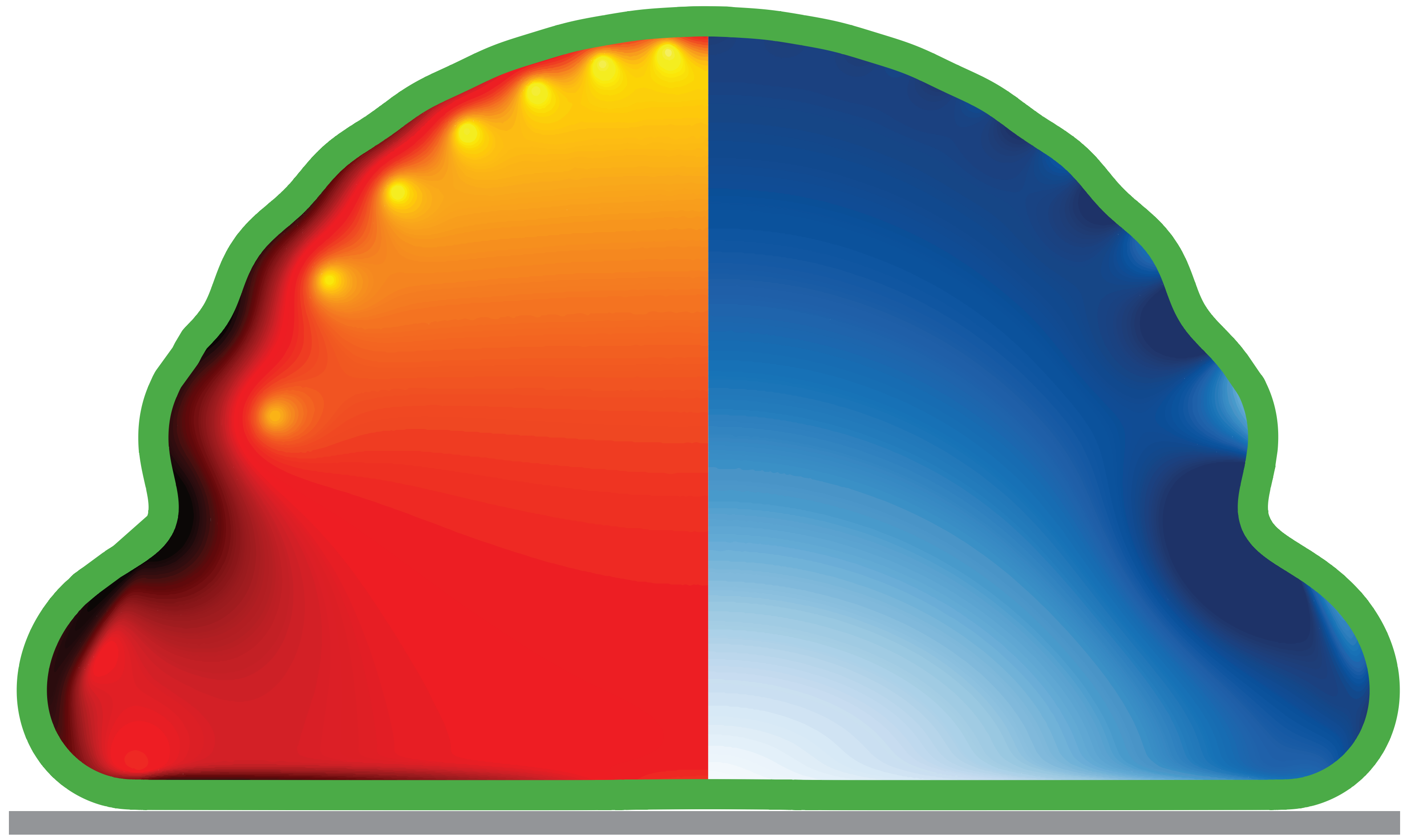
Elastic solid

$$\boldsymbol{\sigma} = G\left(\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{I}}\right)$$

On the menu today



1. Sheets



2. Drops

CoMPhy Lab

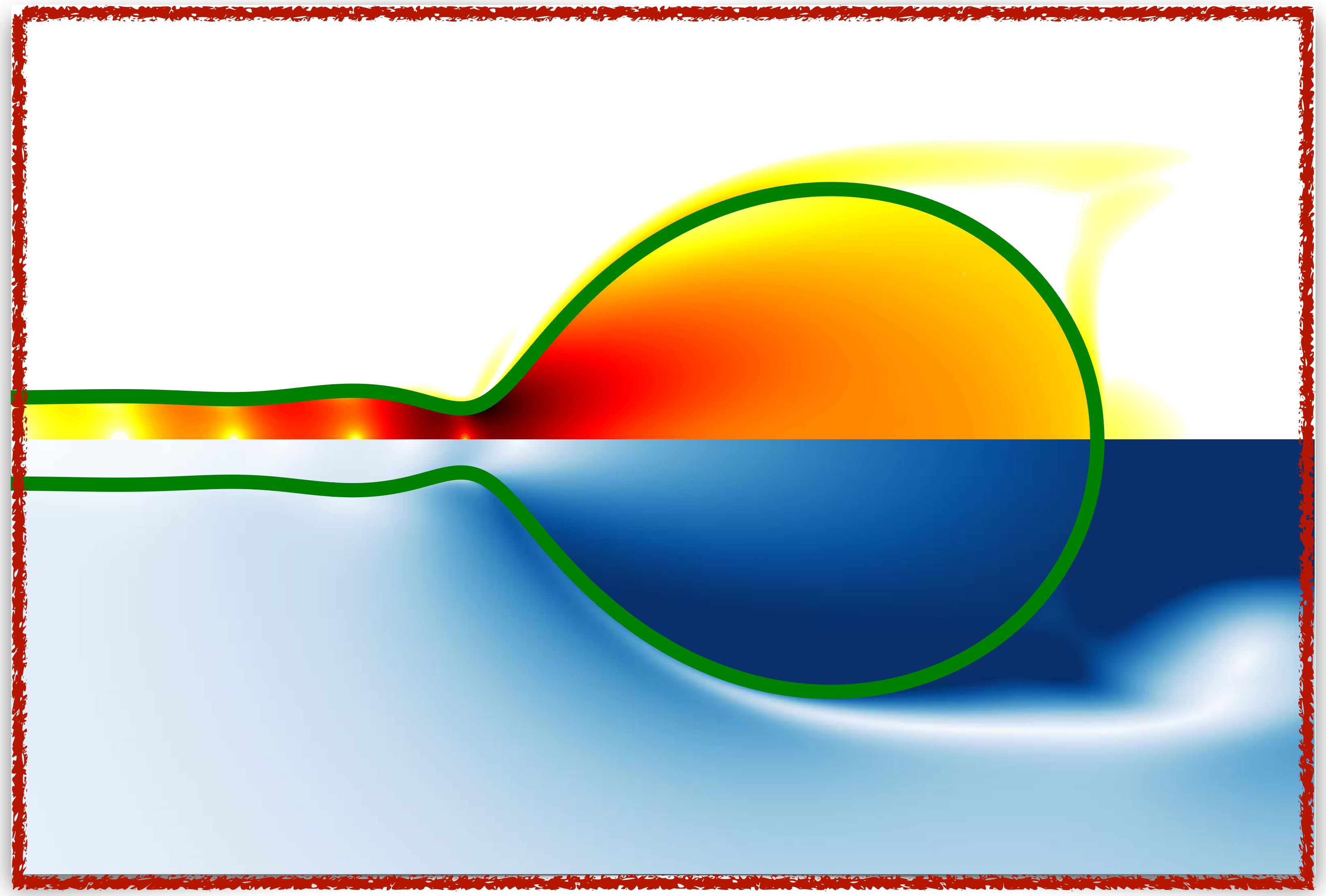
Computational Multiphase Physics Lab

Basilisk C by

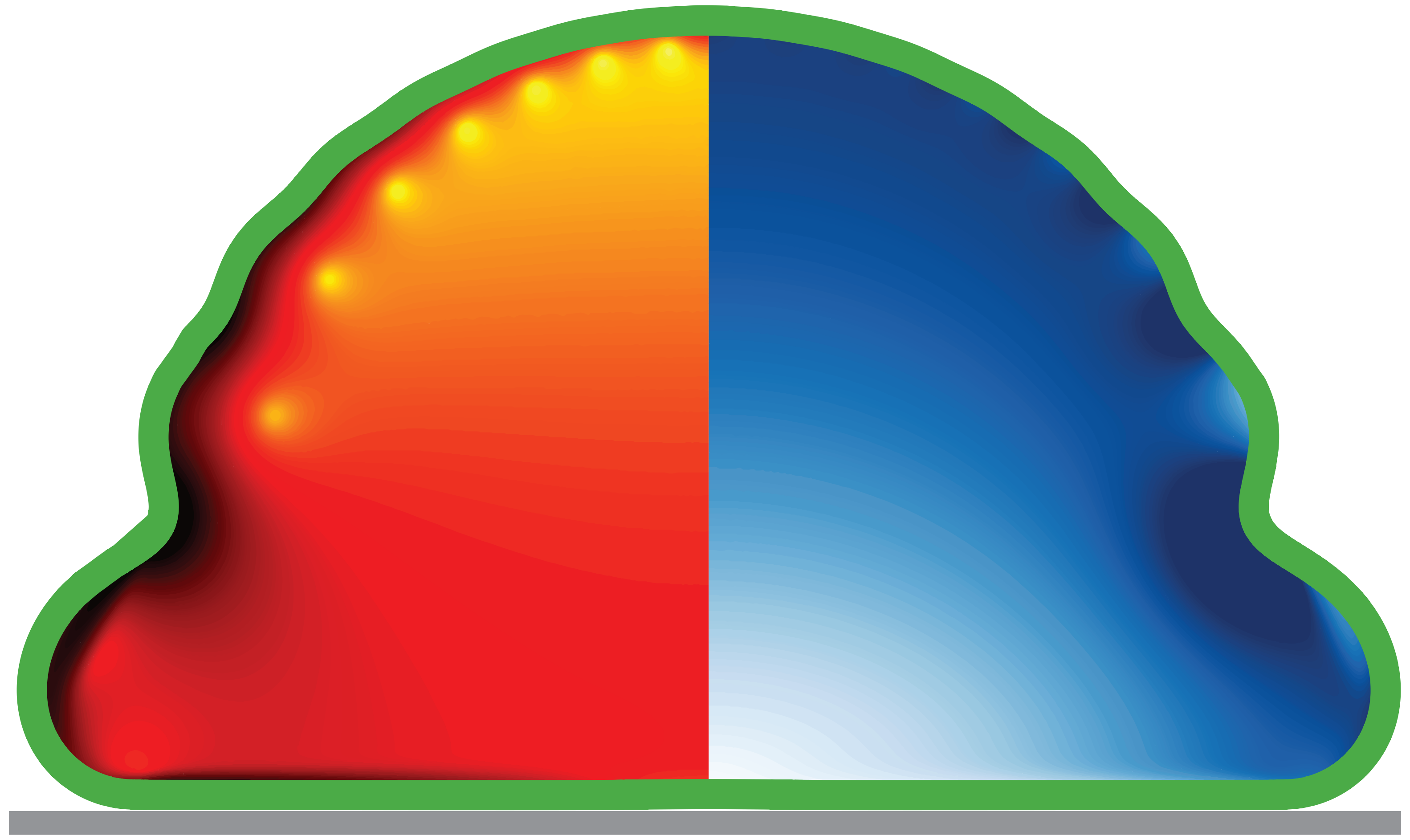
S. Popinet & team

#ilovefs

On the menu today

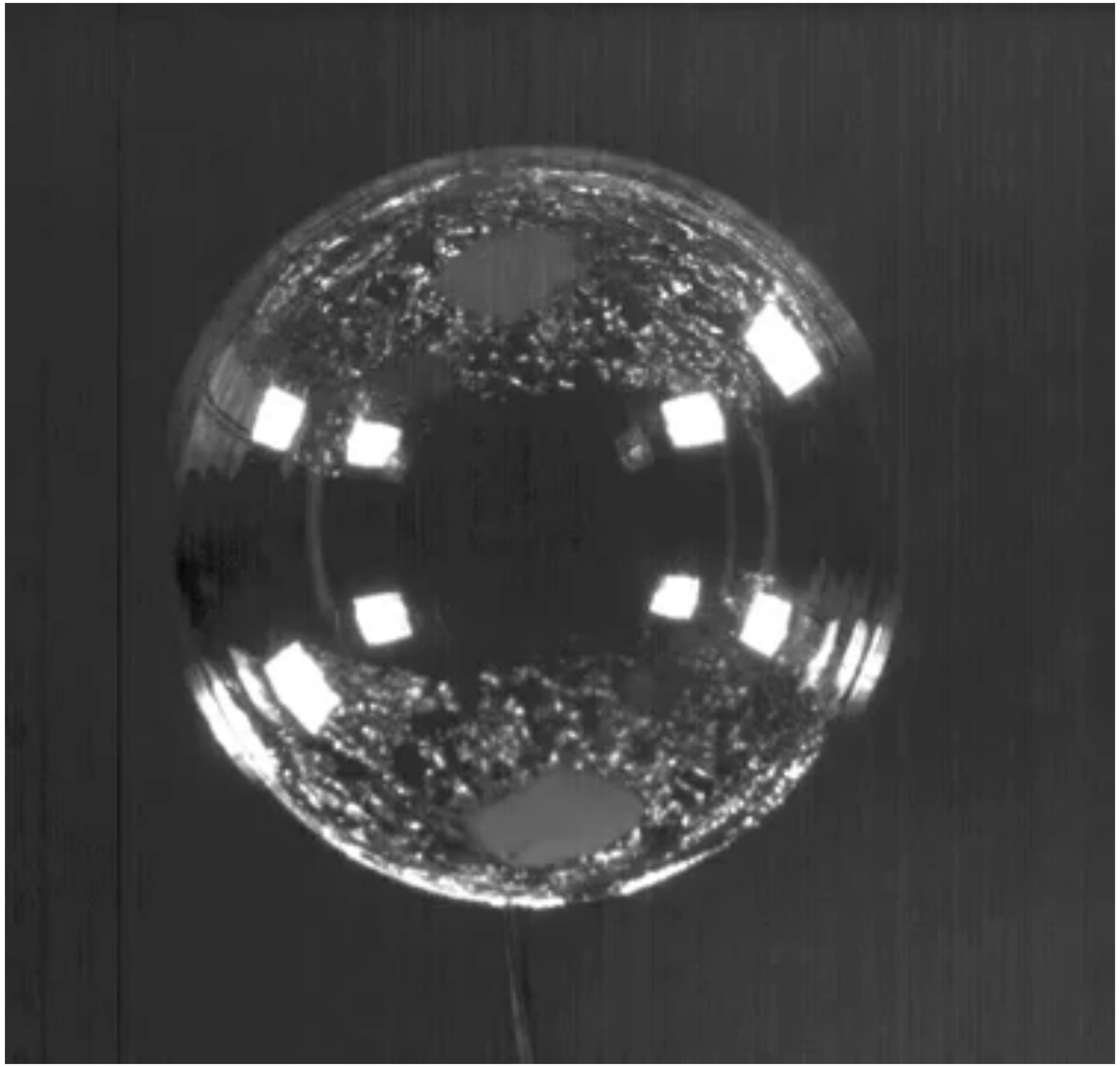


1. Sheets



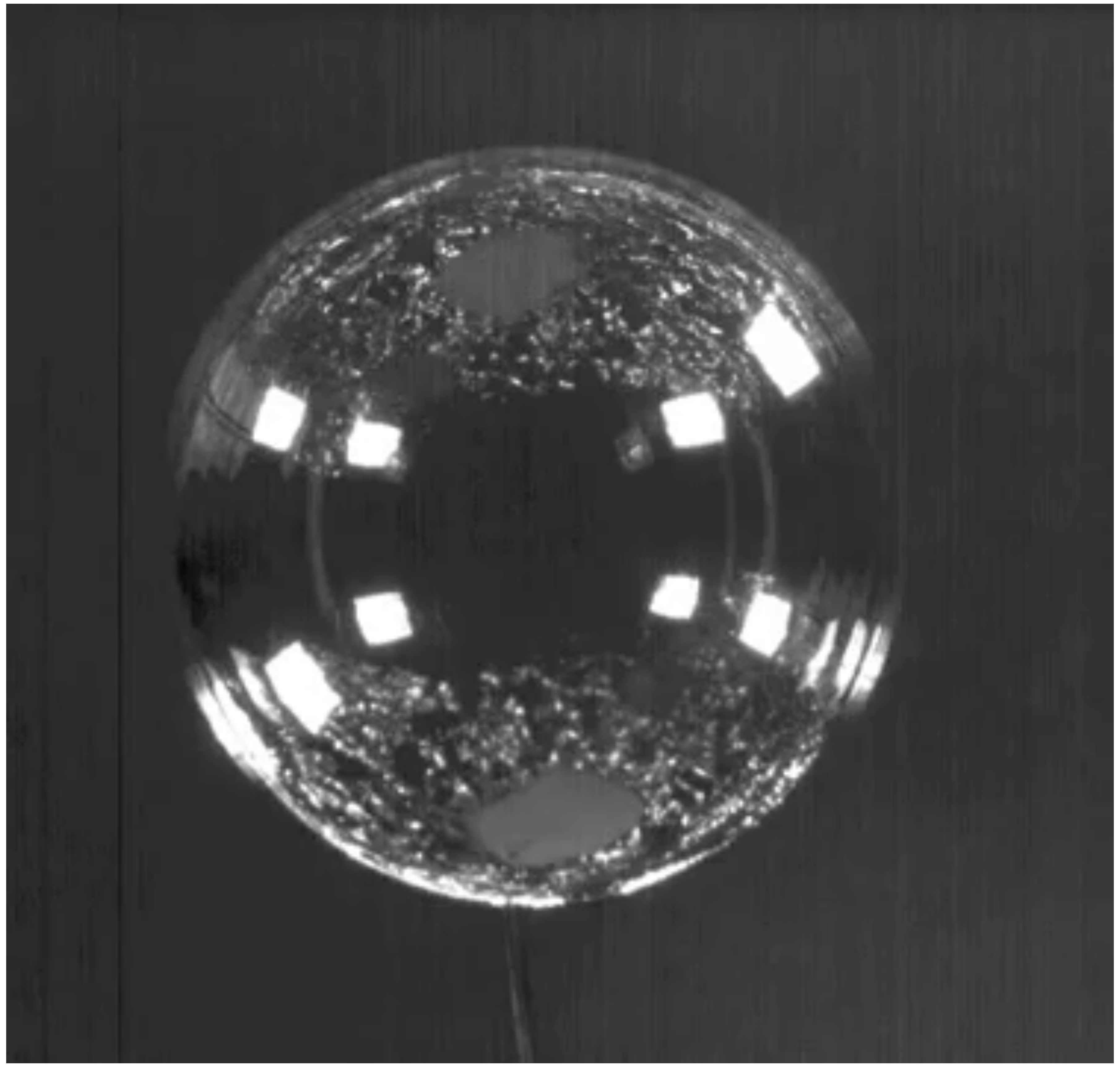
2. Drops

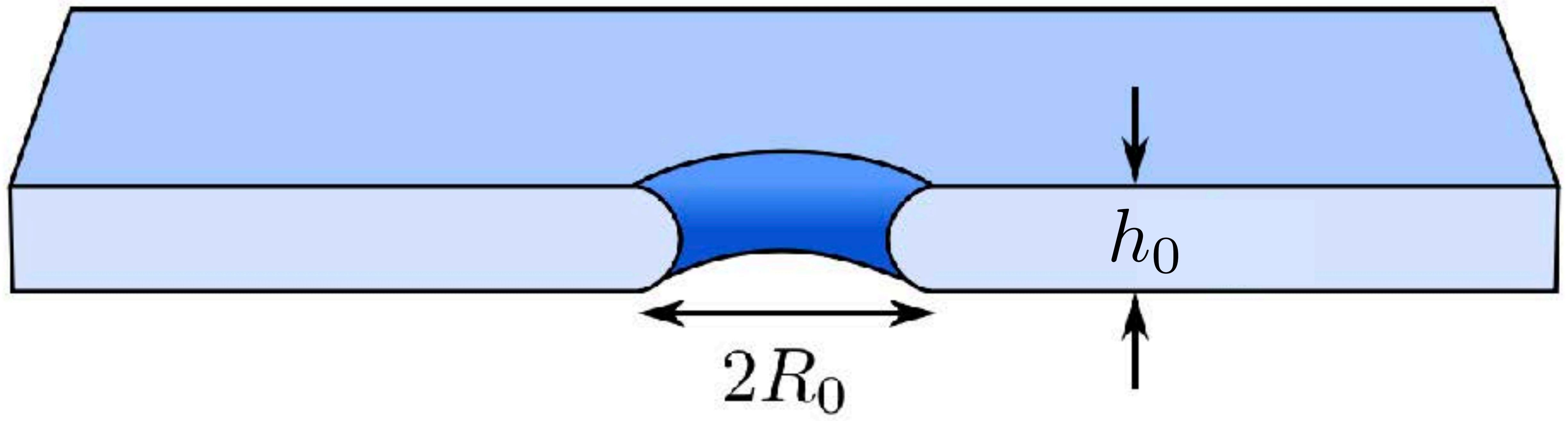
Bursting soap bubble



Video from https://fyfluiddynamics.com/2011/10/high-speed-video-of-a-soap-bubble-being-popped/

Bursting soap bubble





Detlef Lohse | Udo Sen | Pallav Kant | Jacco Snoeijer | Vincent Bertin | Alex Oratis

Classical Taylor-Culick retraction

$$Oh_f = 0.05$$

$$Oh = \frac{\eta}{\sqrt{\rho_f (2\gamma_{af}) h_0}}$$

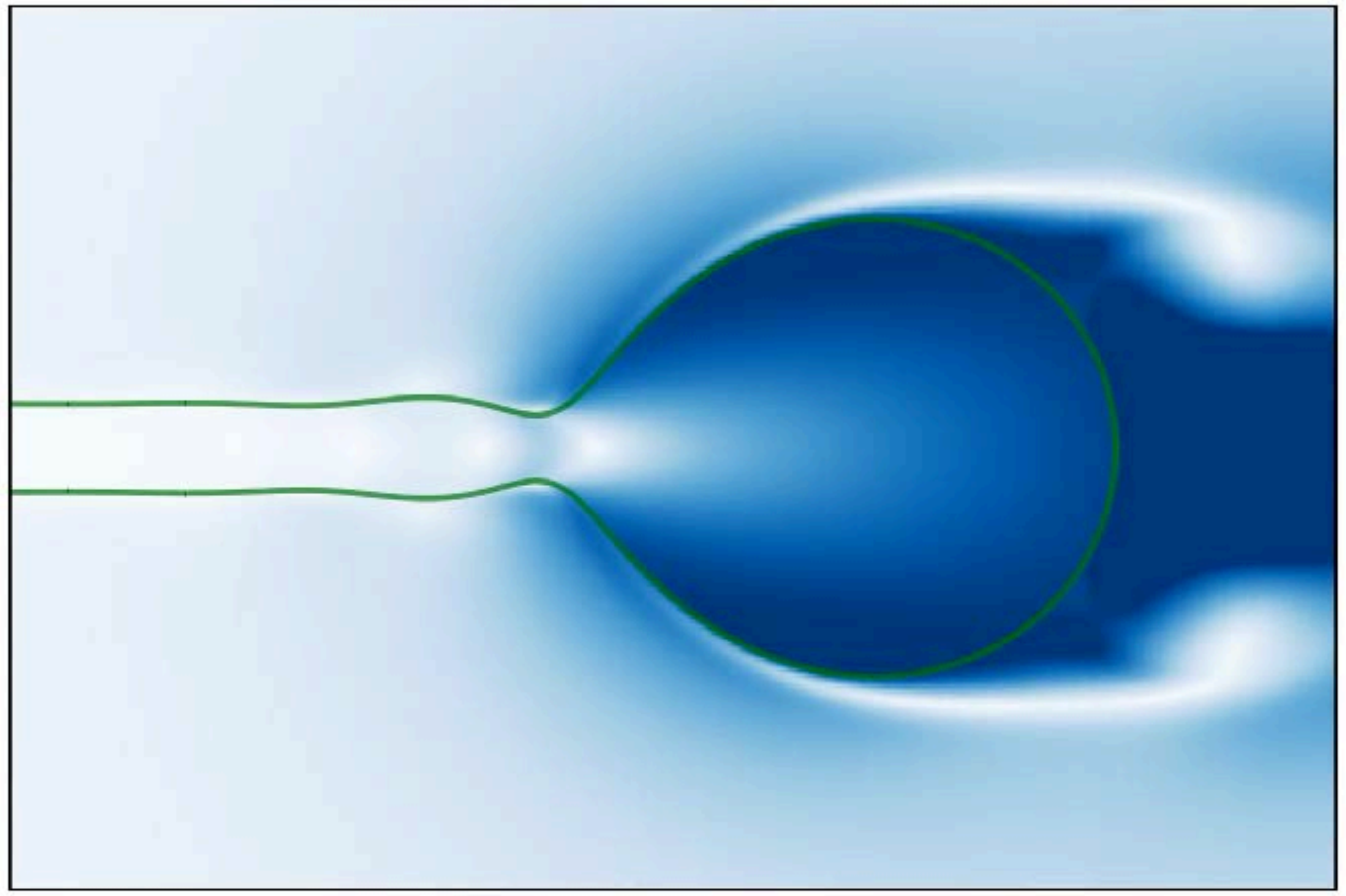
$$t/\tau_\gamma = 42.900$$

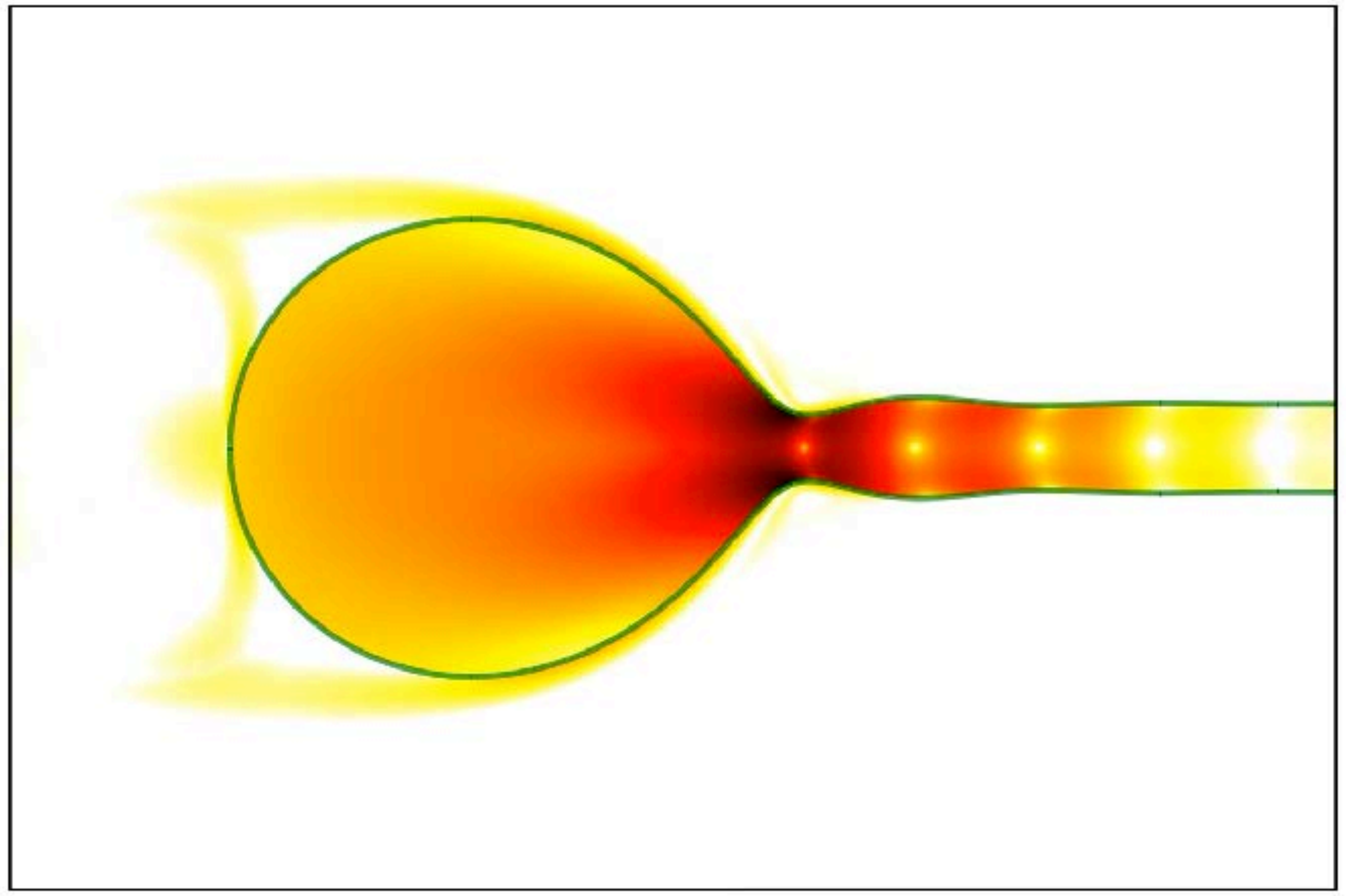
$\|\boldsymbol{v}\|/v_\gamma$

$\log_{10}\left(2Oh\left(\boldsymbol{\mathcal{D}} : \boldsymbol{\mathcal{D}}\right)\tau_\gamma^2\right)$

$$\boldsymbol{\mathcal{D}} = \left(\boldsymbol{\nabla v} + (\boldsymbol{\nabla v})^{\mathrm{T}}\right)/2$$

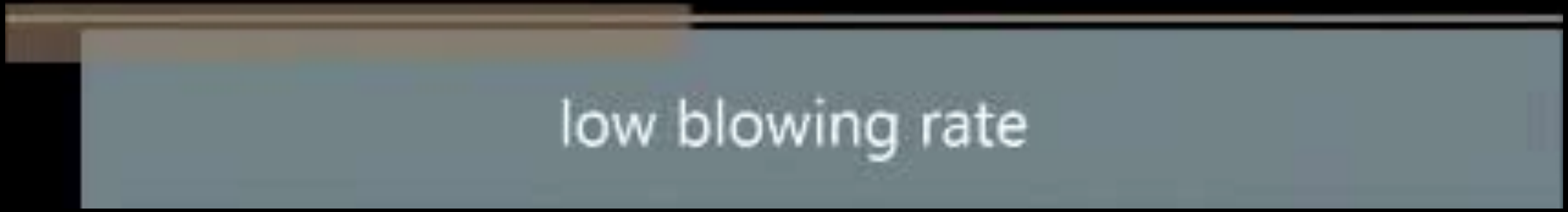
$$\tau_\gamma = \sqrt{\frac{\rho_f h_0^3}{2\gamma_{af}}} \quad v_\gamma = \sqrt{\frac{2\gamma_{af}}{\rho_f h_0}}$$





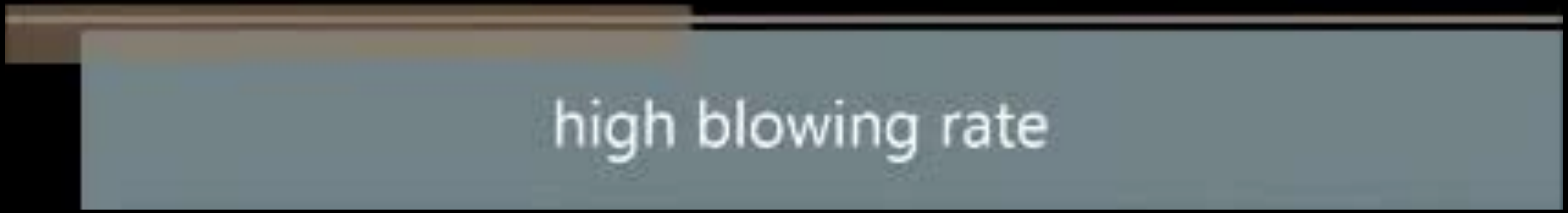
What happens if the film is **non-Newtonian**?

Friday afternoon experiment



D. Tammaro, R. Pasquino, M. M. Villone, G. D'Avino, V. Ferraro, E. Di Maio, A. Langella, N. Grizzuti & P. L. Maffettone, Langmuir, 34:19, 5646-5654 (2018)

Friday afternoon experiment



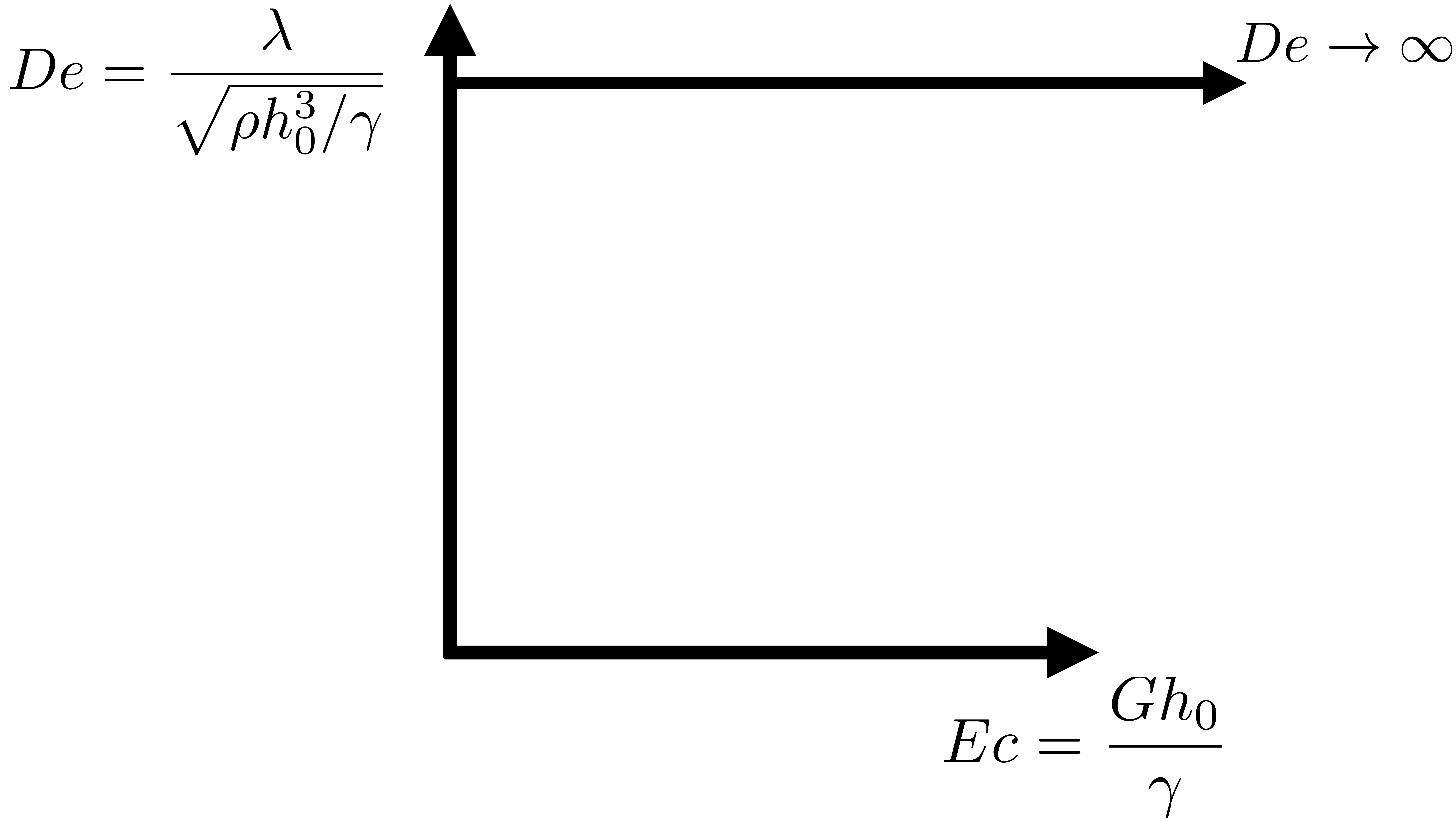
D. Tammaro, R. Pasquino, M. M. Villone, G. D'Avino, V. Ferraro, E. Di Maio, A. Langella, N. Grizzuti & P. L. Maffettone, Langmuir, 34:19, 5646-5654 (2018)

This is still **viscous liquid**

$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty$$

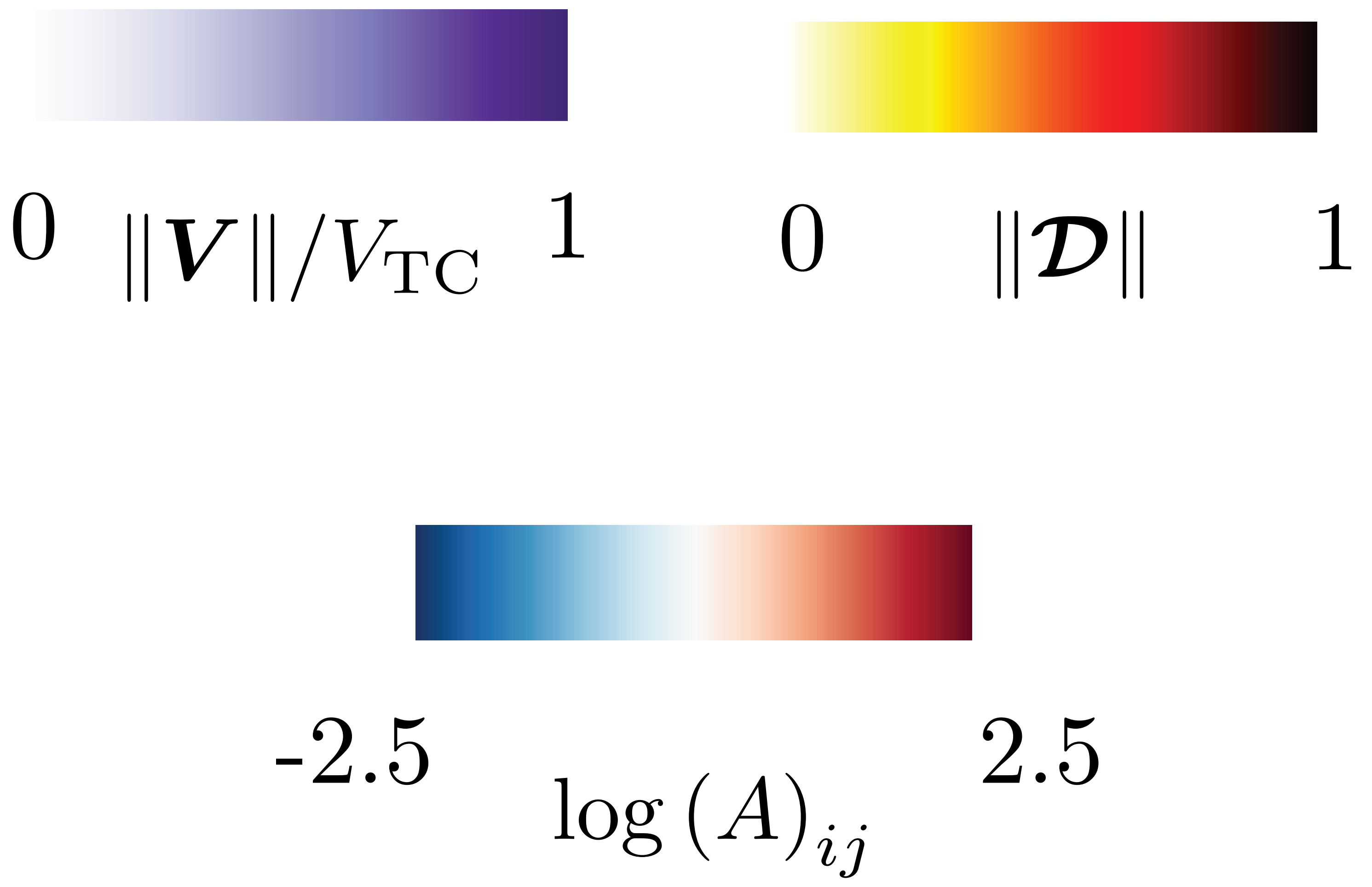
$$Ec = \frac{G h_0}{\gamma} = 0$$

No relaxation limit



$$Ec = \frac{Gh_0}{\gamma} = 0.01$$

$t/t_\gamma = 0.00$



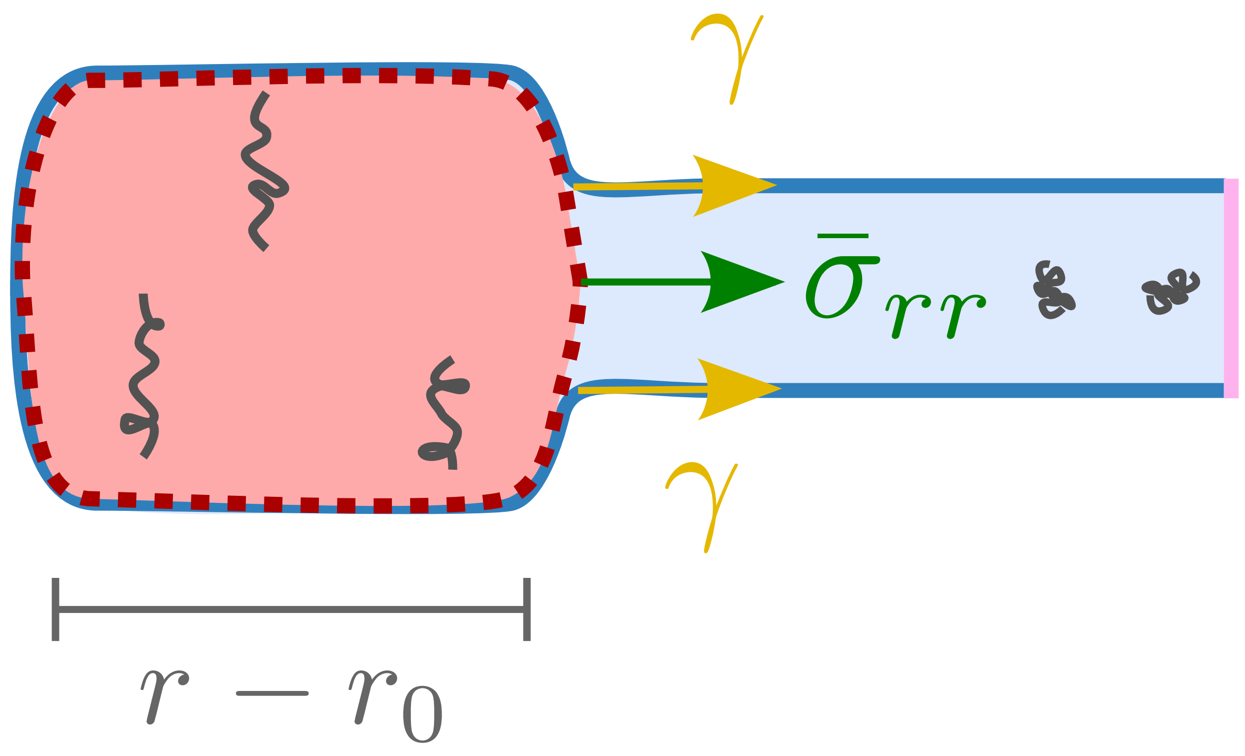
$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty$$

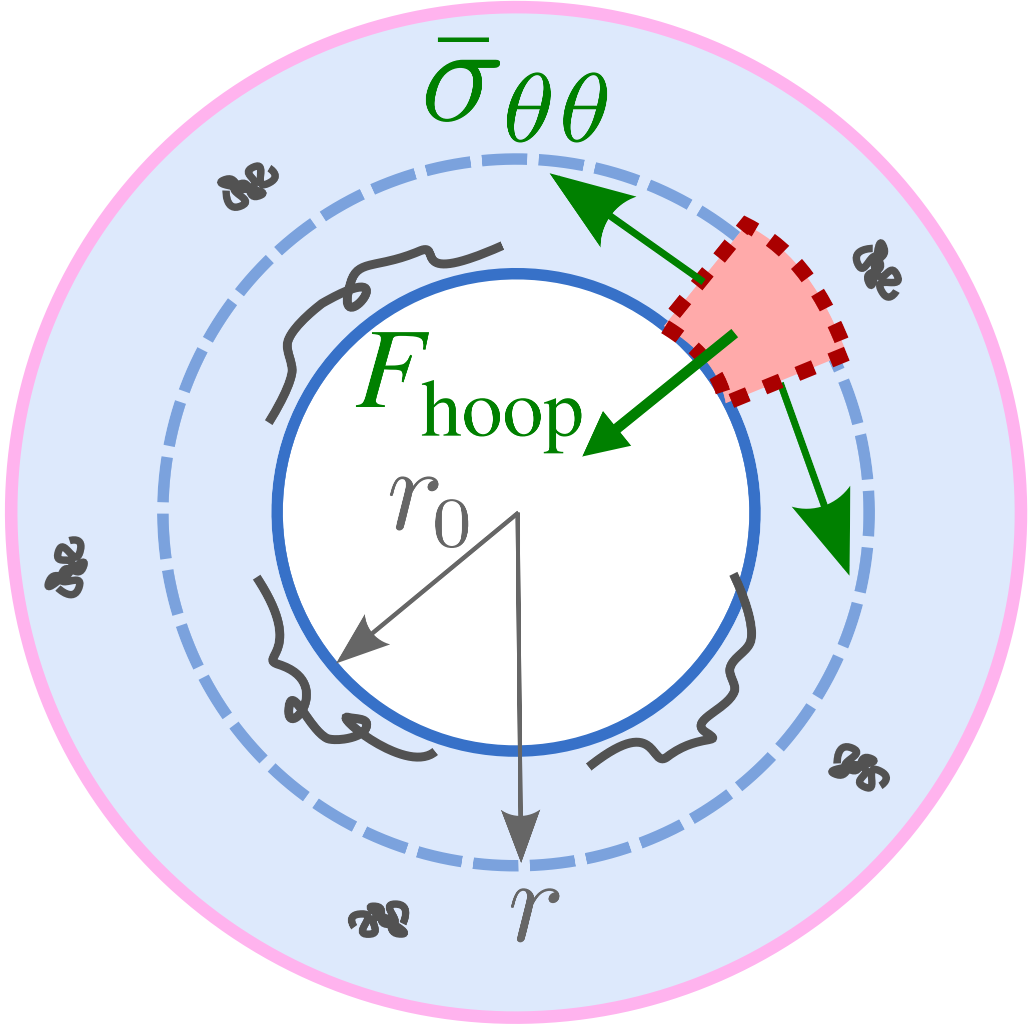
$$Ec = \frac{Gh_0}{\gamma} = 0.10$$

$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty$$

What is going on?

Vertical stretching





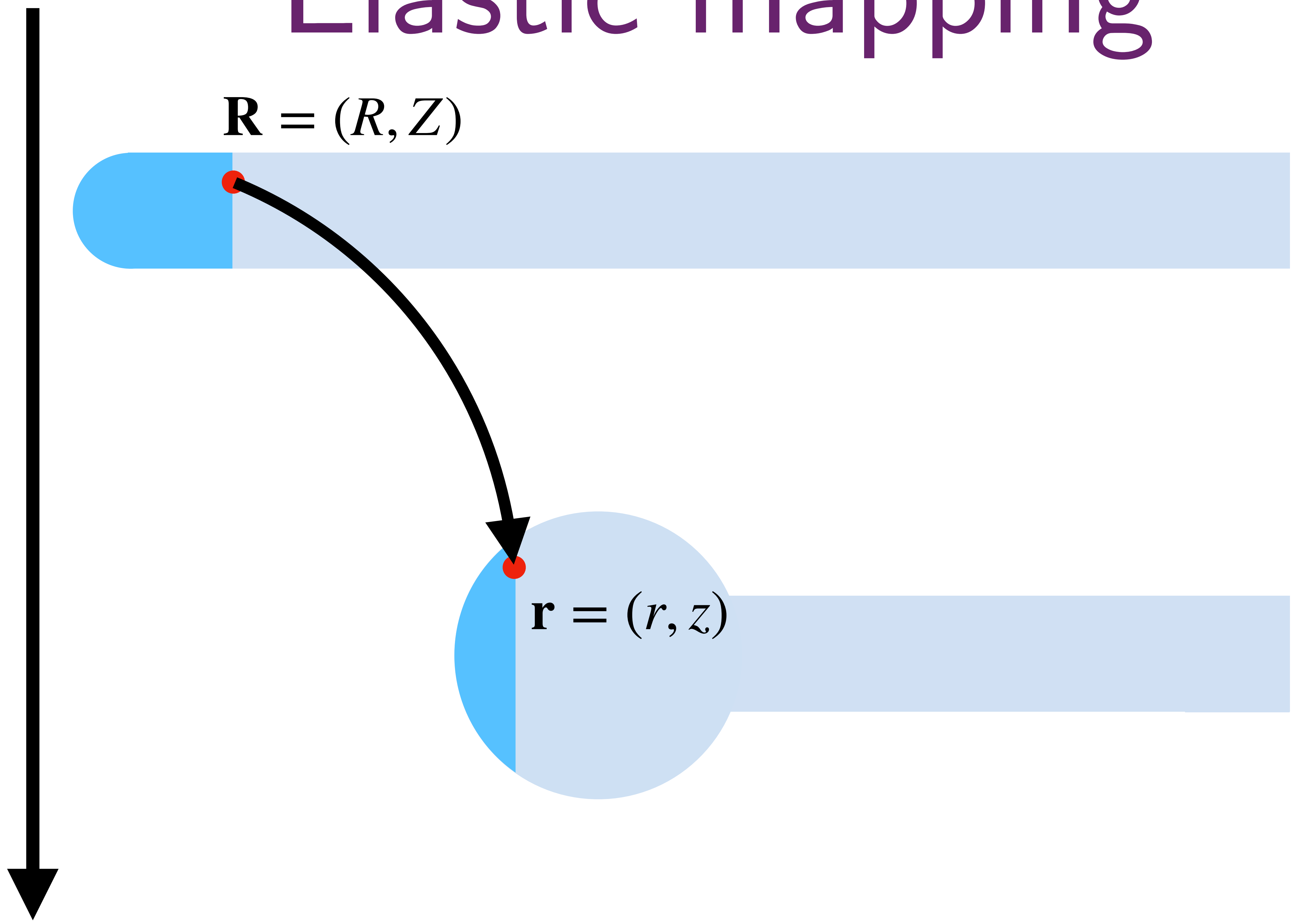
Azimuthal stretching

$$\frac{d}{dt}\left(\rho h_0 \pi r_{\text{tip}}^2 \frac{dr_{\text{tip}}}{dt}\right) = \left(2\pi r_{\text{tip}}\right) 2\gamma - F_{\text{hoop}}$$

$$F_{\text{hoop}} = 2\pi \int_{r_{\text{tip}}}^{\infty} \int_0^{h(r)} \sigma_{\theta\theta} dh dr$$

Only **hoop** force matters for the radial dynamics

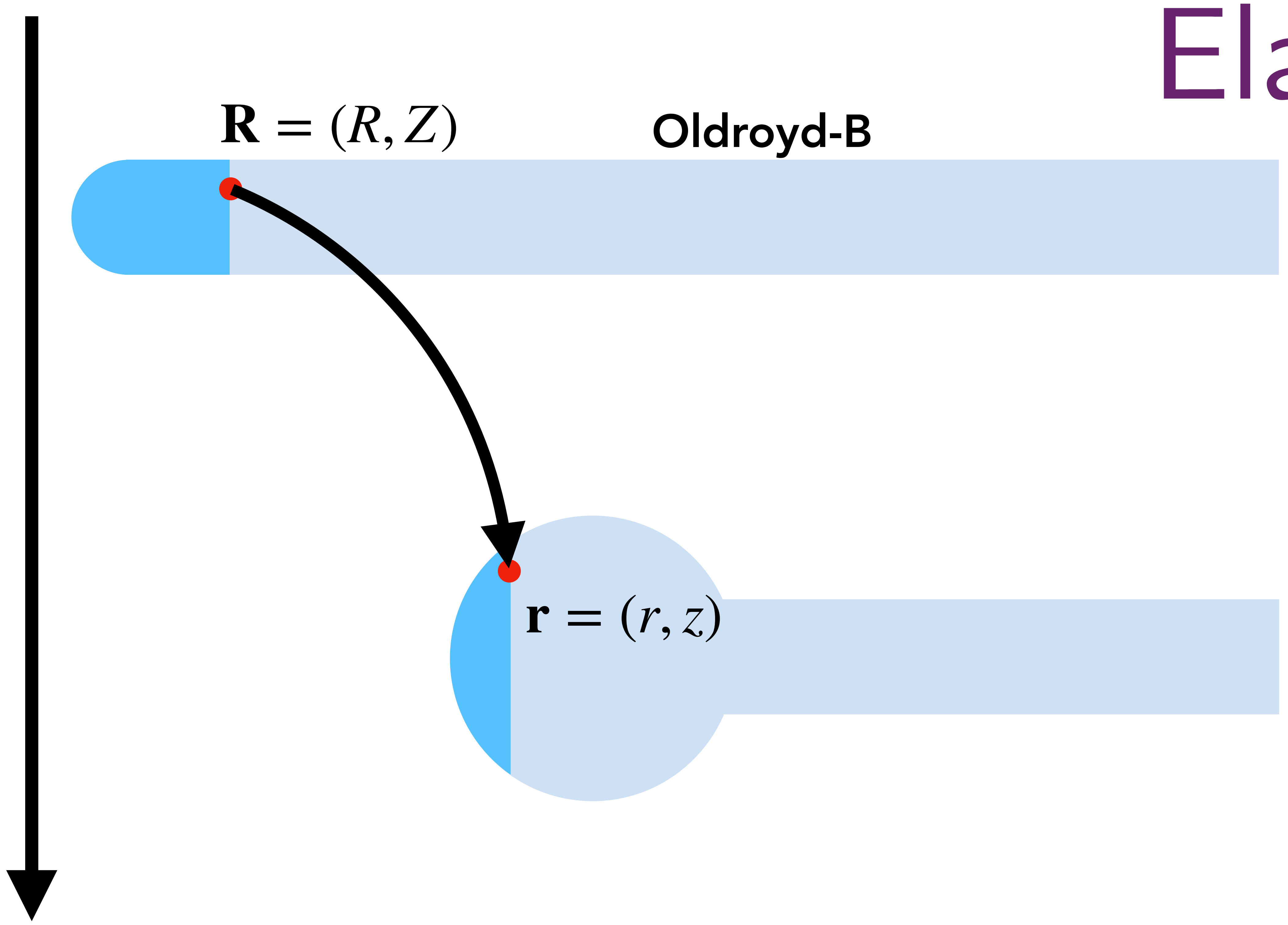
Elastic mapping



- **Kinematic**
- **Elastic mapping** $$\frac{Z}{H} = \frac{z}{h(r)}$$ Homogeneous vertical stretching
- **Shear-free** $$H_0(R^2 - \tilde{R}_0^2) = 2\int_{r_0}^{r} h(\bar{r})\bar{r}\mathrm{d}\bar{r}$$ Volume conservation

J. H. Snoeijer, A. Pandey, M. A. Herrada & J. Eggers, Proc. R. Soc. A, 476:2243, 20200419 (2020)

Elastic stresses



$$\boldsymbol{\sigma} = G(\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{I}})$$ Conformation tensor

$$\boldsymbol{\mathcal{F}} = \frac{\partial \mathbf{r}}{\partial \mathbf{R}}$$

$$\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{F}} \cdot \boldsymbol{\mathcal{F}}^{\mathrm{T}}$$ Elastic limit

$$\sigma_{\theta\theta} = G\left(\frac{r^2}{R^2} - 1\right) = G\left(\frac{r^2}{\tilde{R}_0^2 + \frac{2}{H_0}\int_{r_0}^{r} h(\bar{r})\bar{r}\mathrm{d}\bar{r}} - 1\right)$$

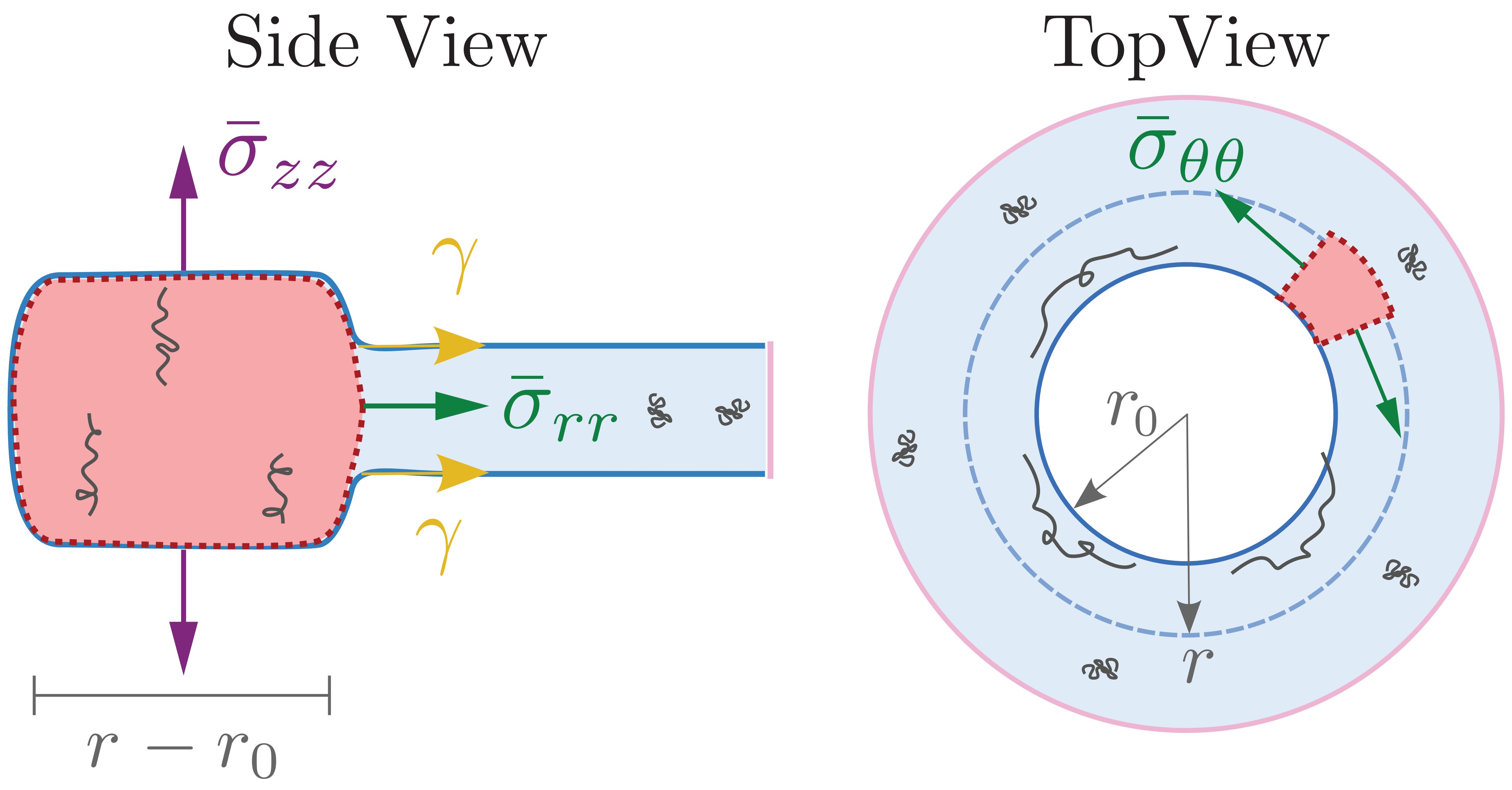
$$F_{\mathrm{hoop}} = 2\pi \int_{r_0}^{r} \sigma_{\theta\theta}\, h(\bar{r})\mathrm{d}\bar{r}$$

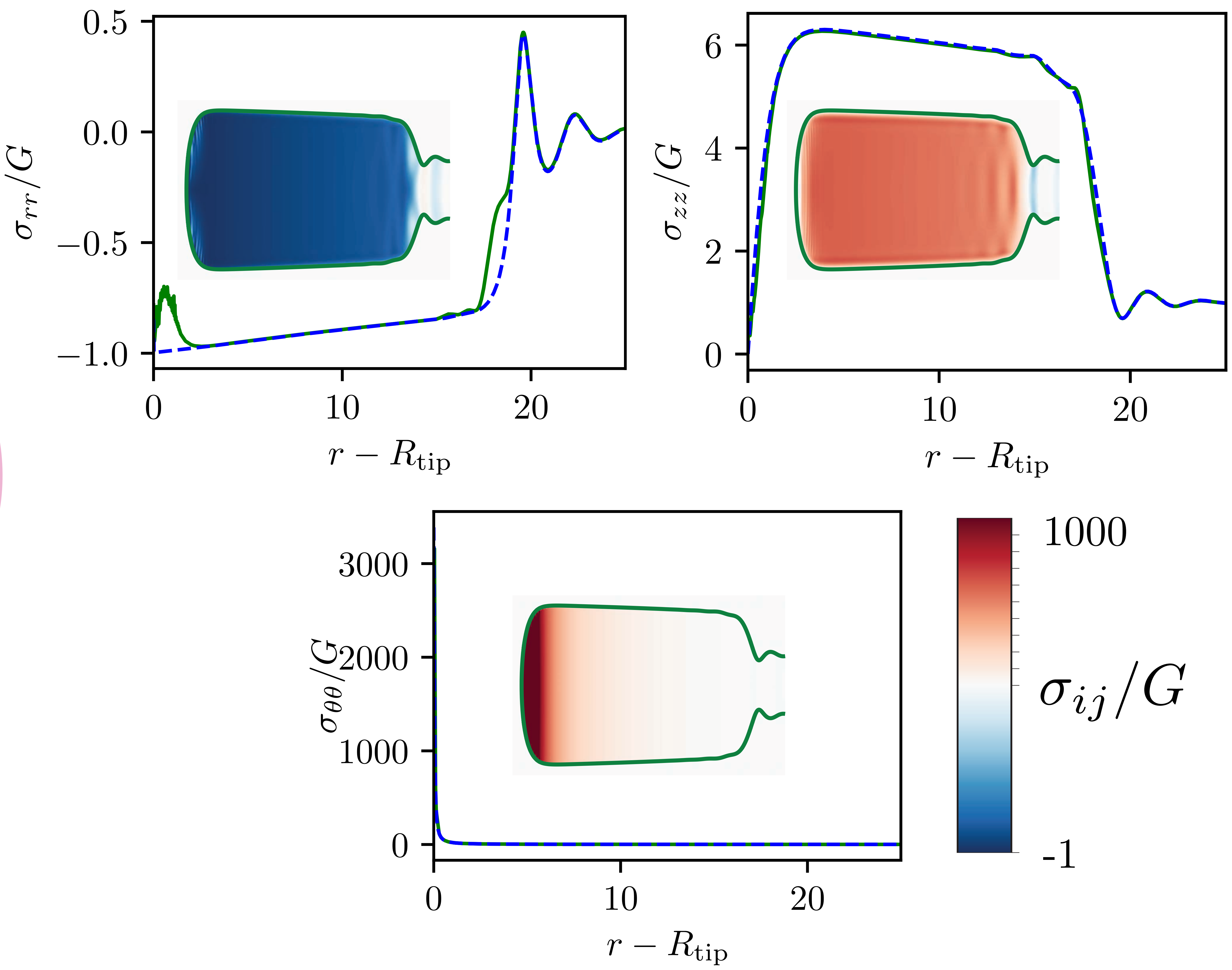
$$= 2\pi G \int_{r_0}^{r} \left(\frac{\bar{r}^2}{\tilde{R}_0^2 + \frac{2}{H_0}\int_{r_0}^{\bar{r}} h(\tilde{r})\tilde{r}\mathrm{d}\tilde{r}} - 1\right) h(\bar{r})\mathrm{d}\bar{r}$$

$$F_{\mathrm{hoop}} = \frac{G r_0 H_0}{2}\left[\ln\left(1 + \frac{r_0^2}{\tilde{R}_0^2}\right) - 1\right]$$

J. H. Snoeijer, A. Pandey, M. A. Herrada & J. Eggers, Proc. R. Soc. A, 476:2243, 20200419 (2020)

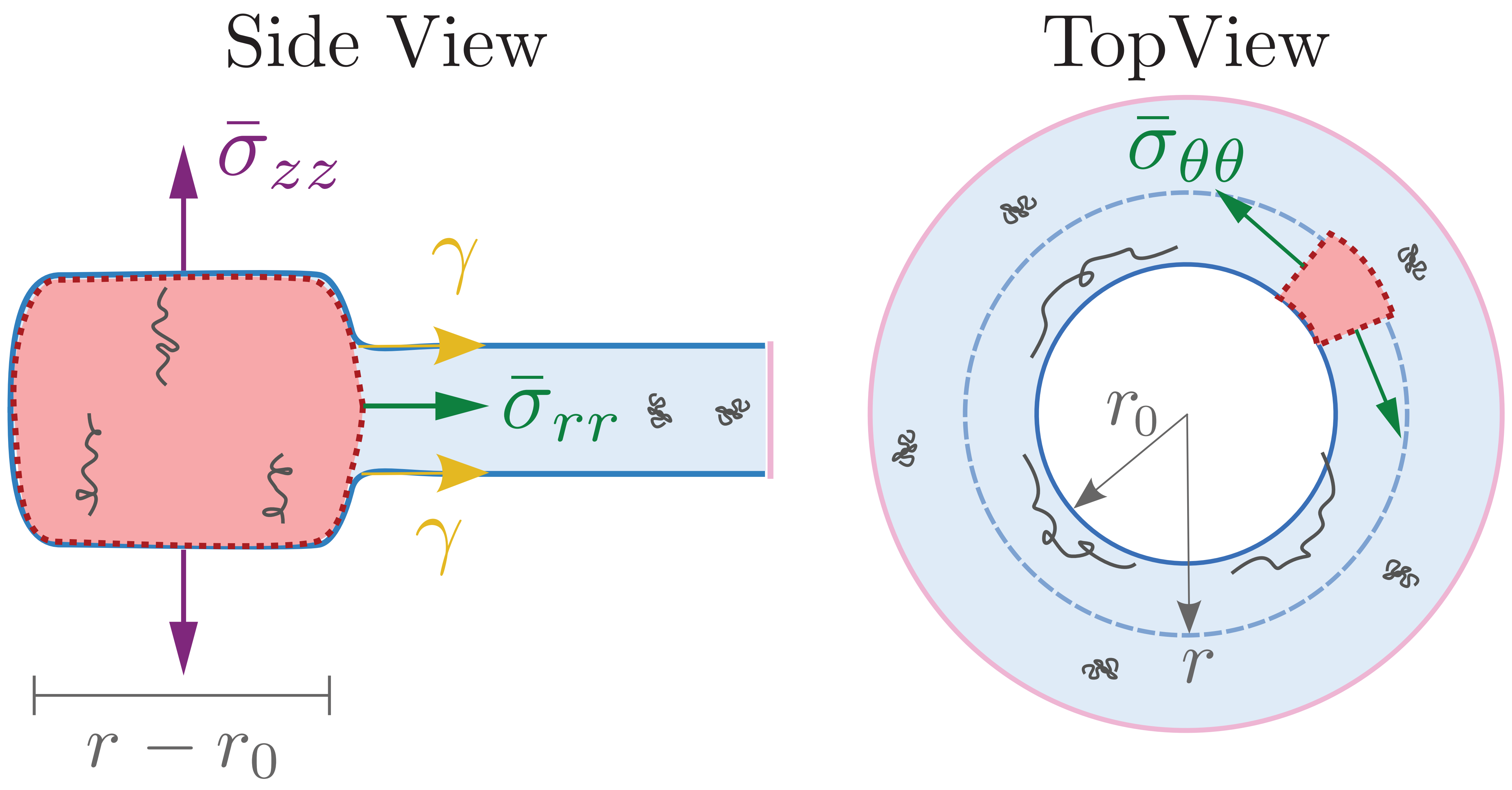
What is going on?

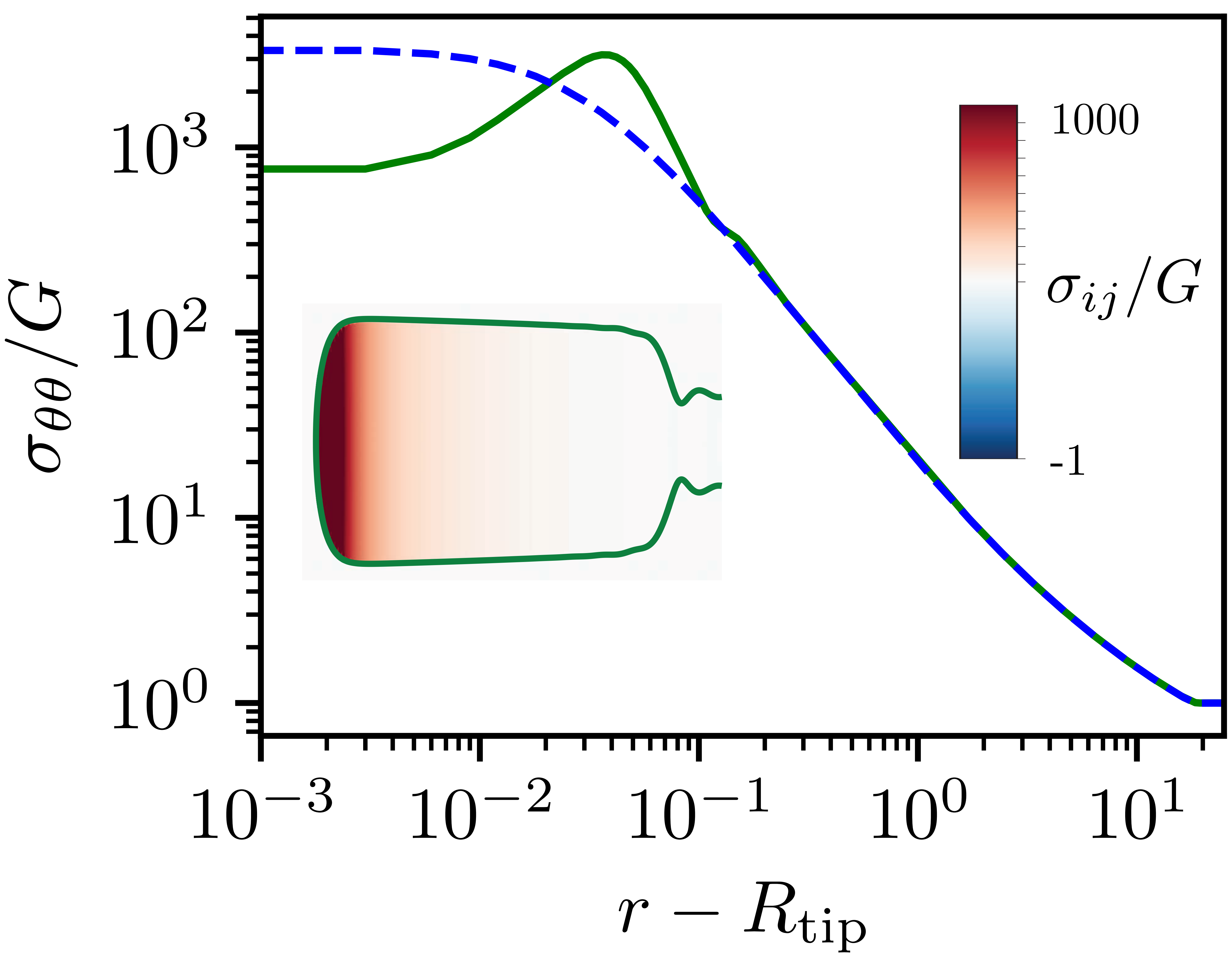




$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty \quad Ec = \frac{G h_0}{\gamma} = 0.10$$

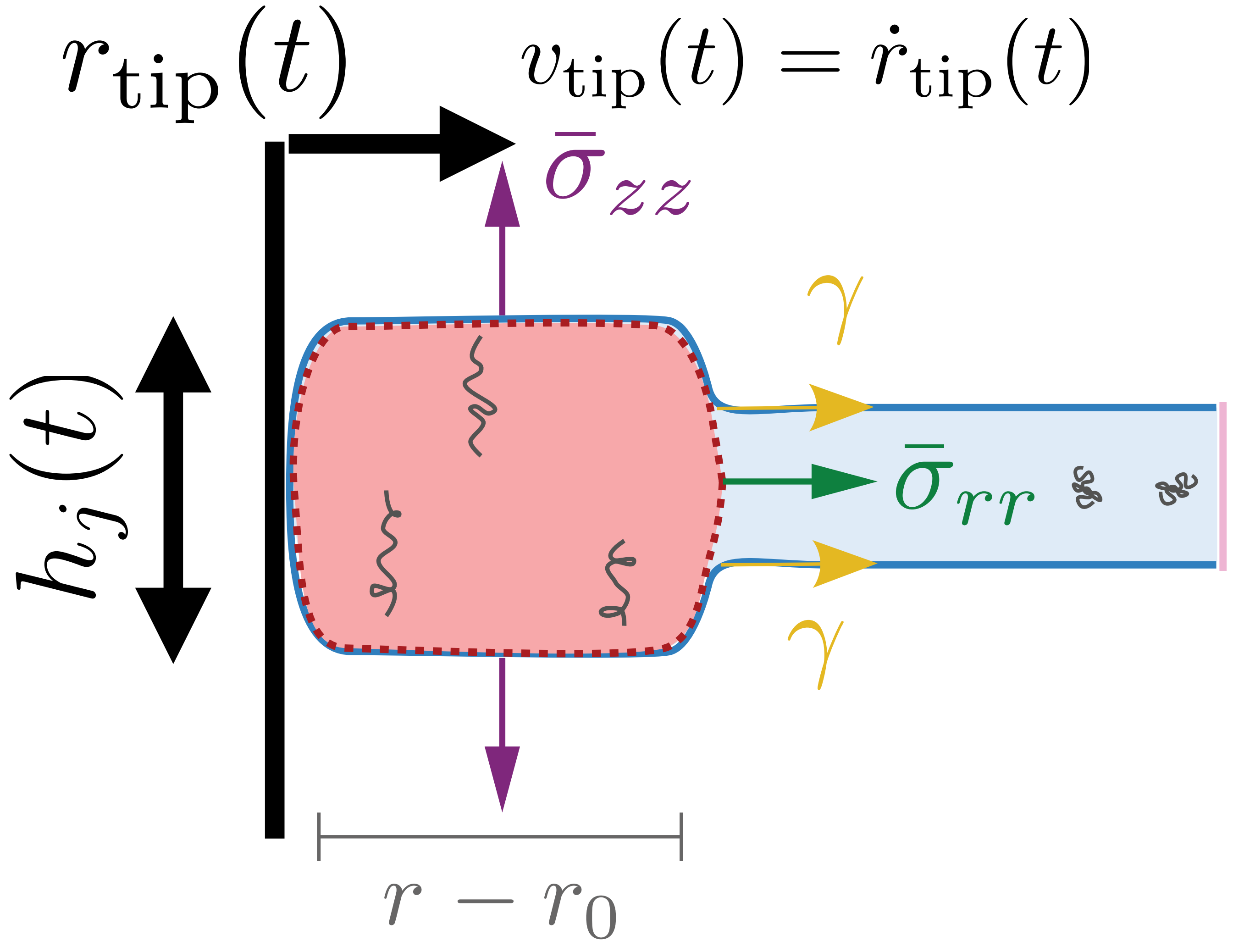
What is going on?

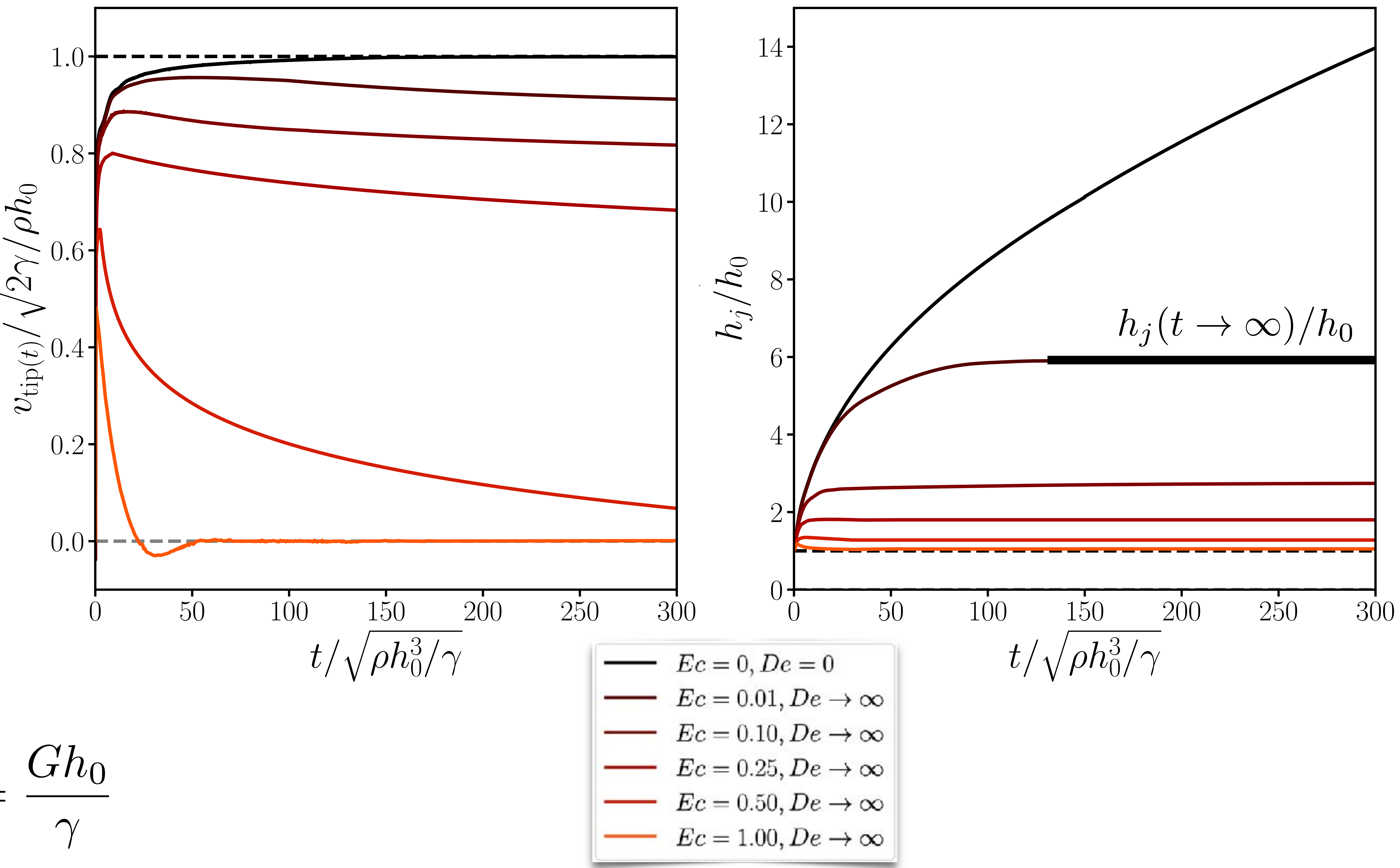




$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty \quad Ec = \frac{G h_0}{\gamma} = 0.10$$

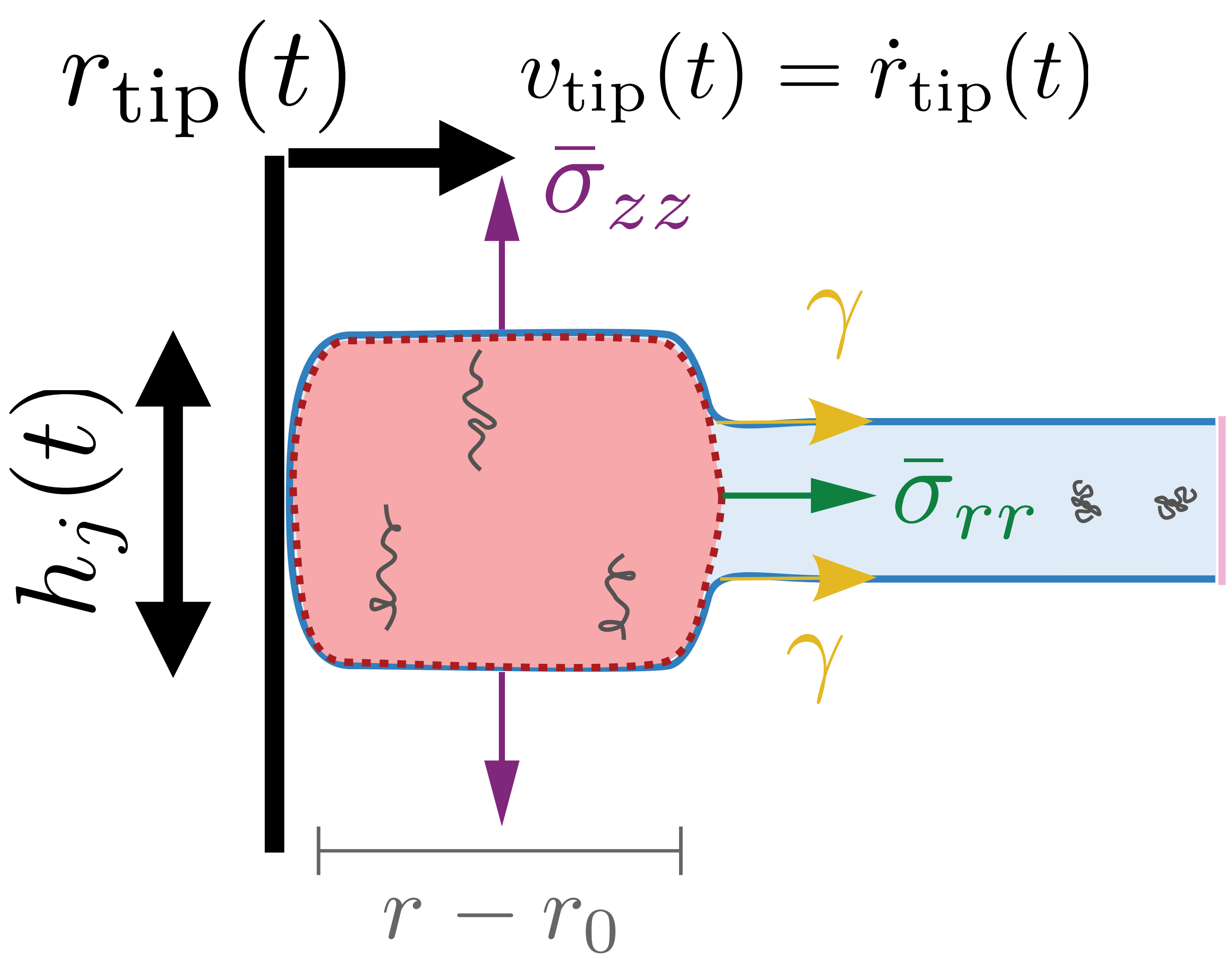
Taylor-Culick retractions: from viscous to elastic

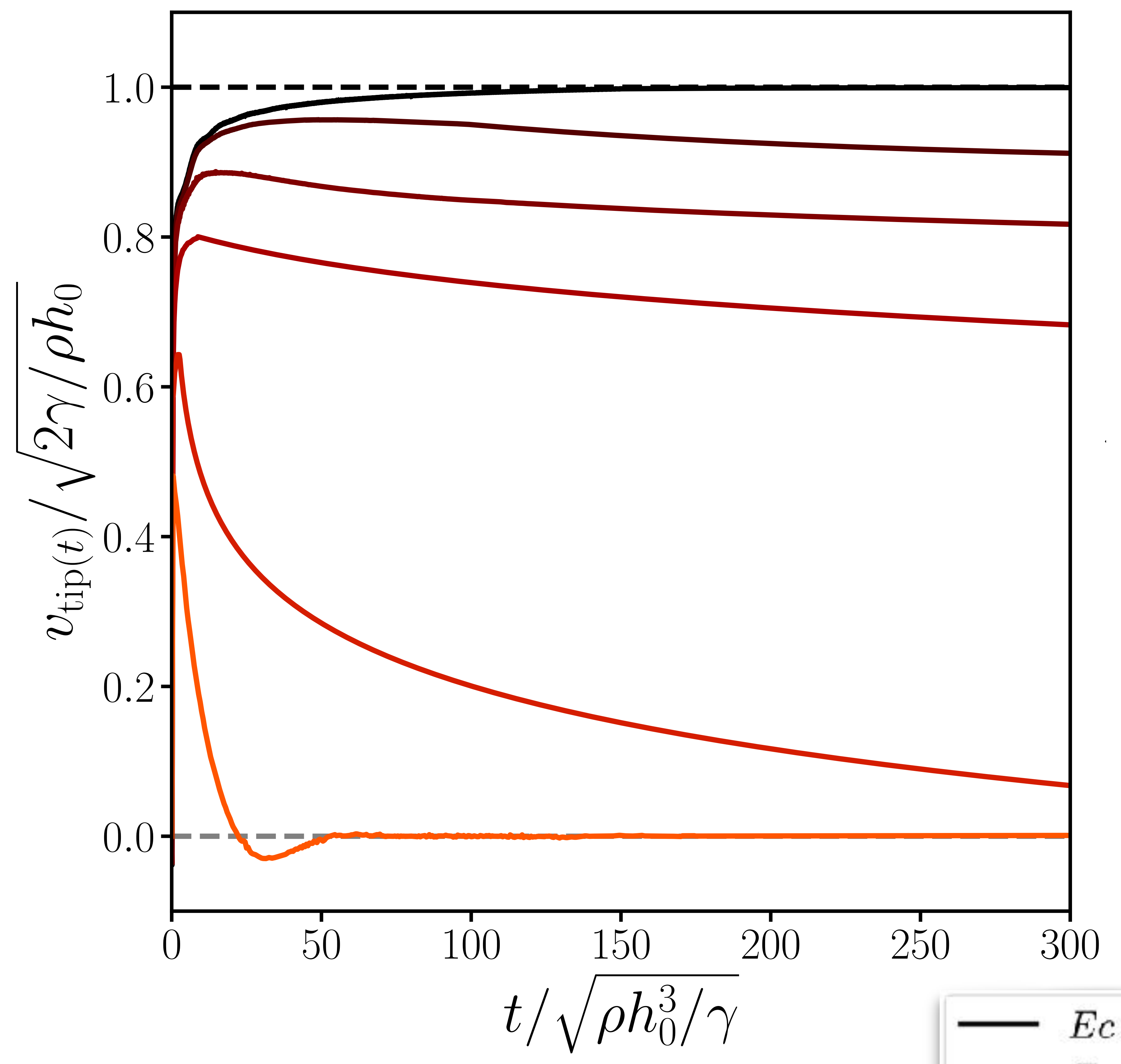


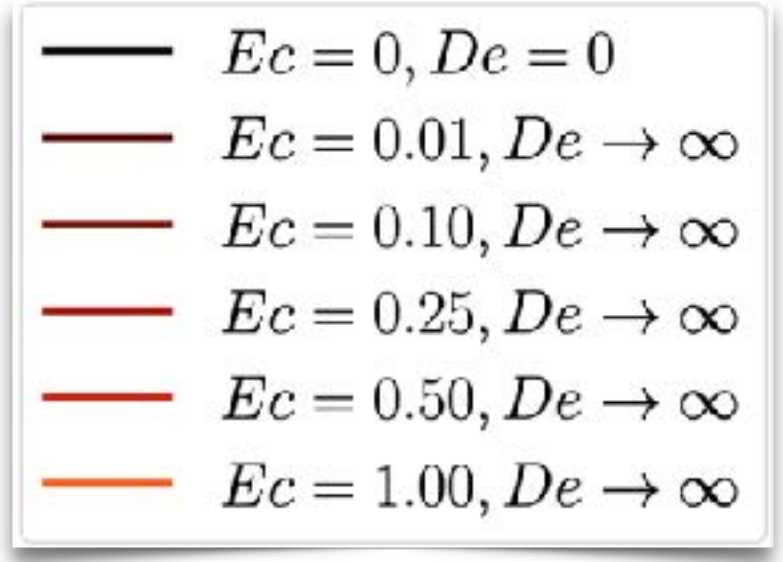


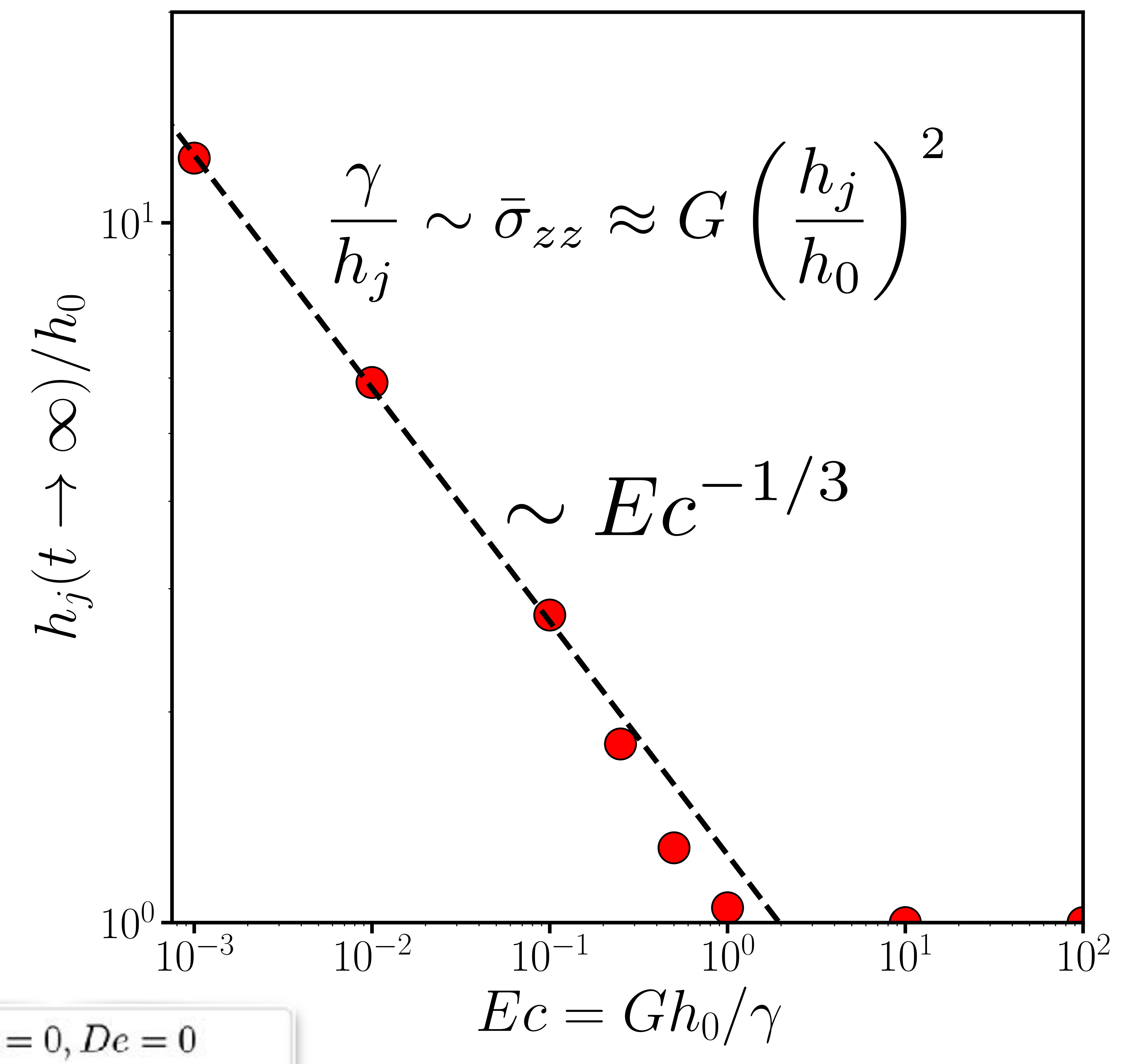
$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty \quad Ec = \frac{G h_0}{\gamma}$$

Taylor-Culick retractions: from viscous to elastic







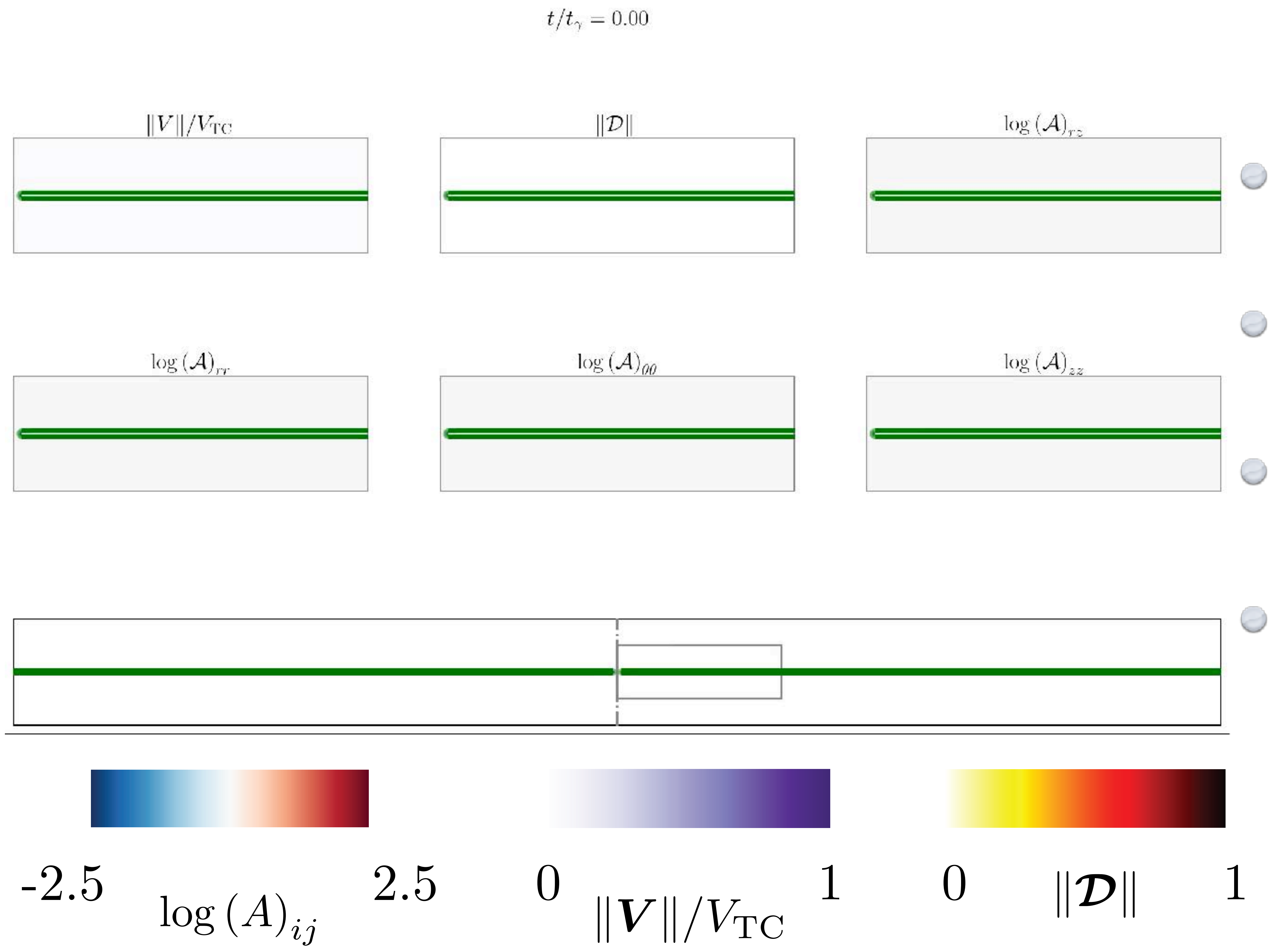


$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty \quad Ec = \frac{Gh_0}{\gamma}$$

$$Ec = \frac{Gh_0}{\gamma} = 1$$

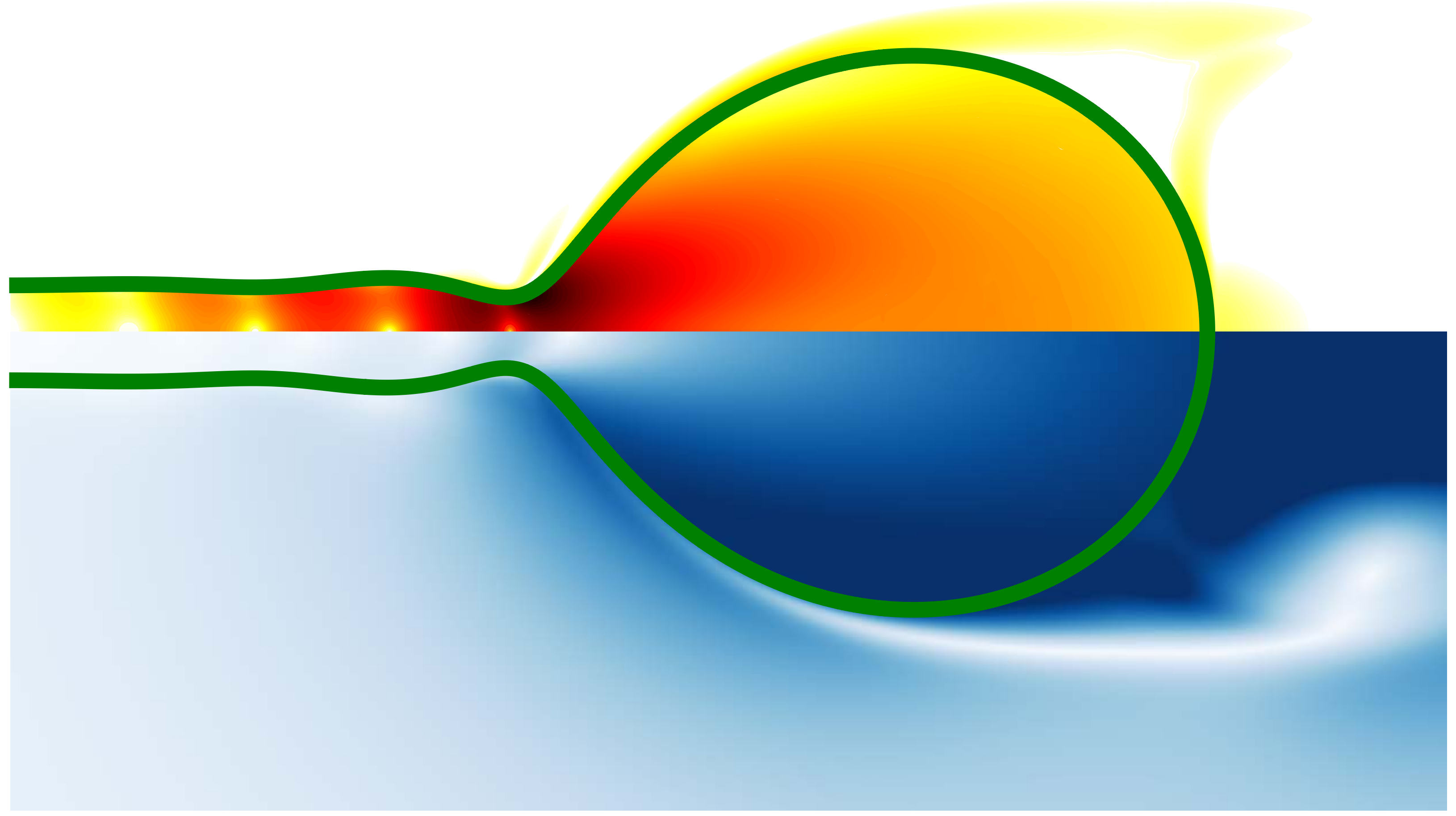
$$De = \frac{\lambda}{\sqrt{\rho h_0^3/\gamma}} \to \infty$$

Summary: bursting viscoelastic sheets

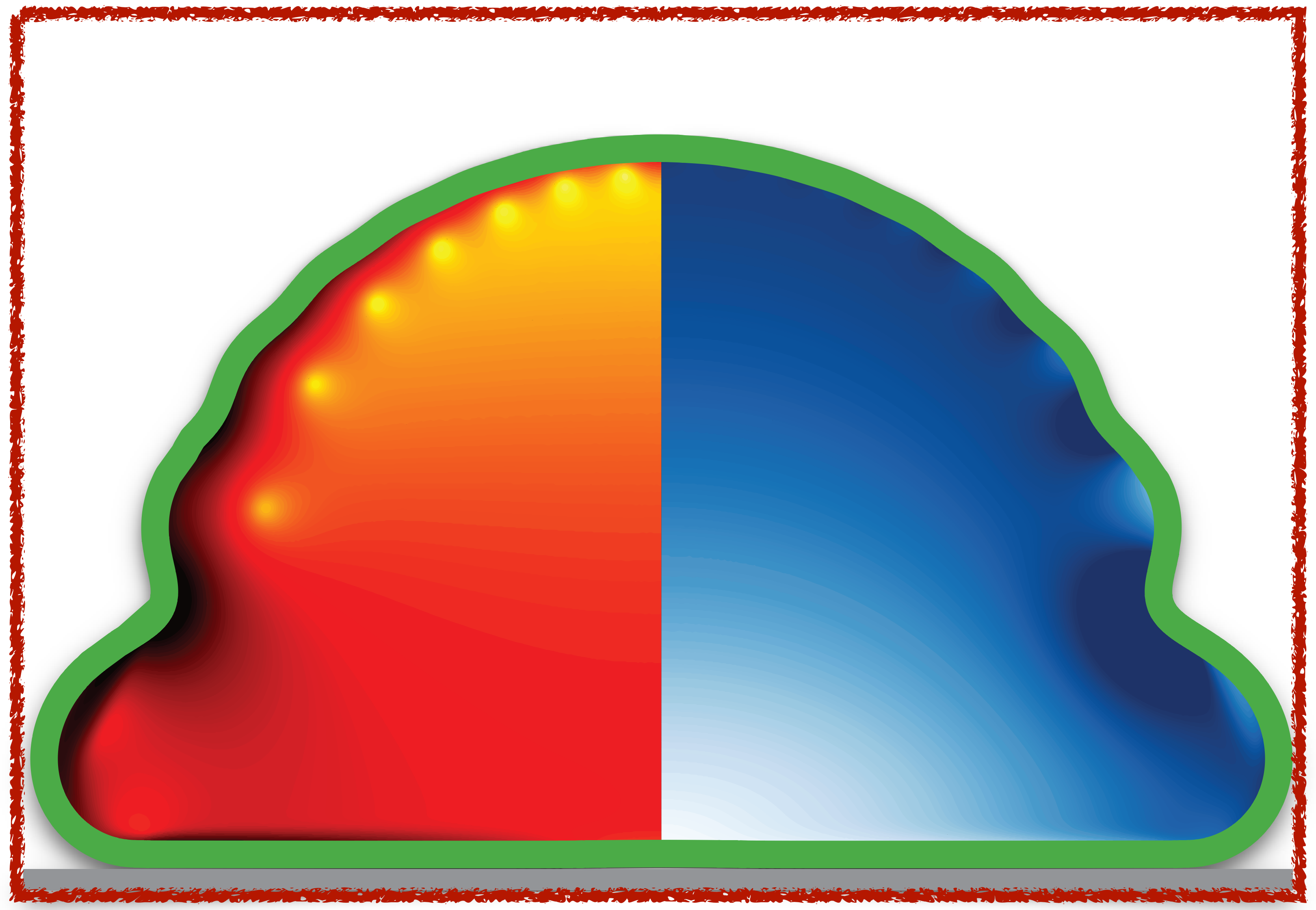


- Singular perturbation for Ec $= 0^+$
- Capillary pressure determines elastic jump height
- Large elastocapillary numbers arrests film retraction
- *Deborah number: solid-like to liquid-like behavior*

On the menu today



1. Sheets



2. Drops

Liquid Ping-Pong in Space: https://www.youtube.com/watch?v=TLbhrMCM4_0

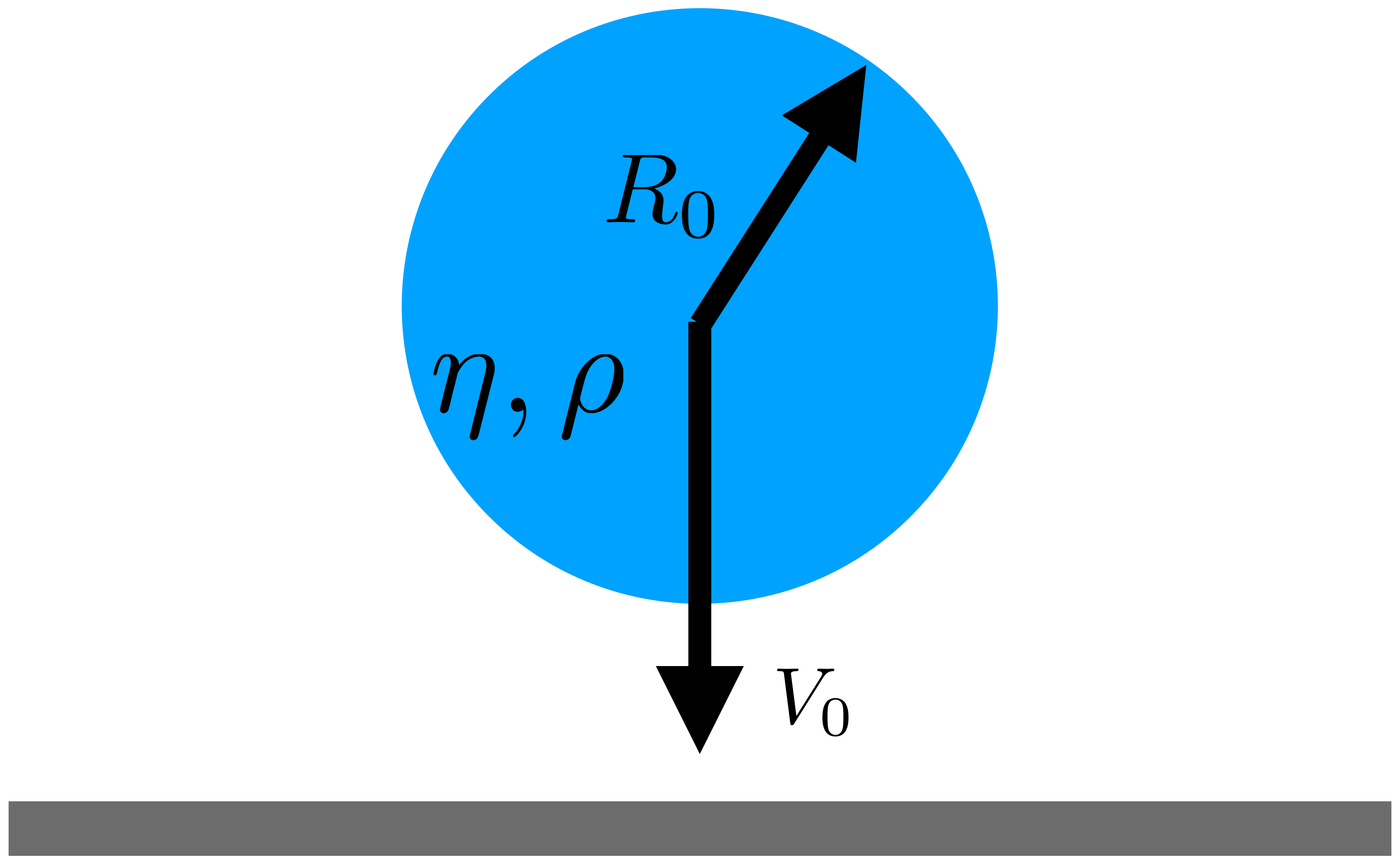
Soft impacts

Saumili Jana

John Kolinski

Detlef Lohse

Today, we look at forces: Wagner vs Hertz

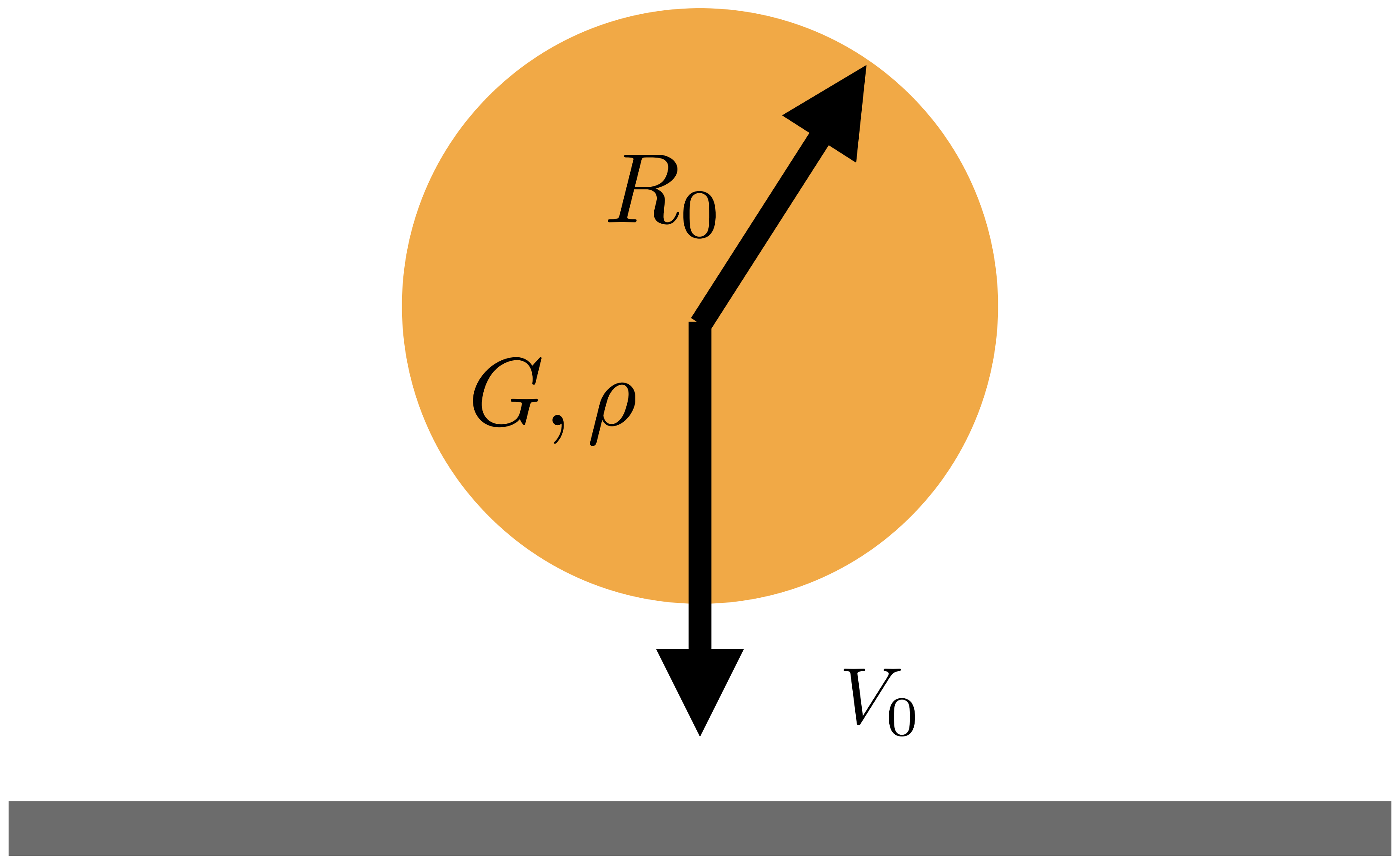


$$\tau_i = \frac{R_0}{V_0}$$

$$r_{\text{foot}} = \sqrt{3 V_0 R_0 t}$$

$$F \sim \rho V_0^2 R_0^2$$

Liquid impacts



$$\tau_i = \frac{R_0}{V_0}$$

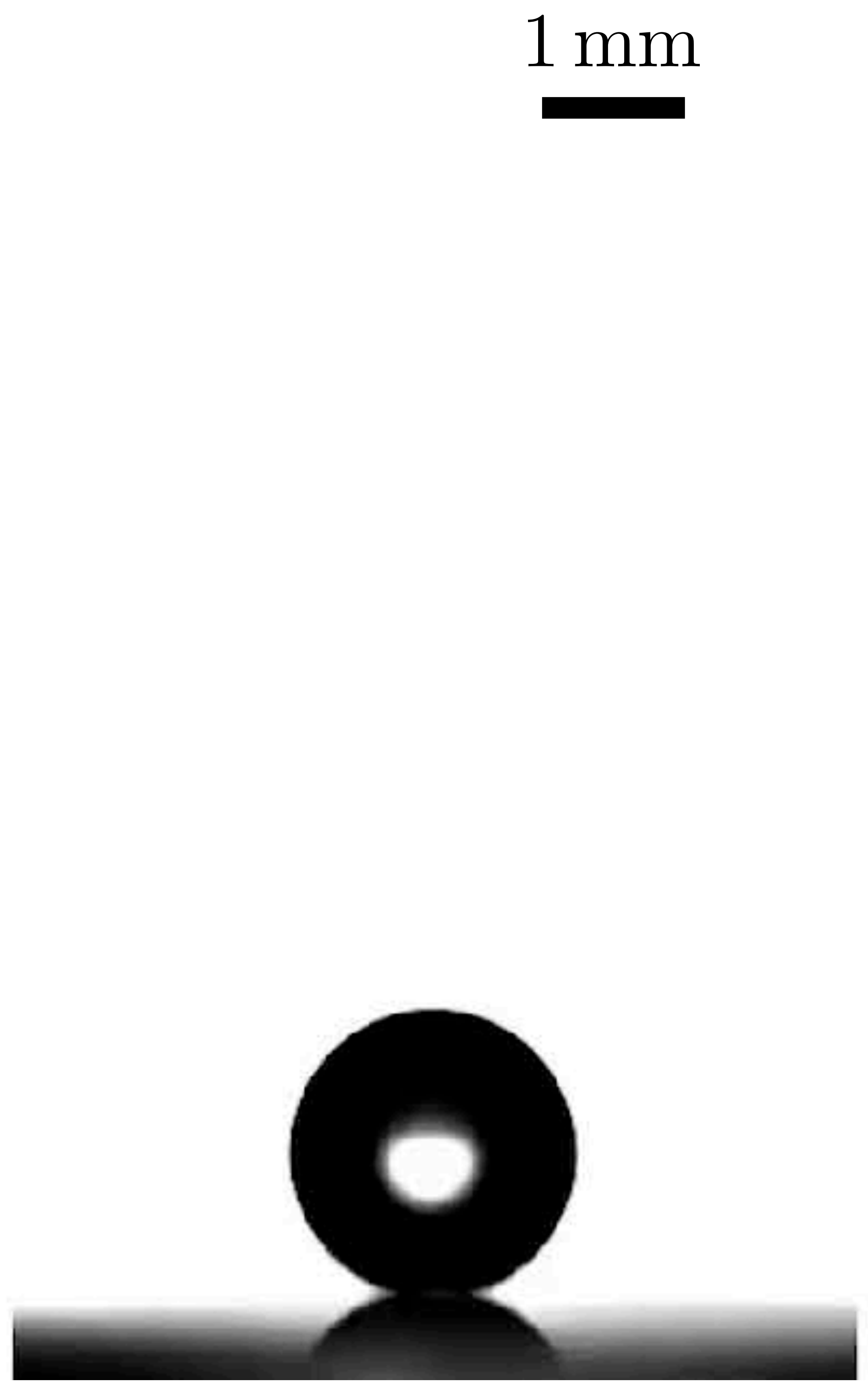
$$r_{\text{foot}} = \sqrt{V_0 R_0 t}$$

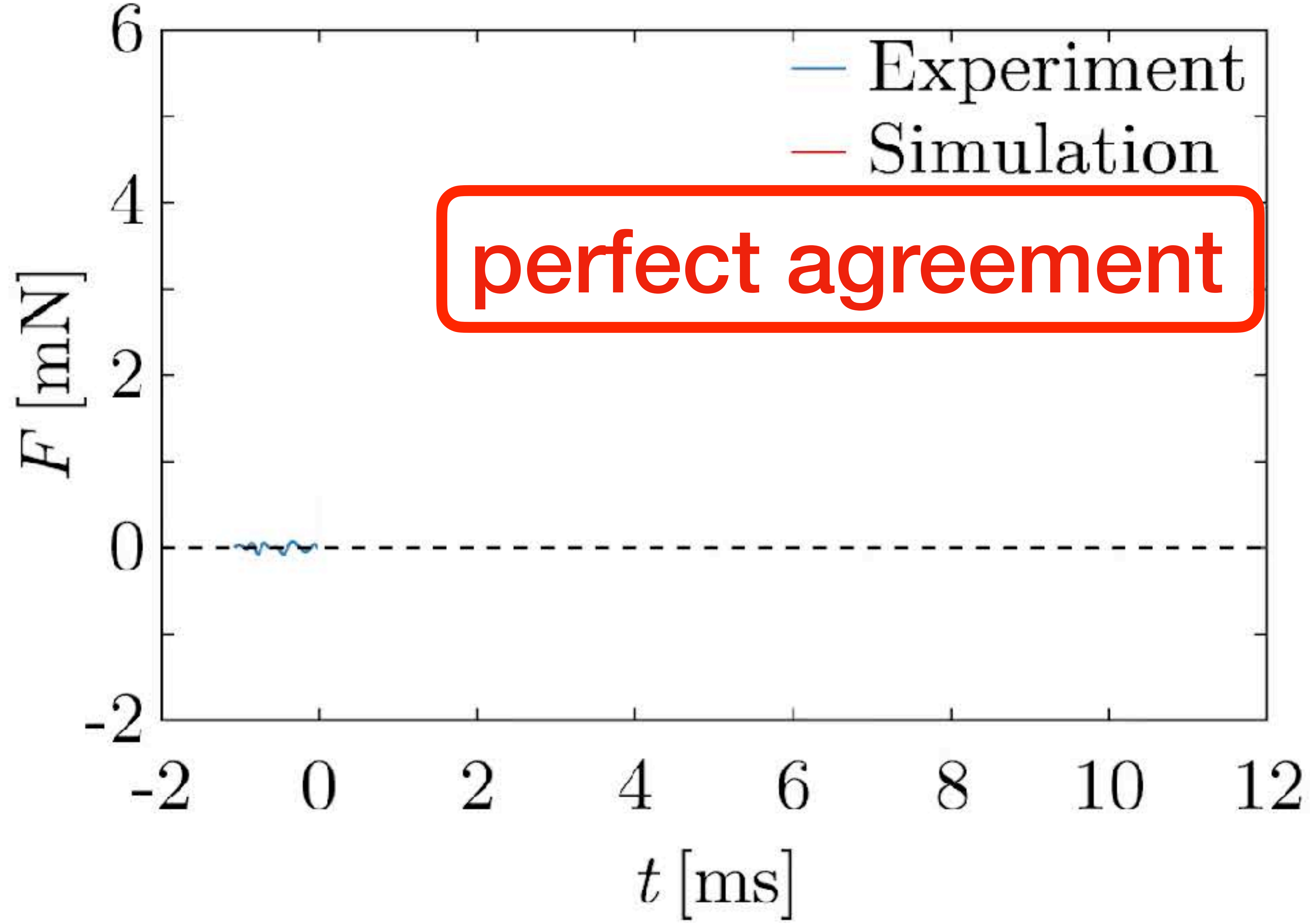
$$F \sim \left(G R_0^2\right)^{2/5} \left(\rho V_0^2 R_0^2\right)^{3/5}$$

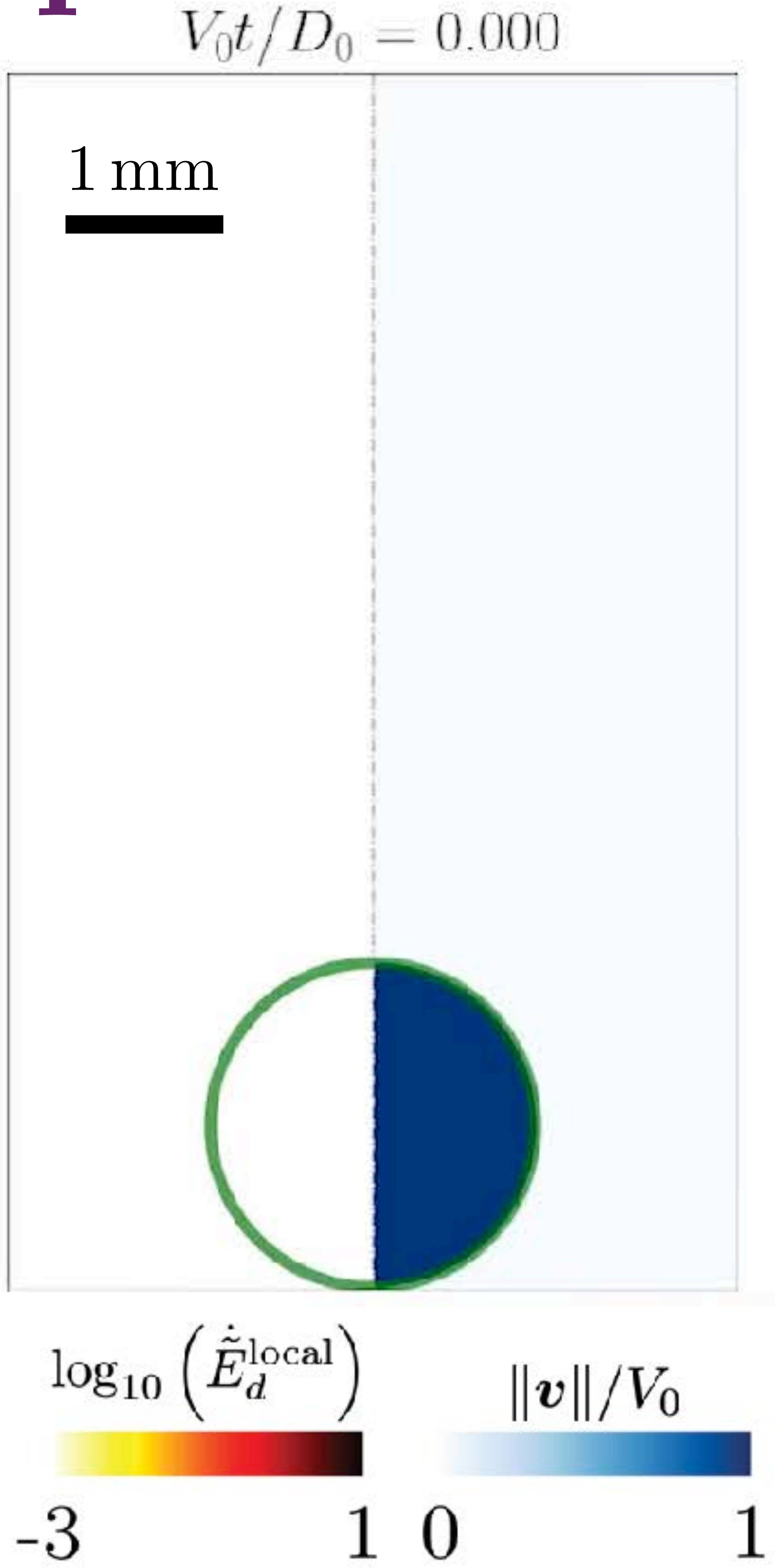
Solid impacts

Let us look at the liquid impacts

Typical result for **impacting drop**





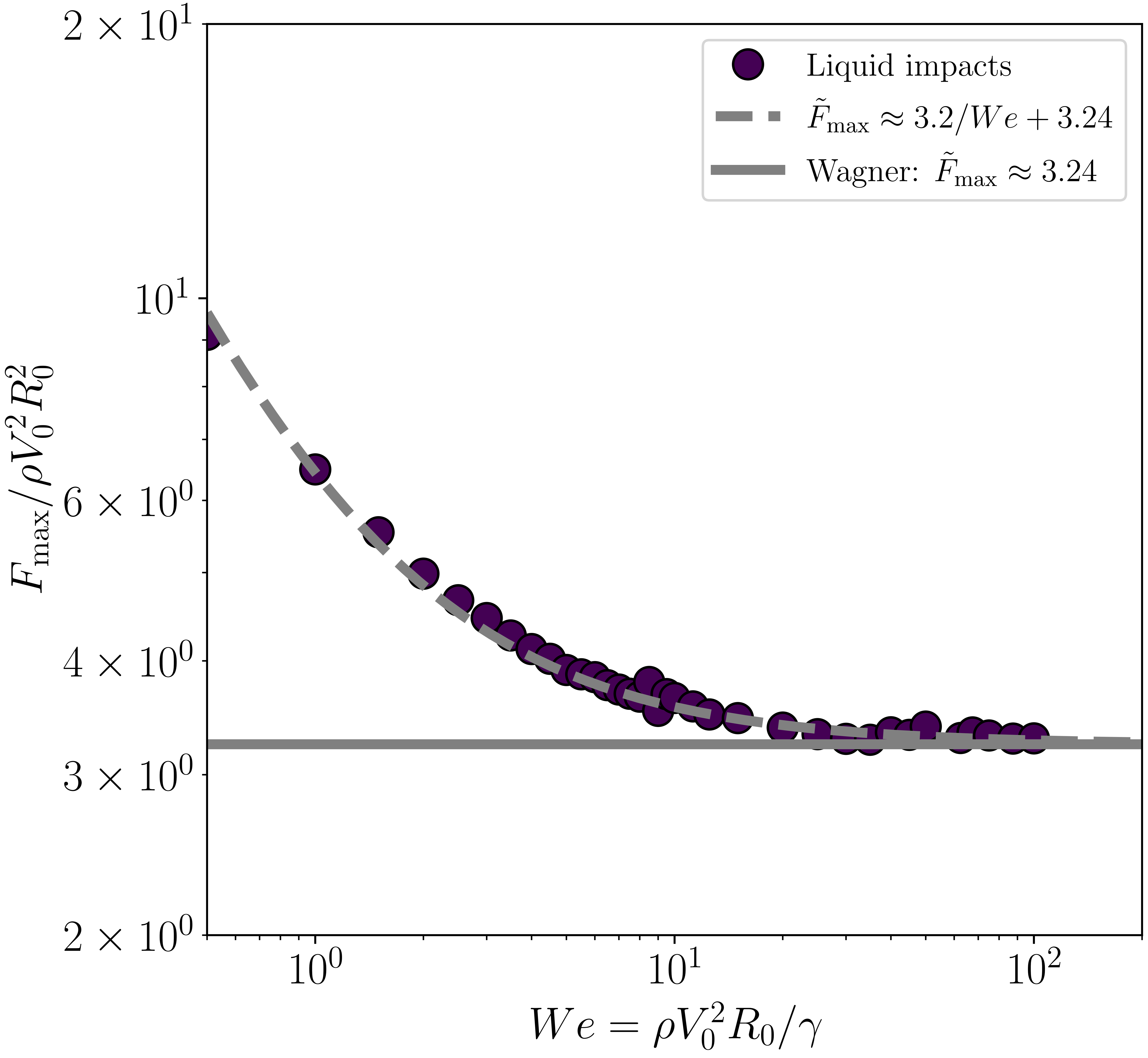


$$Oh = \frac{\eta}{\sqrt{\rho\gamma D_0}} = 0.0025$$

$$We = \frac{\rho V_0^2 D_0}{\gamma} = 40$$

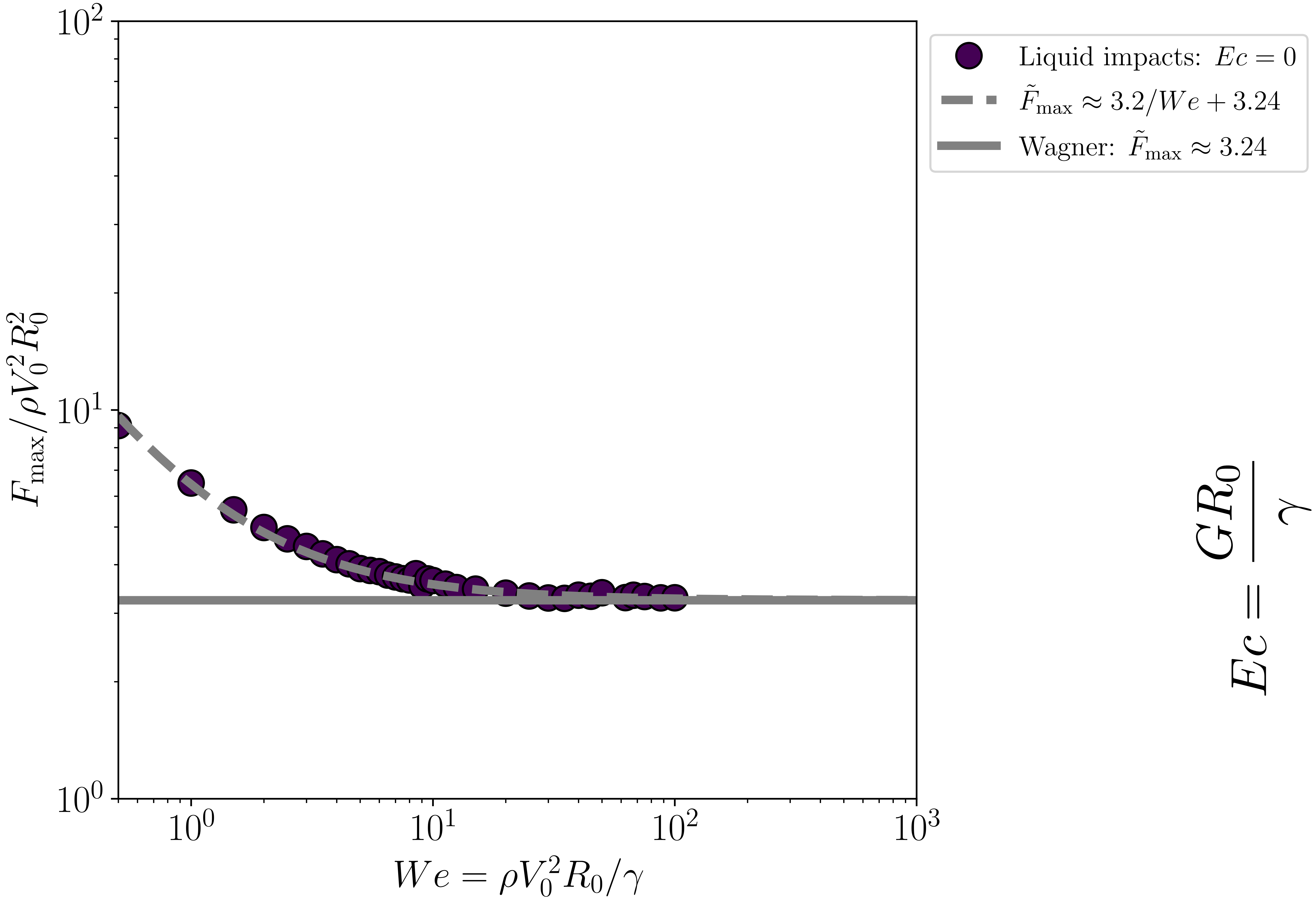
Zhang, Sanjay *et al.*, Phys. Rev. Lett., 129, 104501 (2022)

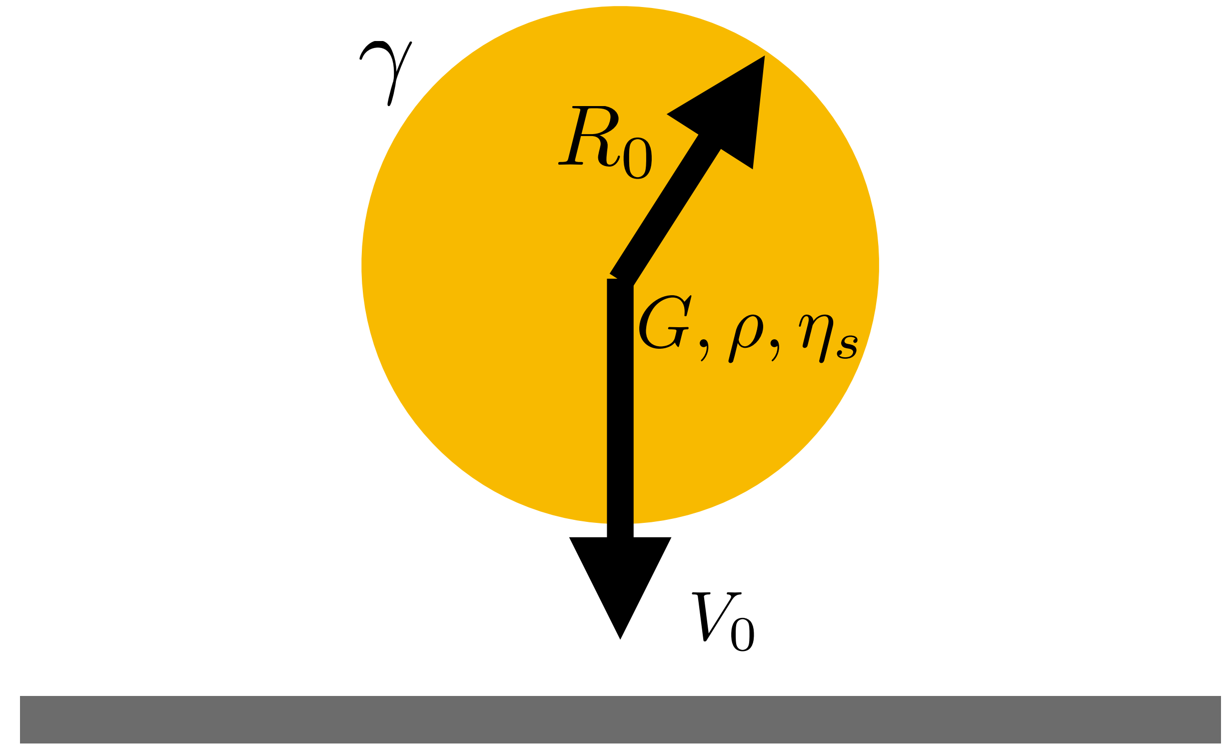
Summary of liquid impacts

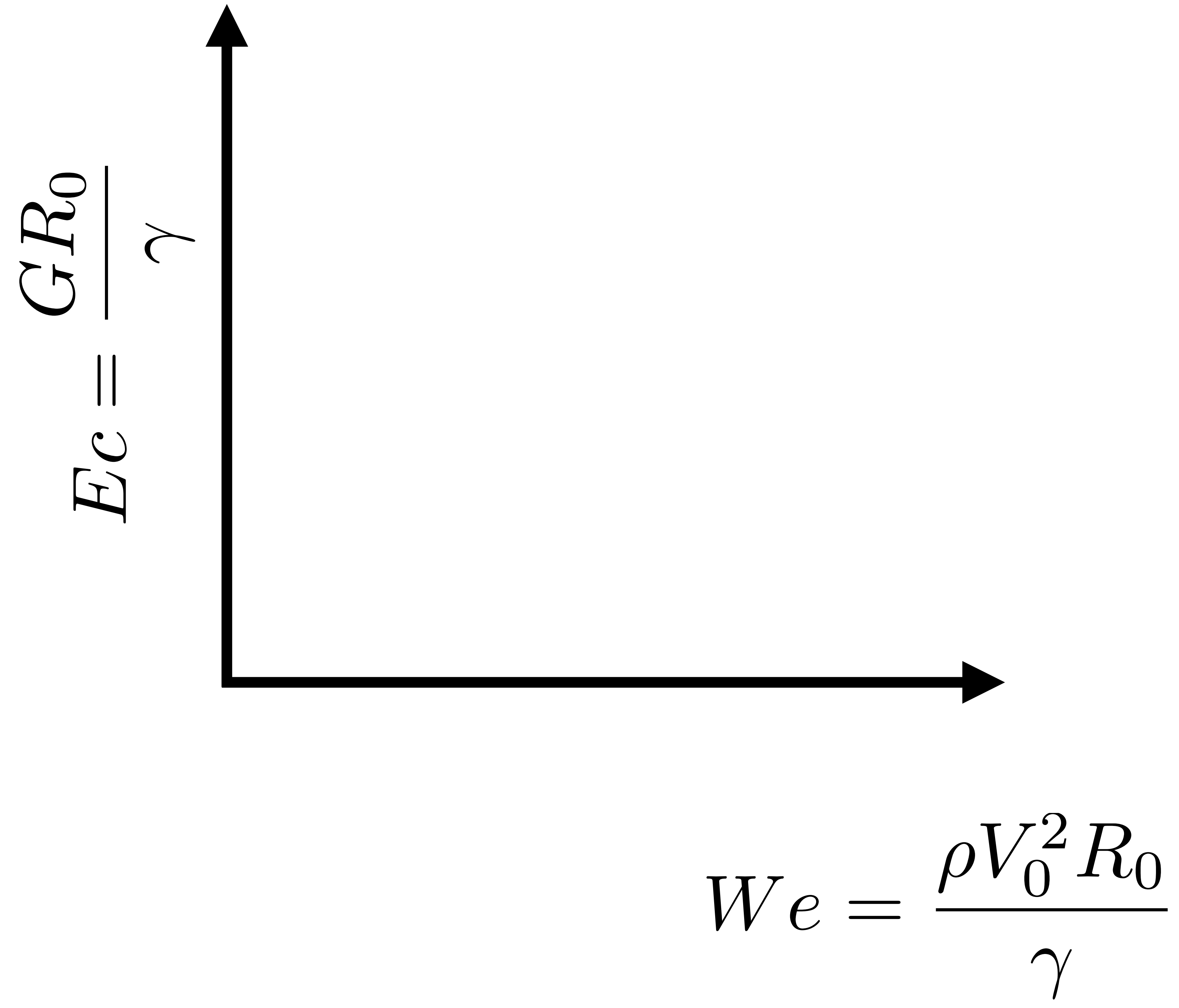


$$Oh = \frac{\eta_s}{\sqrt{\rho \gamma R_0}} = 0.01$$

From liquids to soft solids

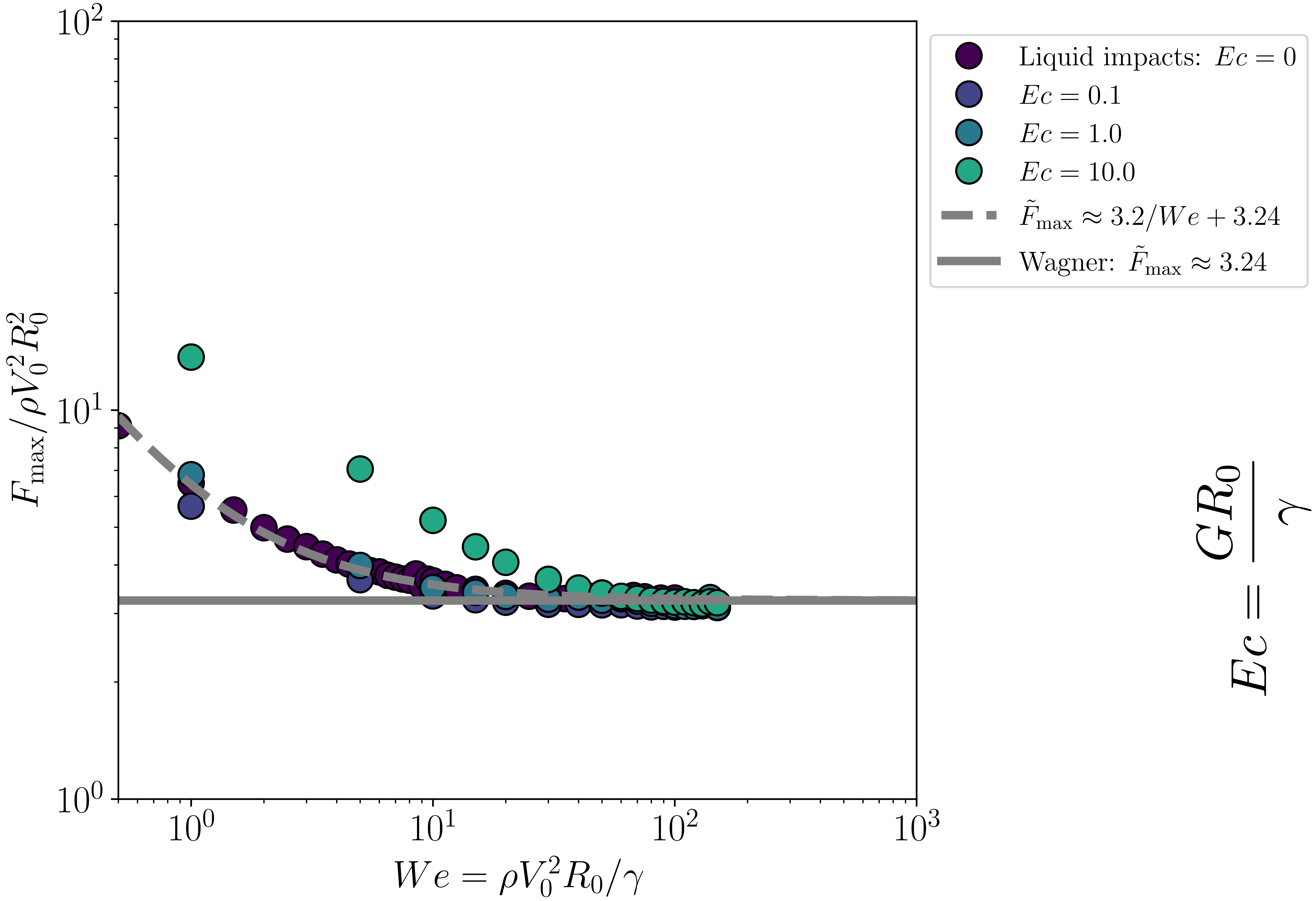


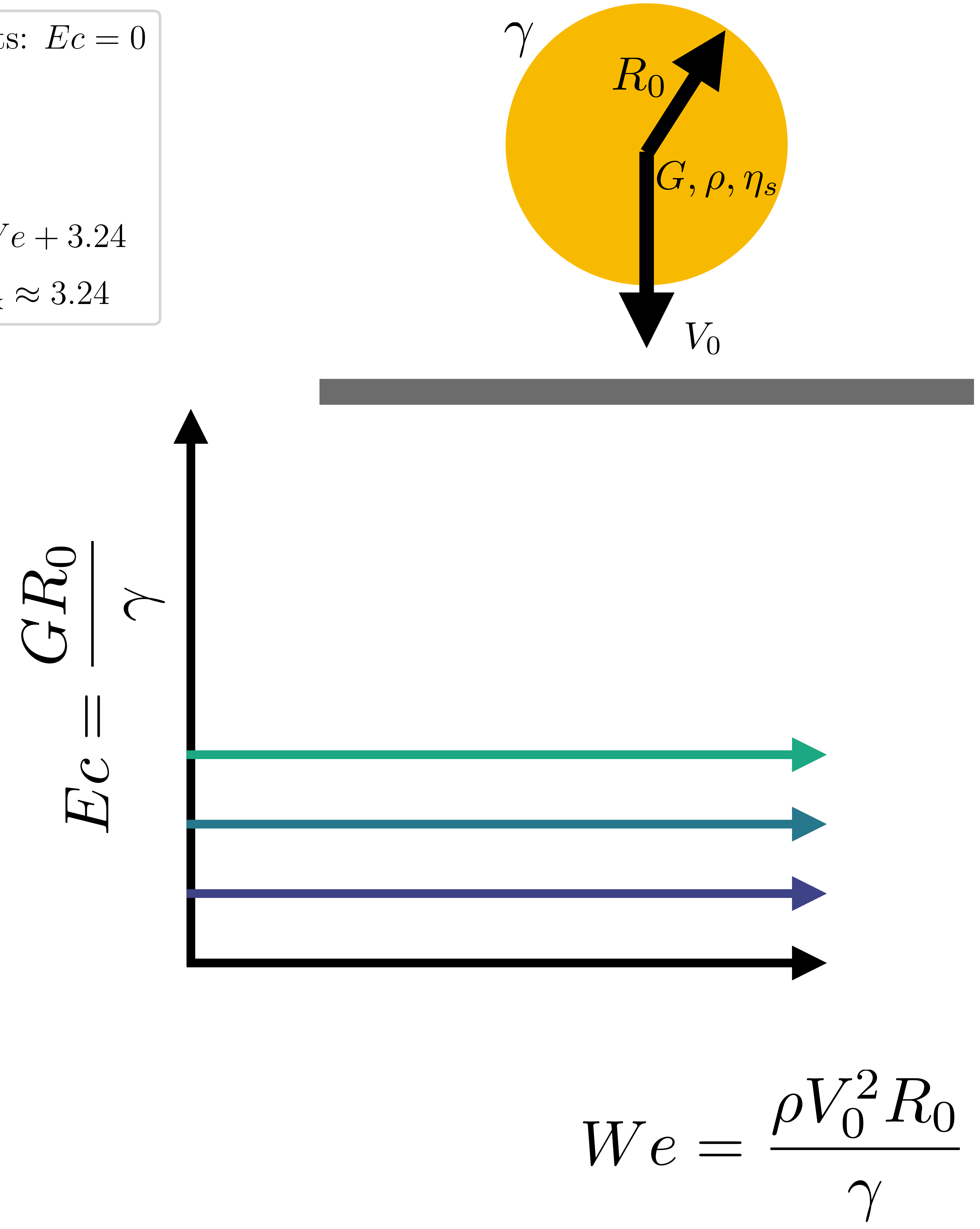




$$Oh = \frac{\eta_s}{\sqrt{\rho\gamma R_0}} = 0.01$$

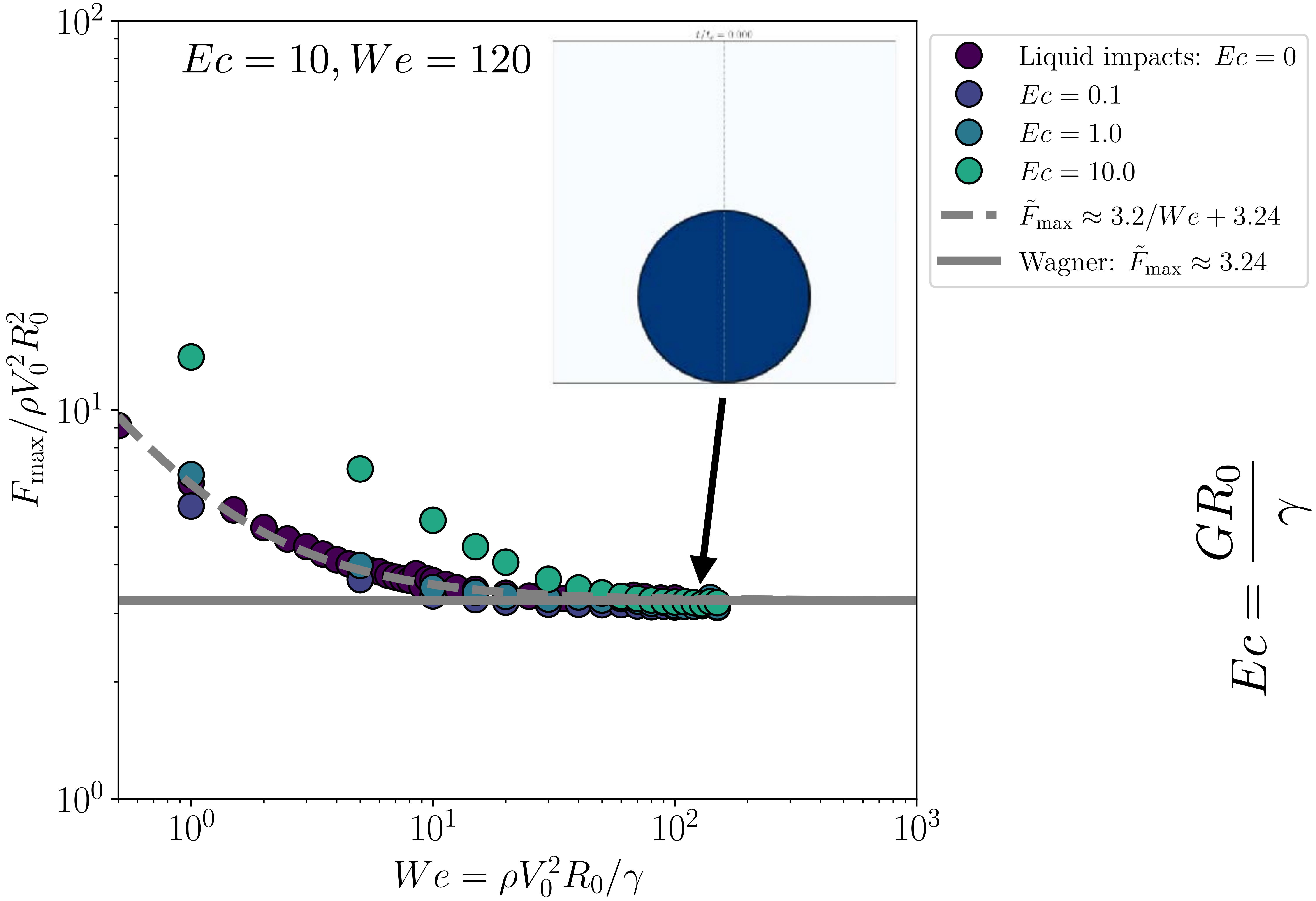
From liquids to soft solids

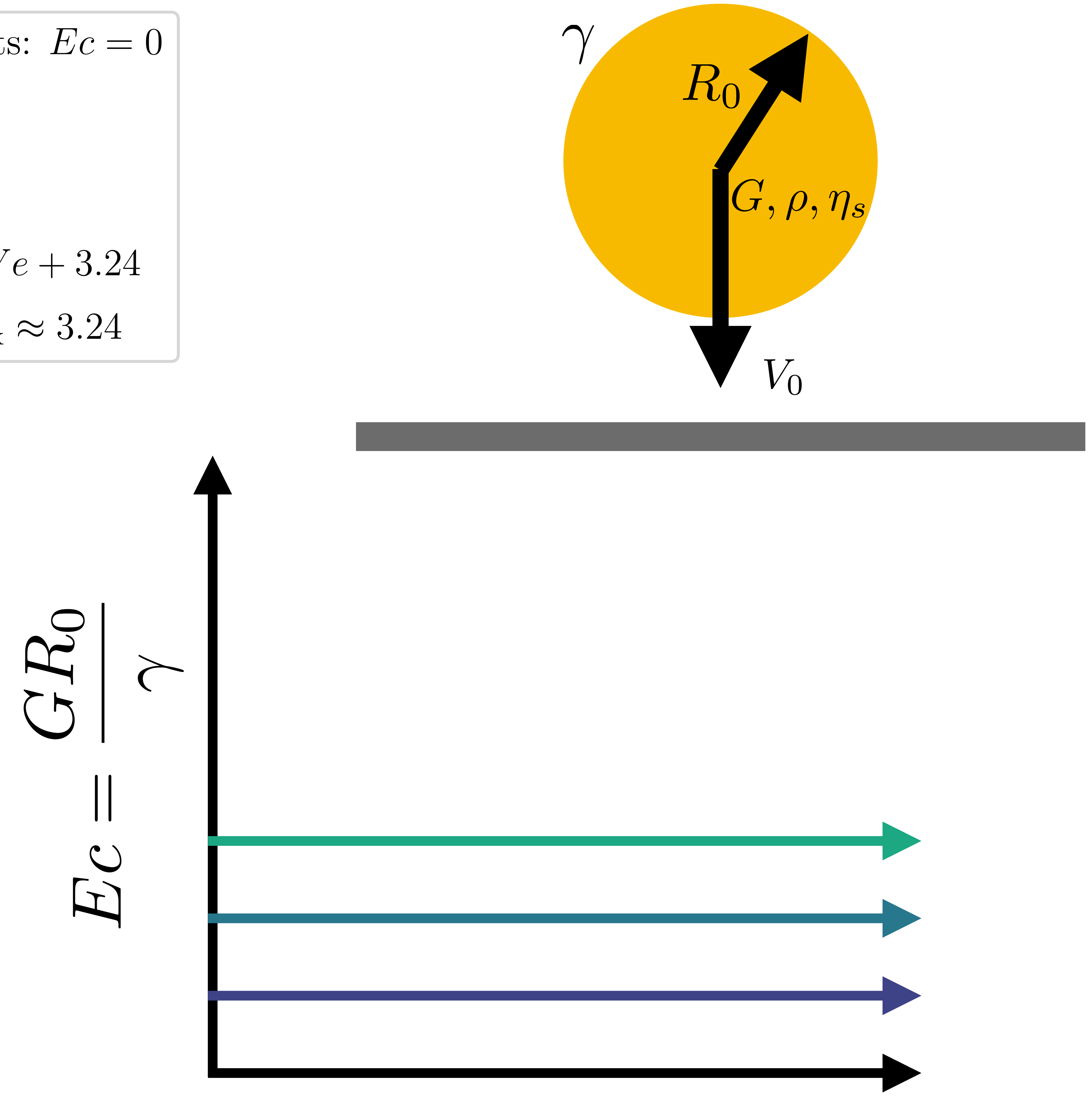




$$Oh = \frac{\eta_s}{\sqrt{\rho \gamma R_0}} = 0.01$$

From liquids to soft solids

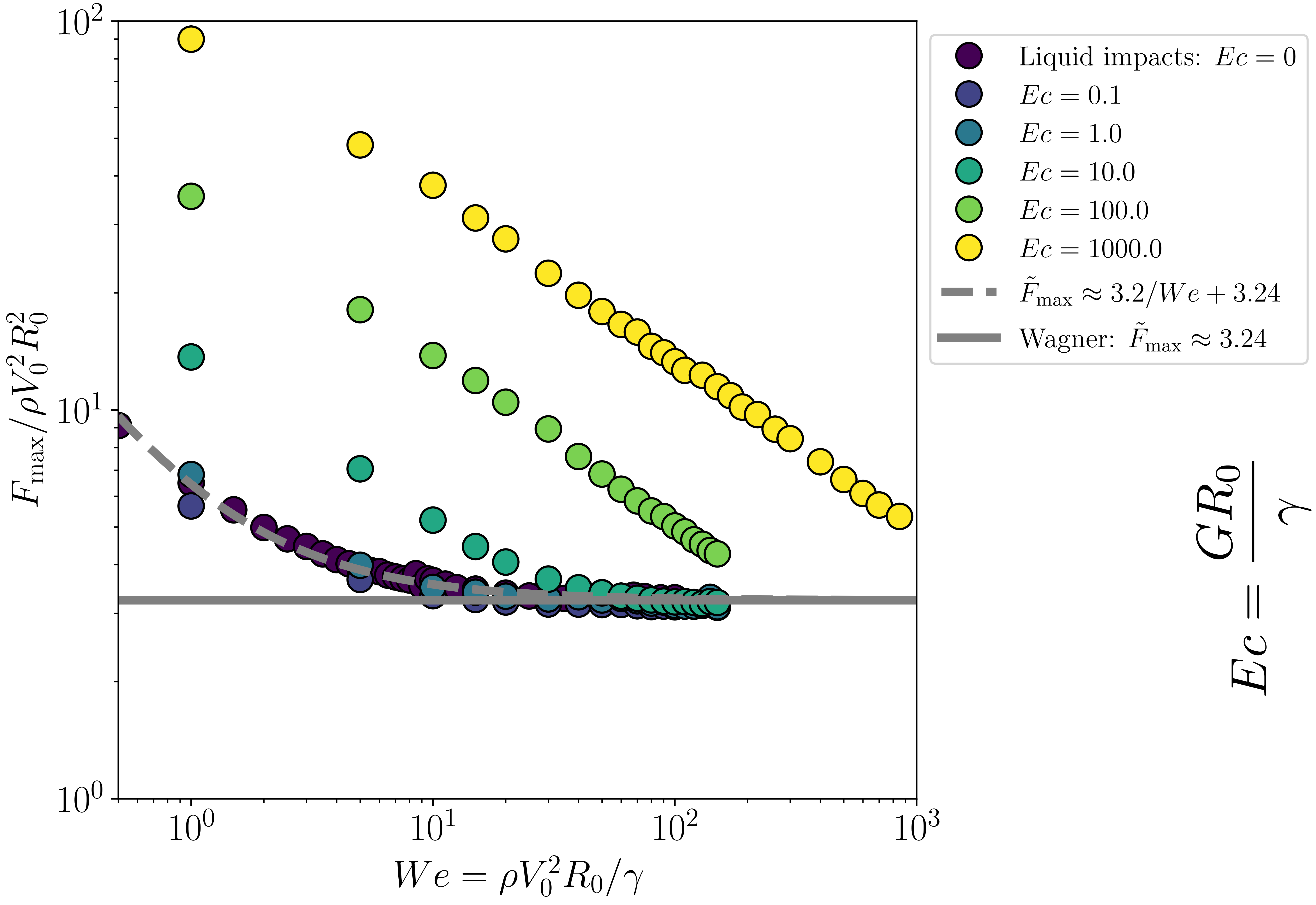


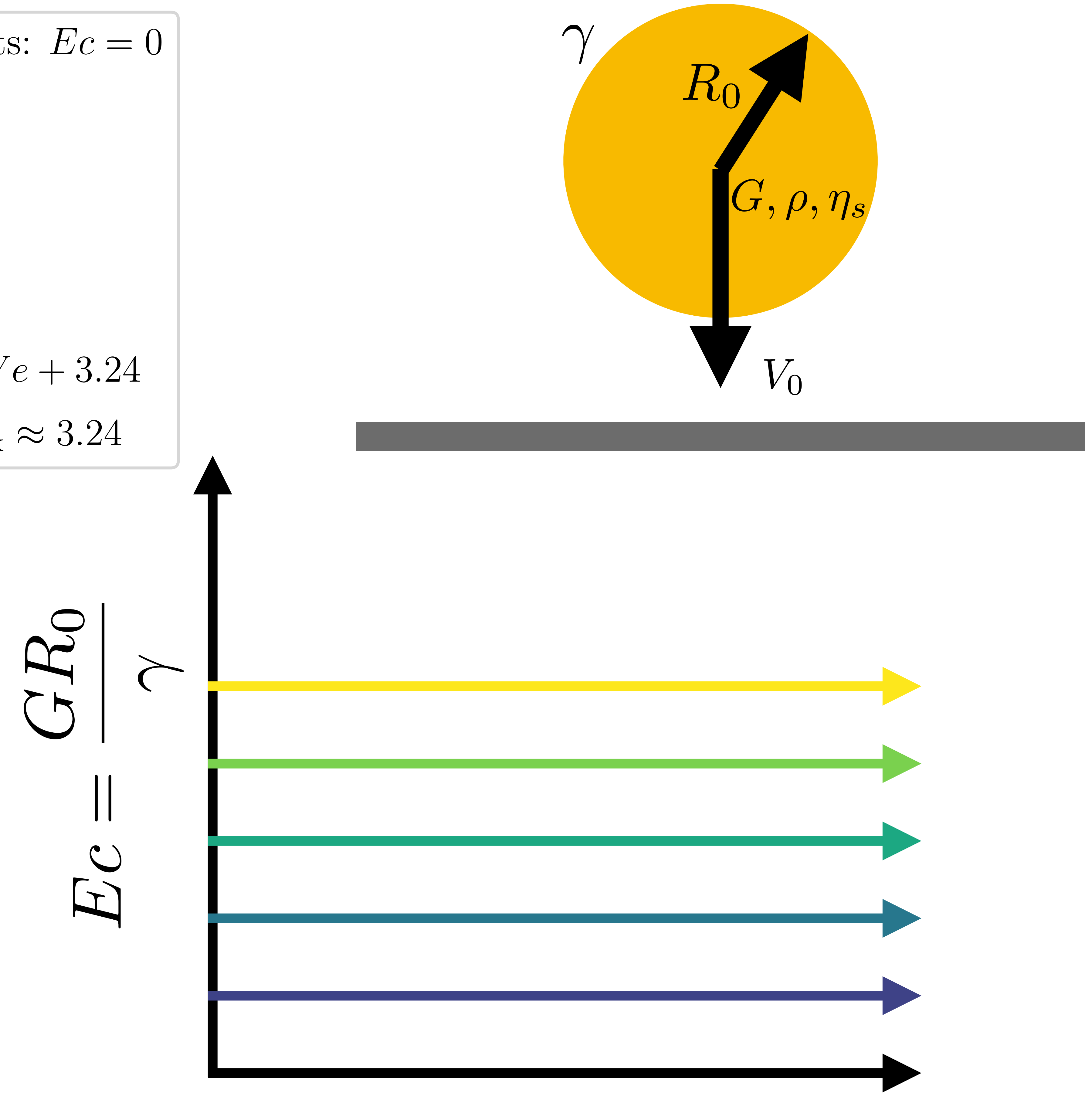


$$We = \frac{\rho V_0^2 R_0}{\gamma}$$

$$Oh = \frac{\eta_s}{\sqrt{\rho\gamma R_0}} = 0.01$$

From liquids to soft solids





$$We = \frac{\rho V_0^2 R_0}{\gamma}$$

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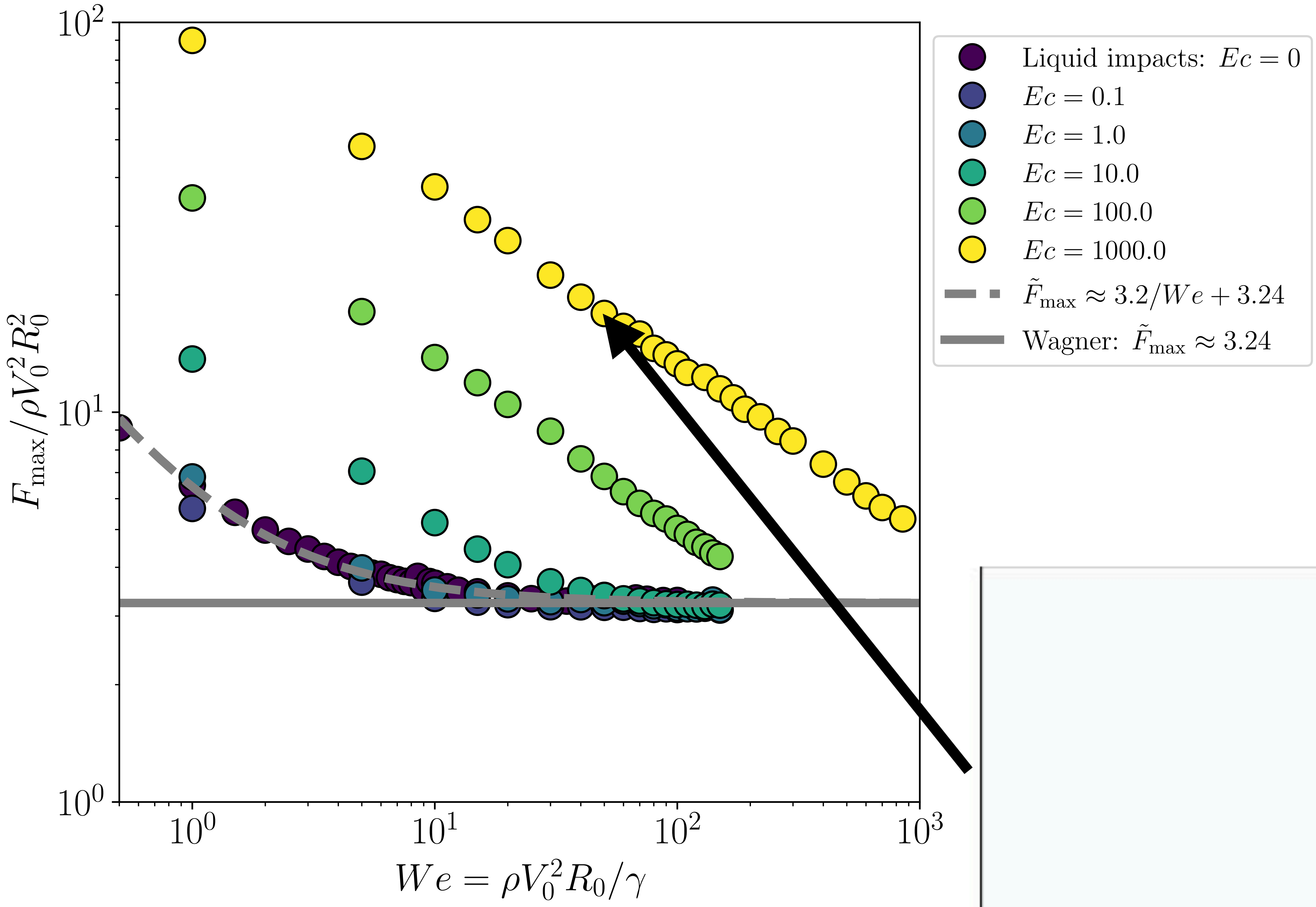
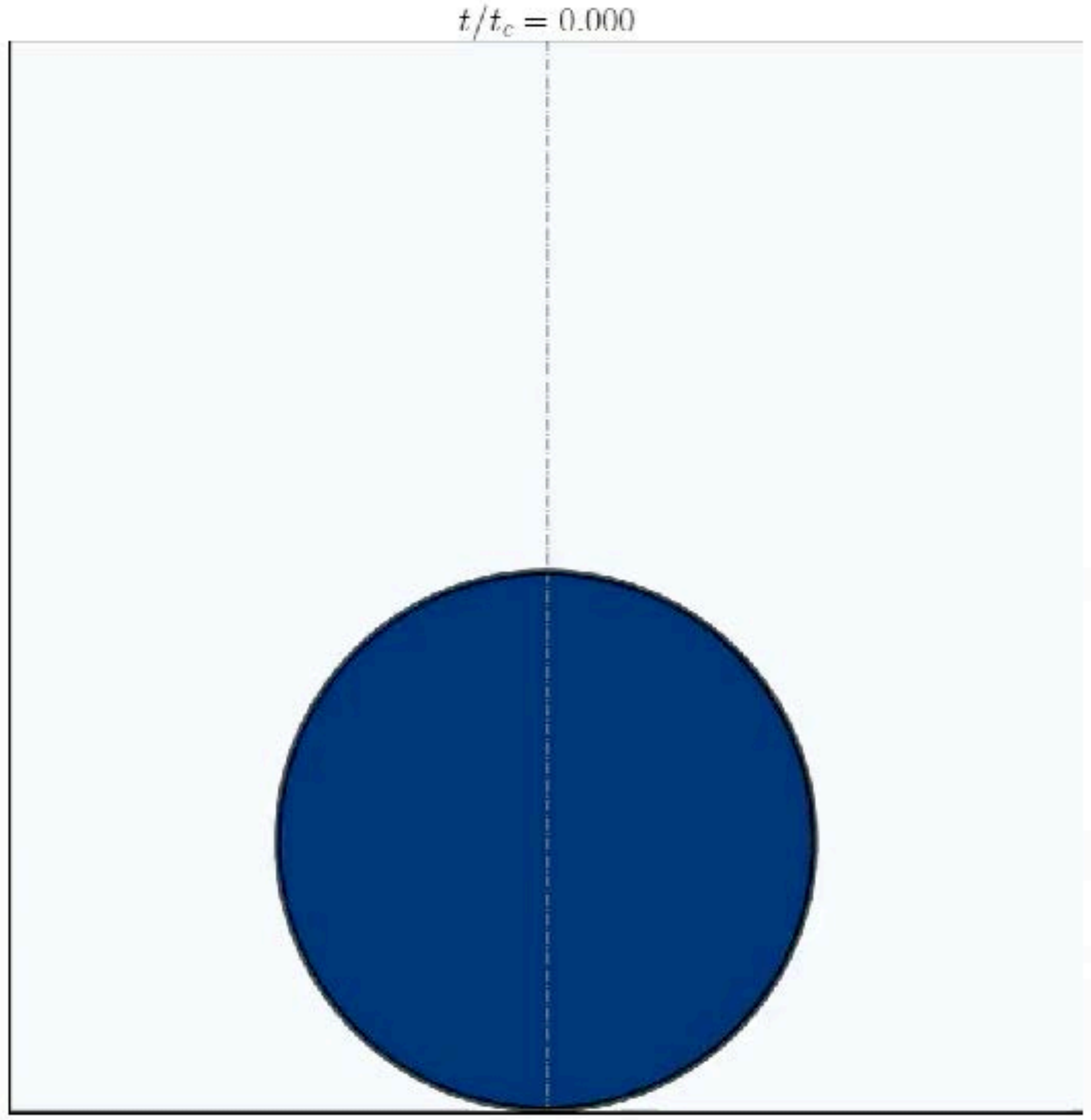
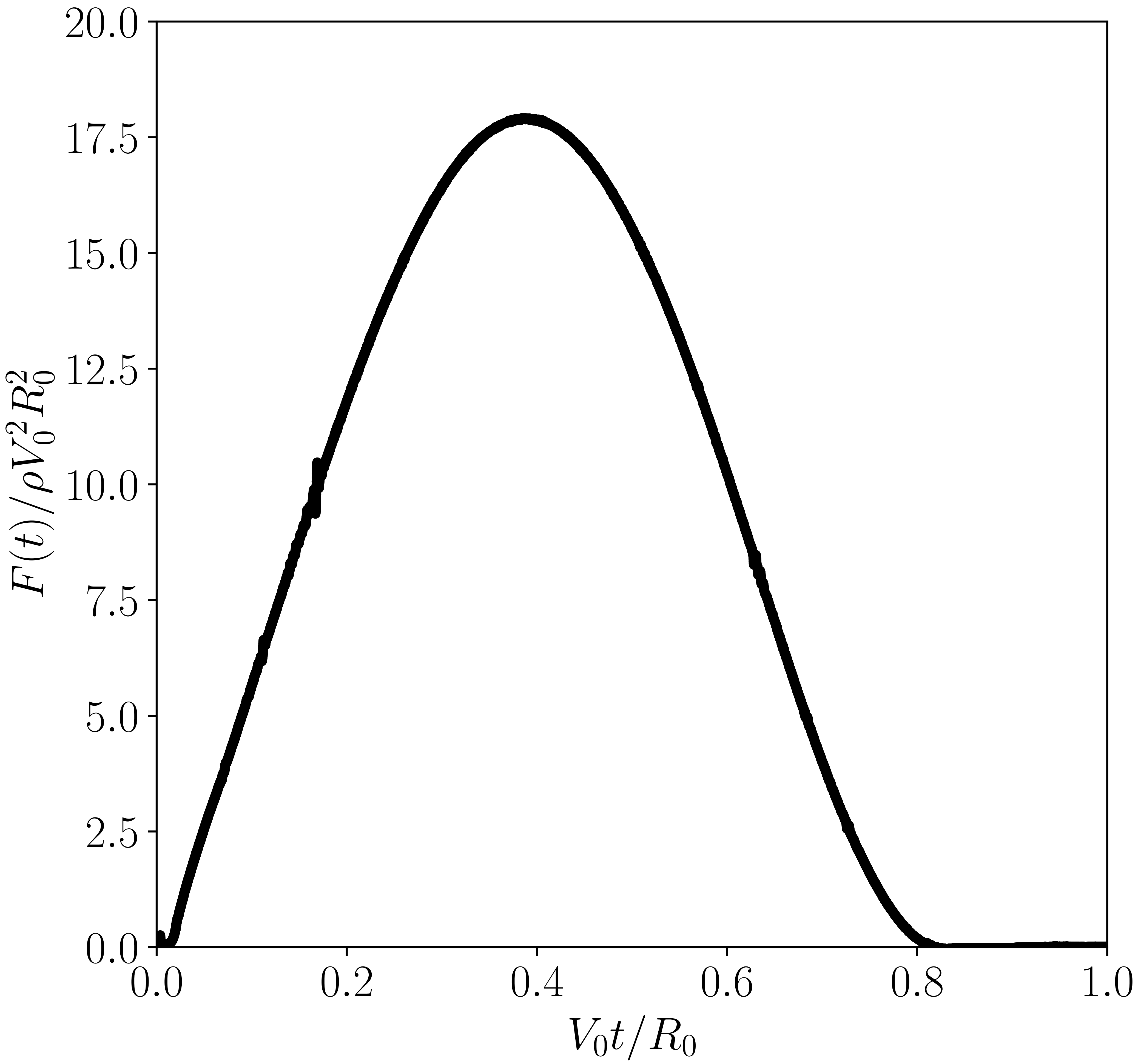
From liquids to soft solids





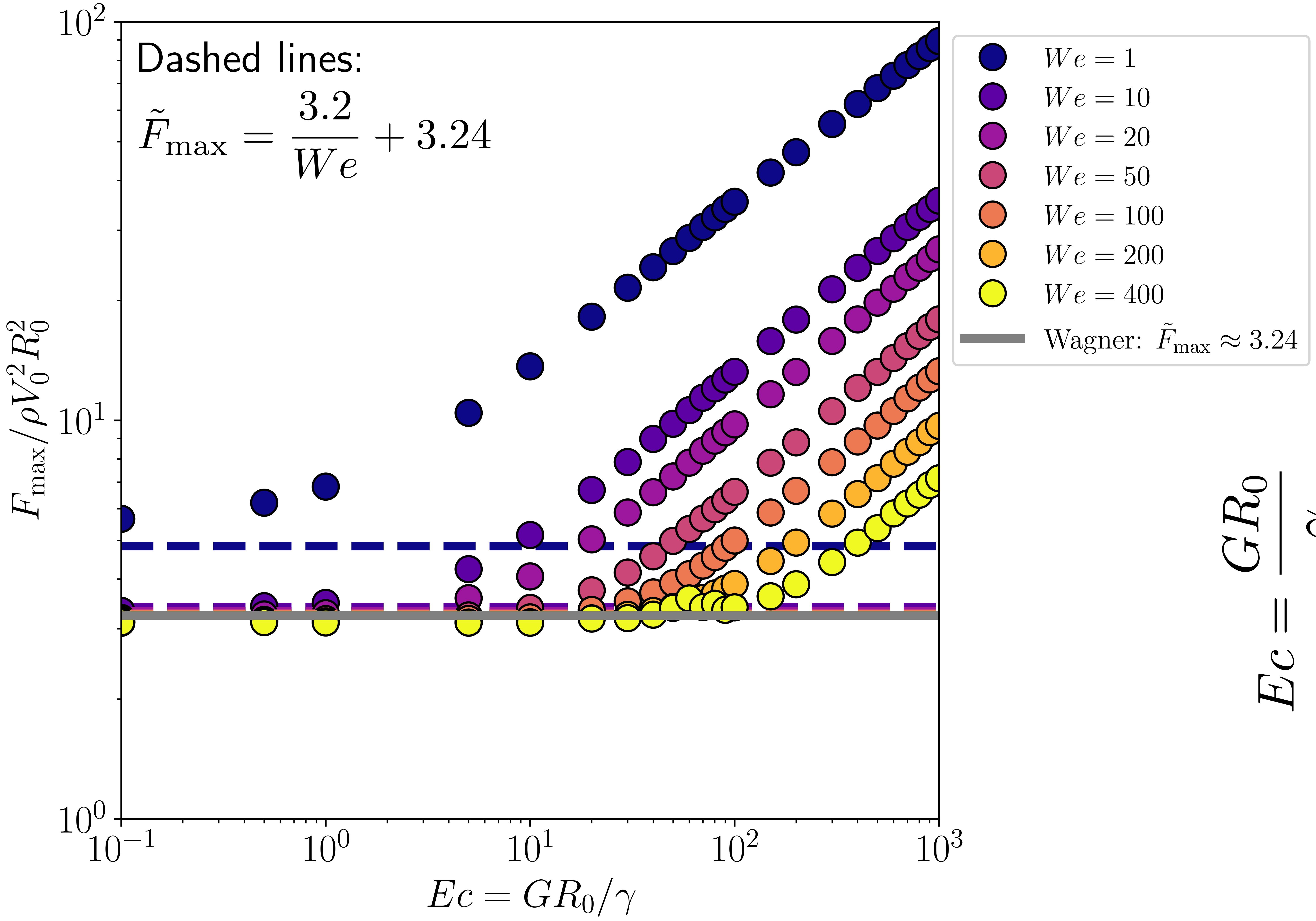


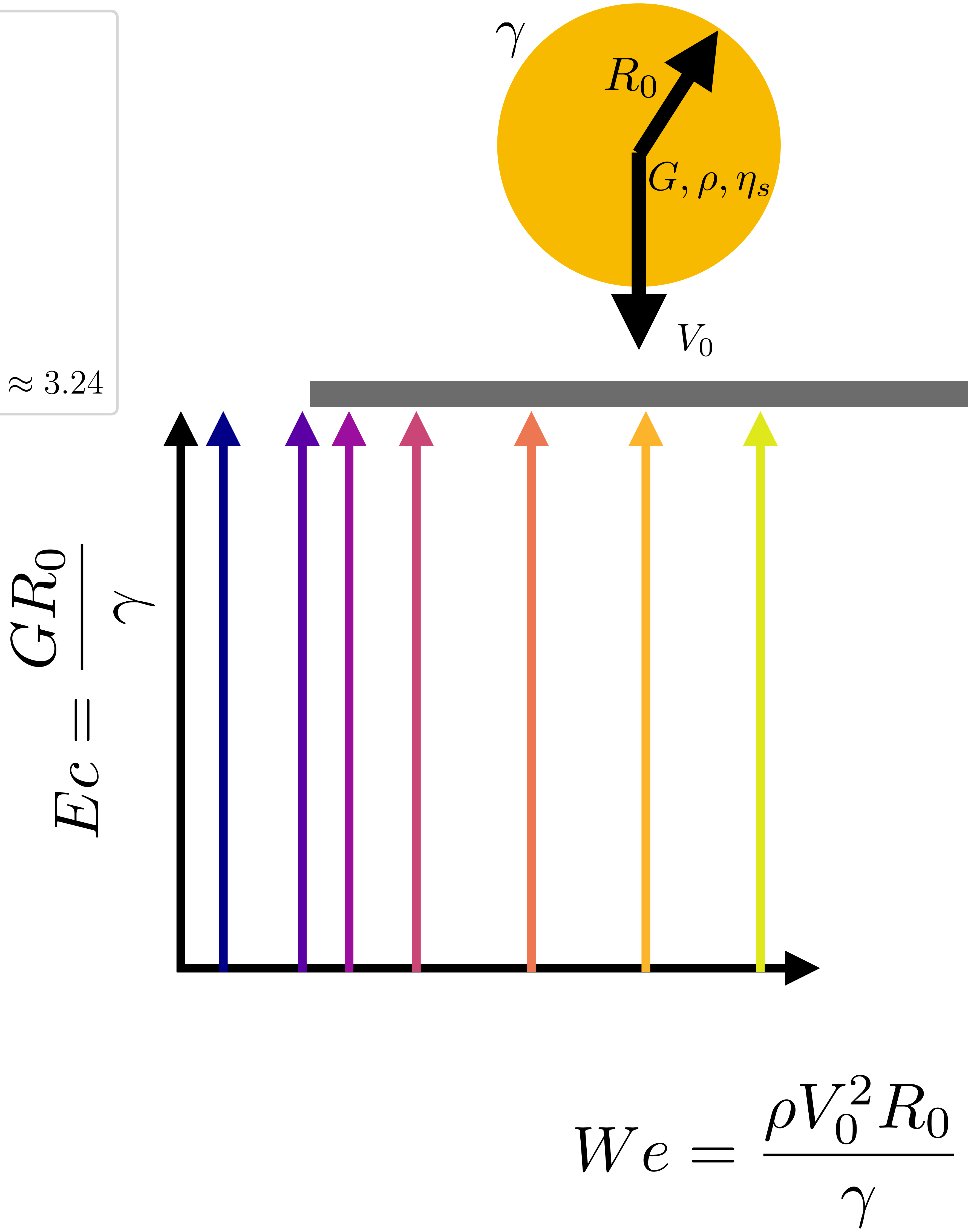
$$Ec = \frac{G R_0}{\gamma} = 1000$$

$$We = \frac{\rho V_0^2 R_0}{\gamma} = 50$$

$$Oh = \frac{\eta_s}{\sqrt{\rho\gamma R_0}} = 0.01$$

From liquids to soft solids





Can we explain this dependance?

Wagner

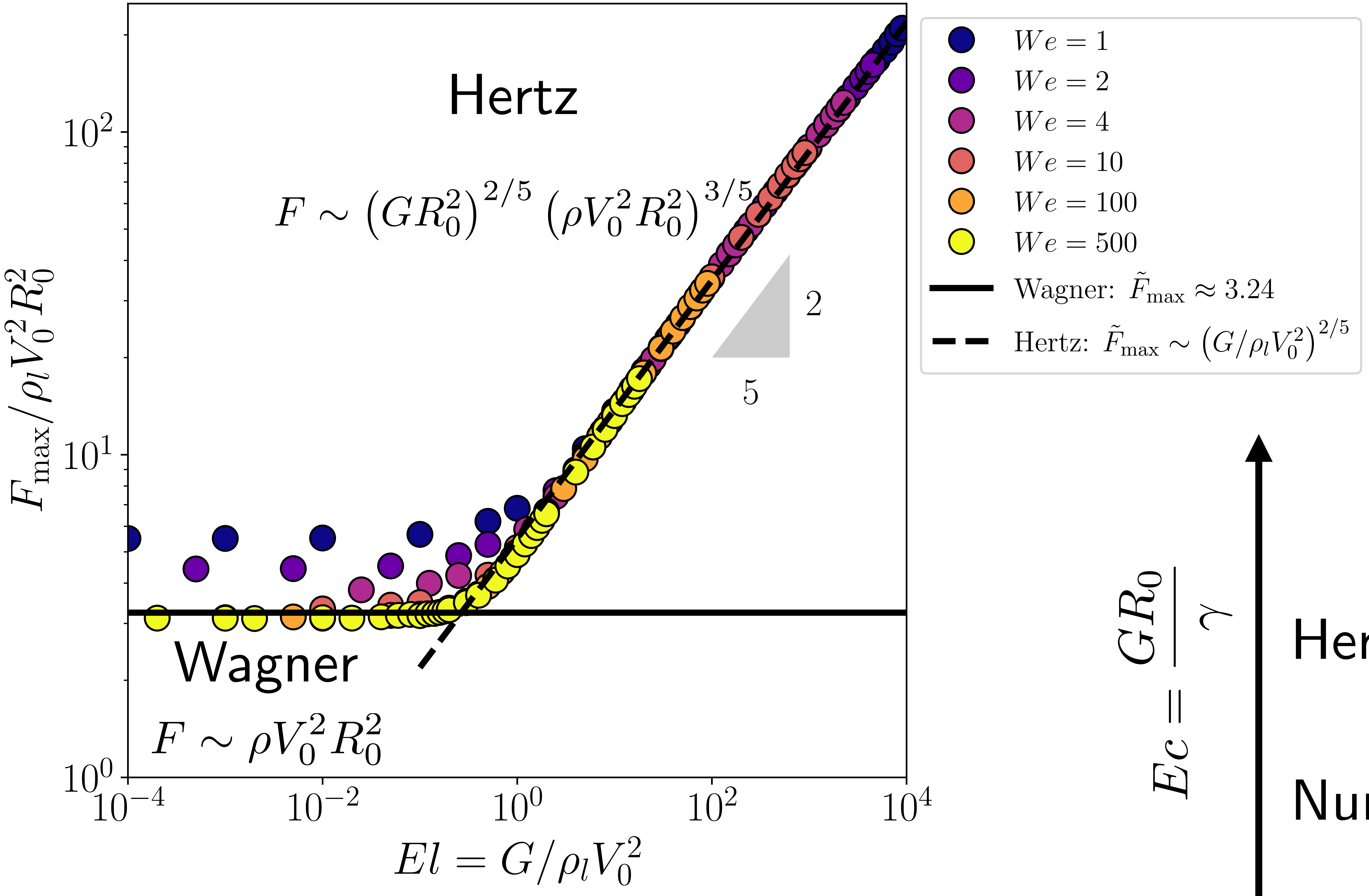
$$F \sim \rho V_0^2 R_0^2$$

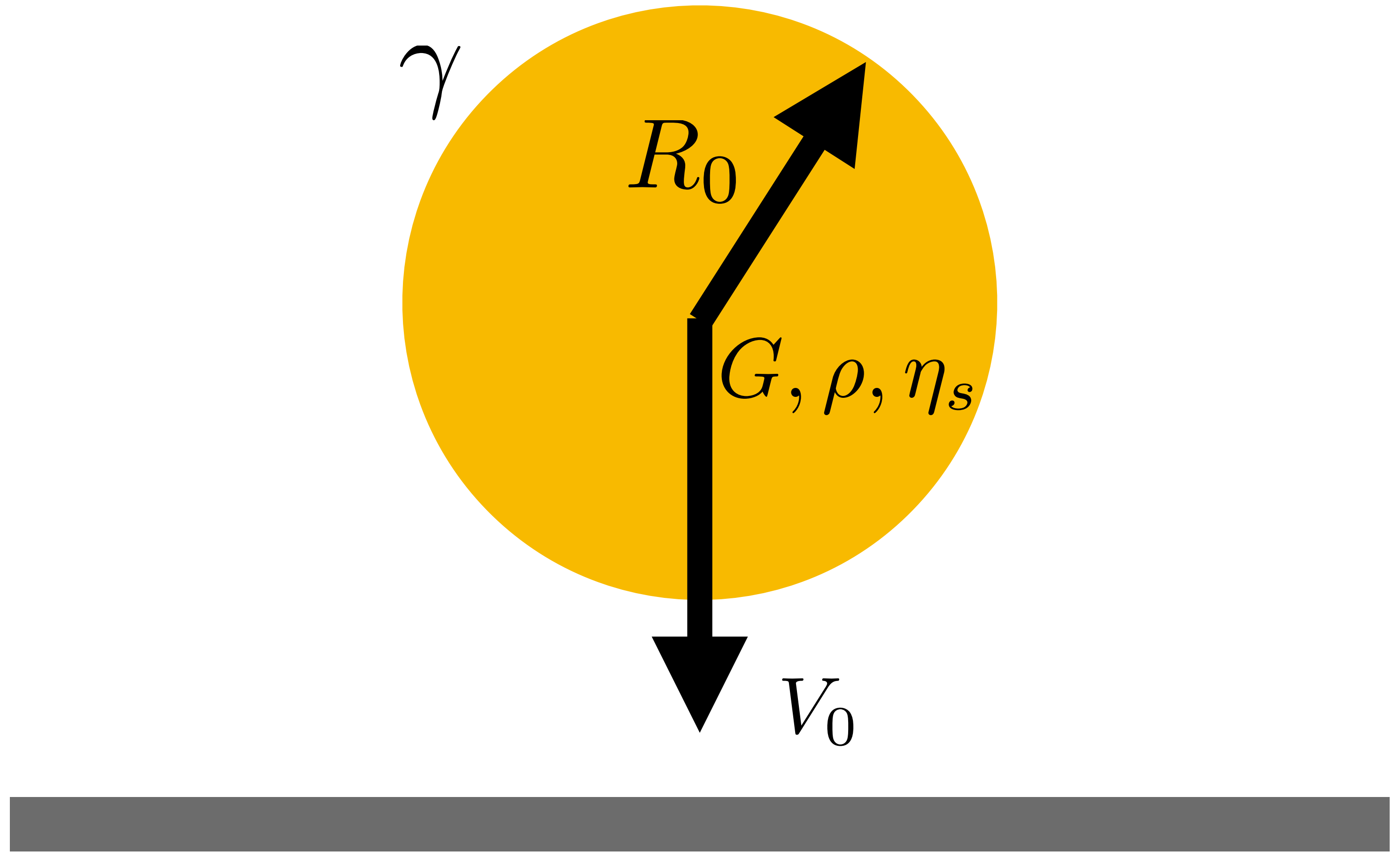
Hertz

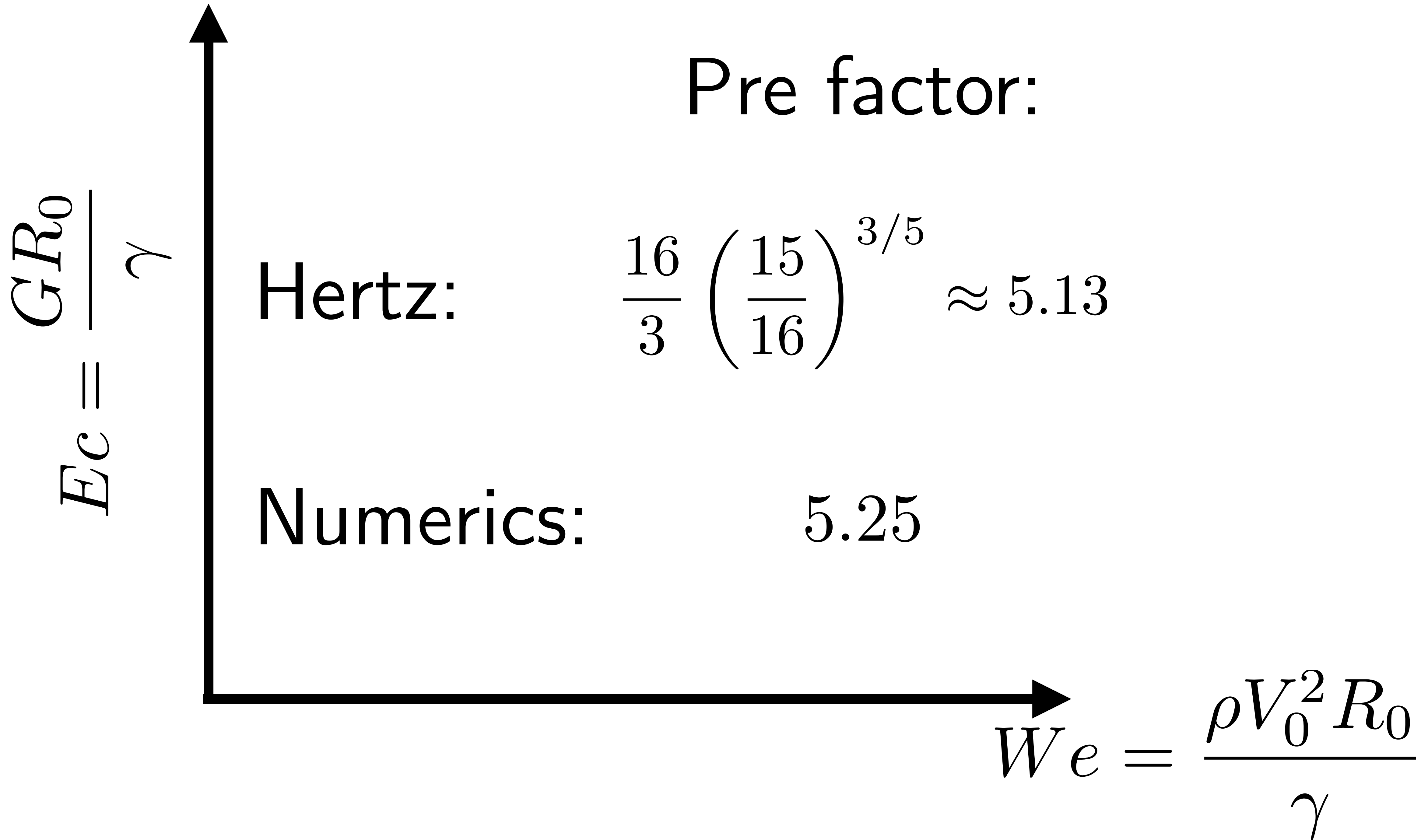
$$F \sim \left(GR_0^2\right)^{2/5} \left(\rho V_0^2 R_0^2\right)^{3/5}$$

$$Oh = \frac{\eta_s}{\sqrt{\rho \gamma R_0}} = 0.01$$

Summary: Wagner vs Hertz





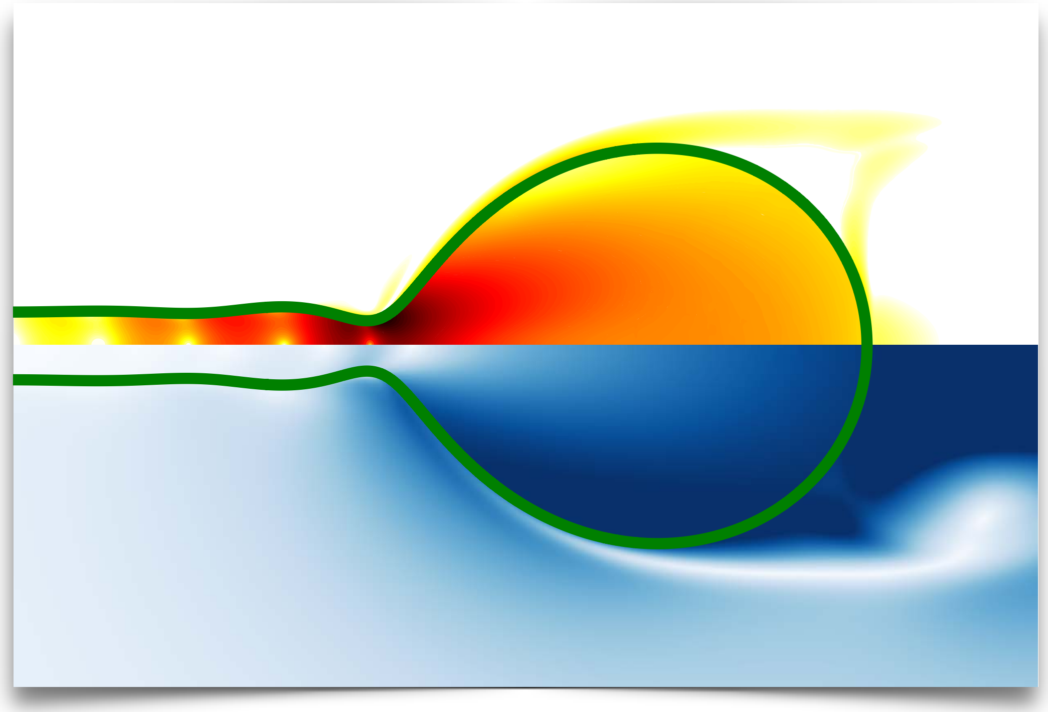


Pre factor:

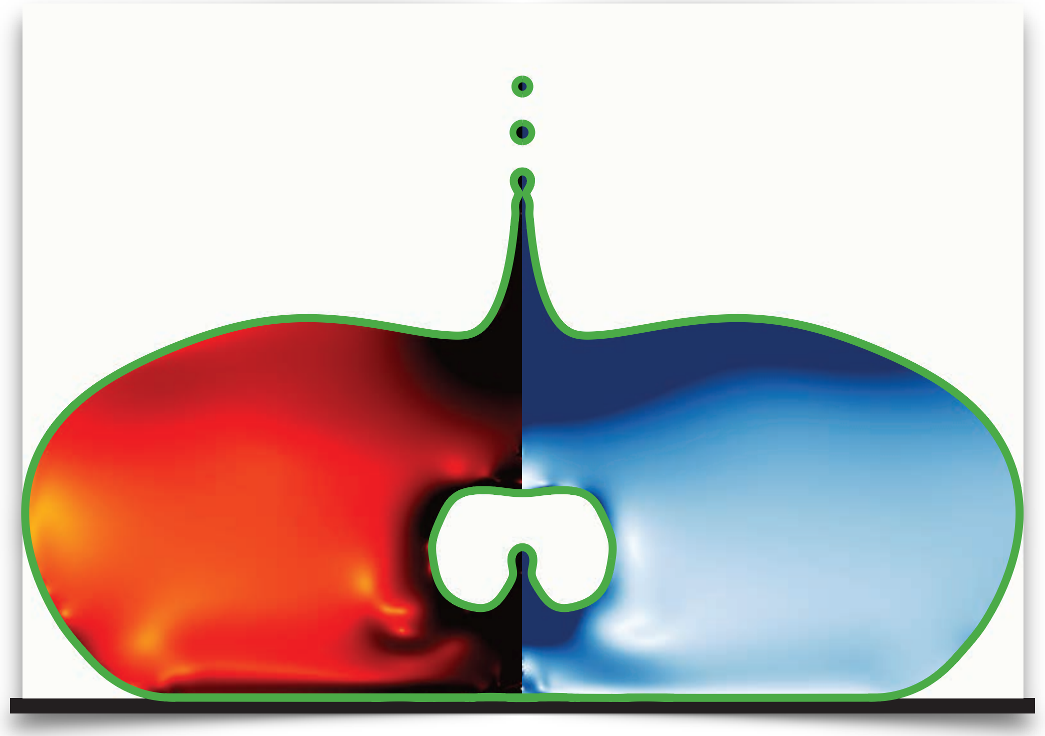
Hertz: $\frac{16}{3}\left(\frac{15}{16}\right)^{3/5} \approx 5.13$

Numerics: 5.25

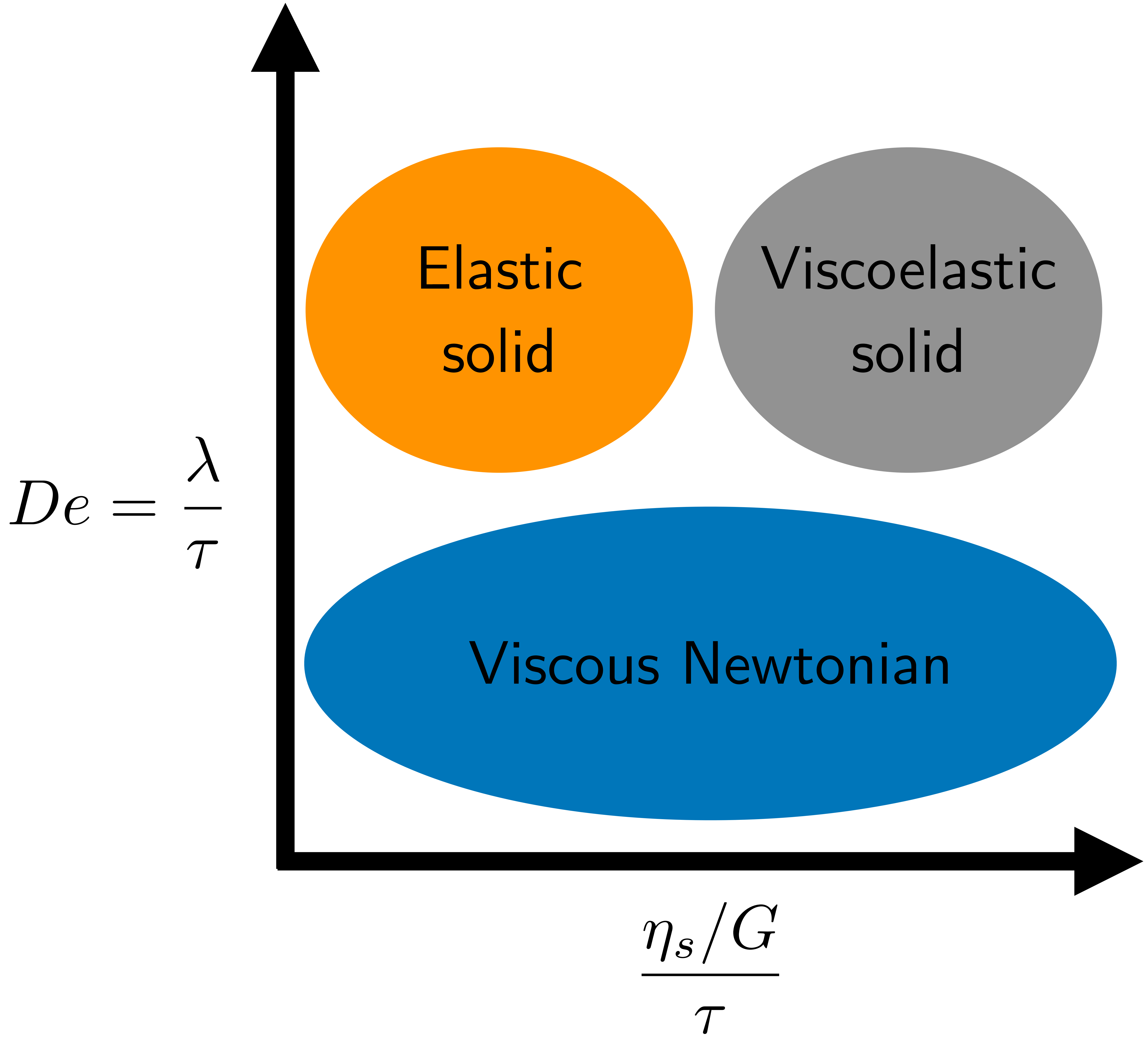
Today's summary



1. Sheets



2. Drops



One more thing ...

Computational **M**ultiphase **Phy**sics Lab

CoMPhy Lab

Durham University

Physics of Fluids

Computational Multiphase Physics Lab

Theory:
- Scaling analysis
- Reduced order models

High-fidelity numerical codes
- **Basilisk C** framework
- Extended to non-Newtonian liquids

CoMPhy Lab

Active experimental collaborations
See: comphy-lab.org/team

Partners:

MIST ASML Canon
KRÜSS

Durham University Physics of Fluids

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CoMPhy Lab

Theory:
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- Reduced order models

Ayush Dixit (PhD student)
- Holey Sheets (1700h)

Aman Bhargava (PhD student)
- Dancing drops (Wed, 1st talk)

Jnan Talukdar (PhD student)
- Singularities with surfactants

Saumili Jana (PhD student)
- Soft Impacts

Computational Multiphase Physics Lab

High-fidelity numerical codes

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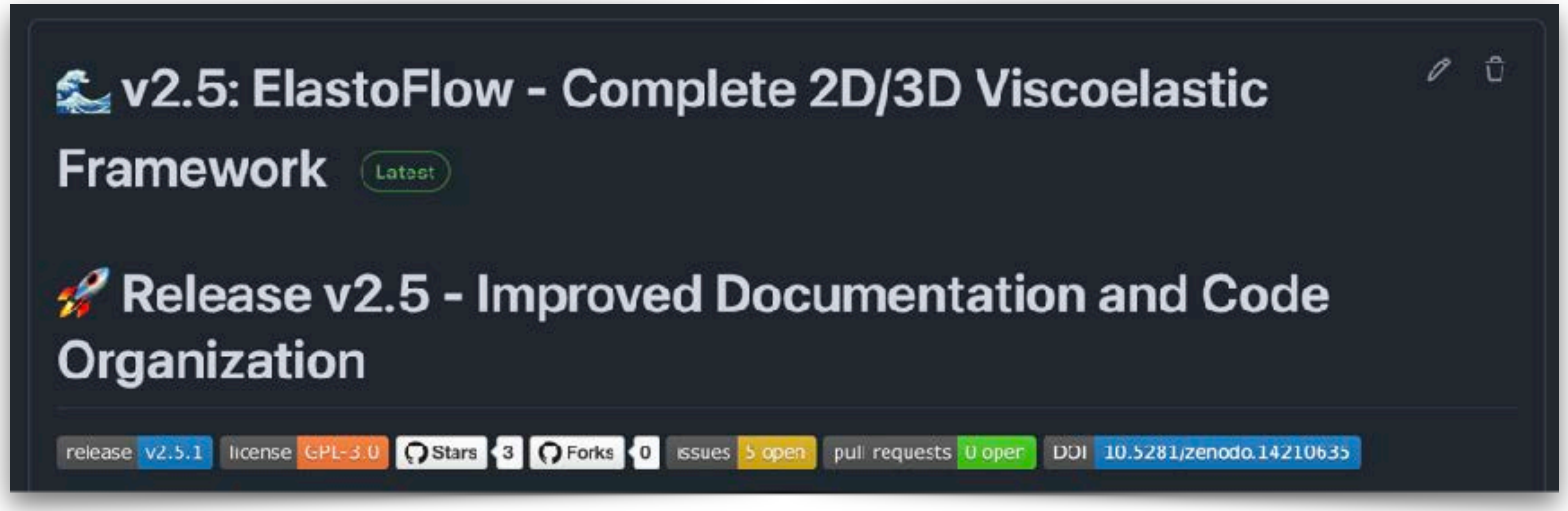
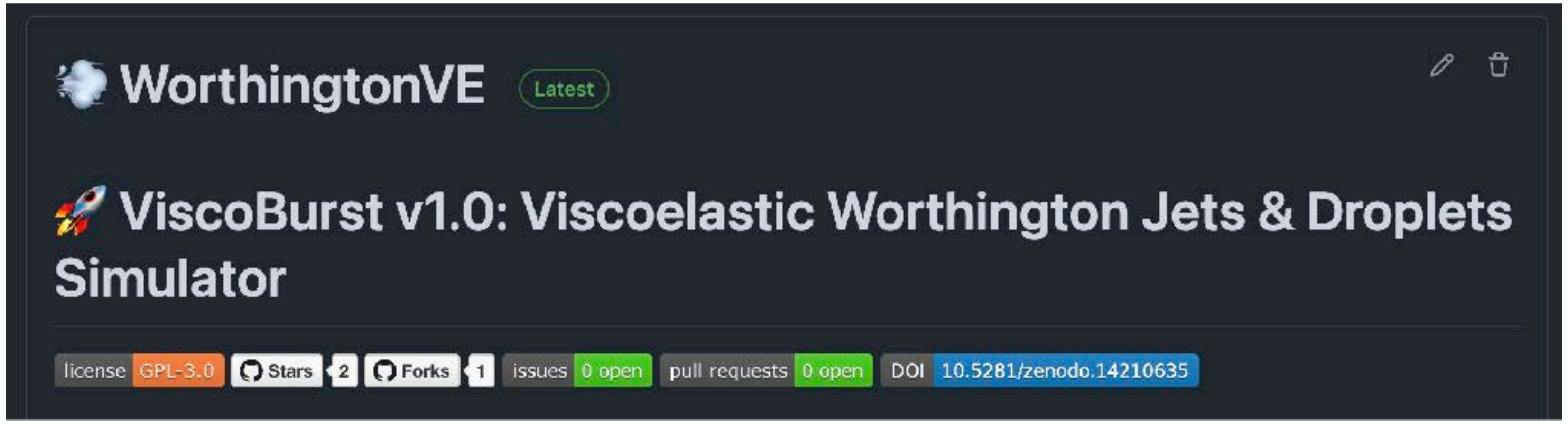
Open source codes are the way to go





Computational Multiphase Physics Lab

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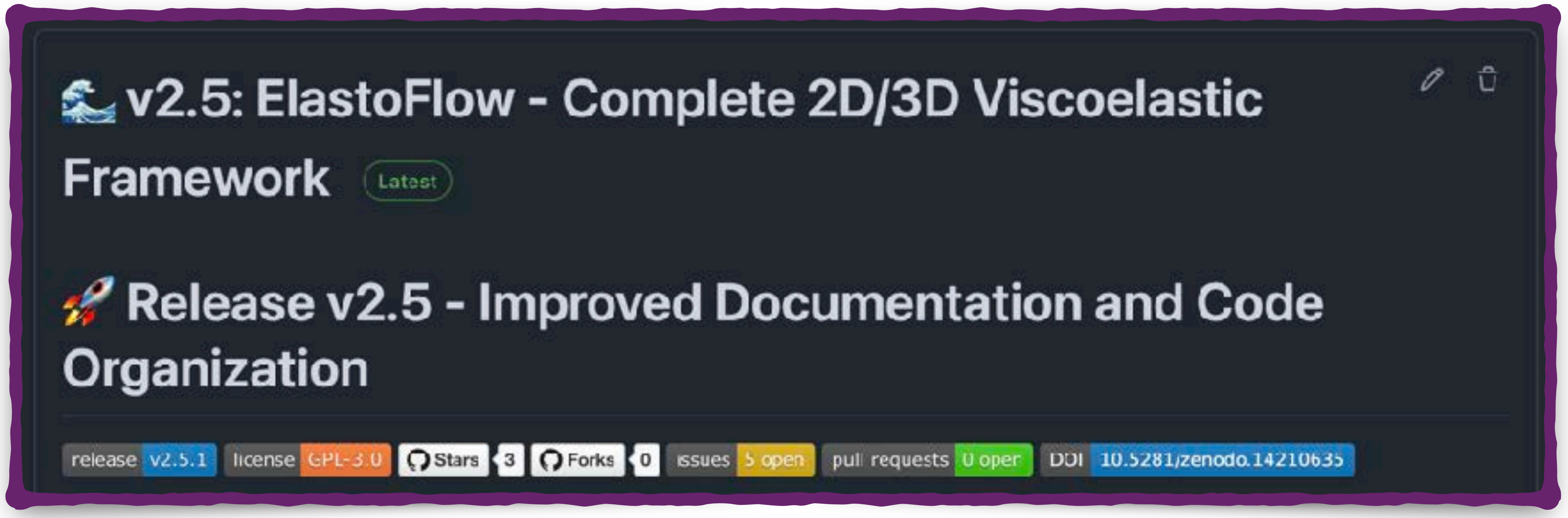
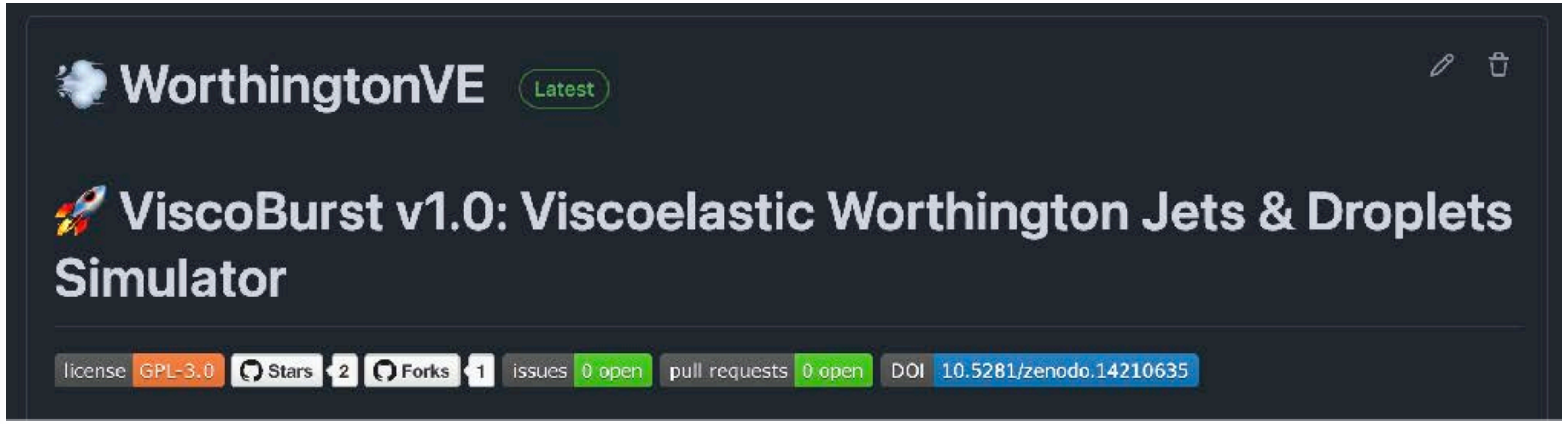
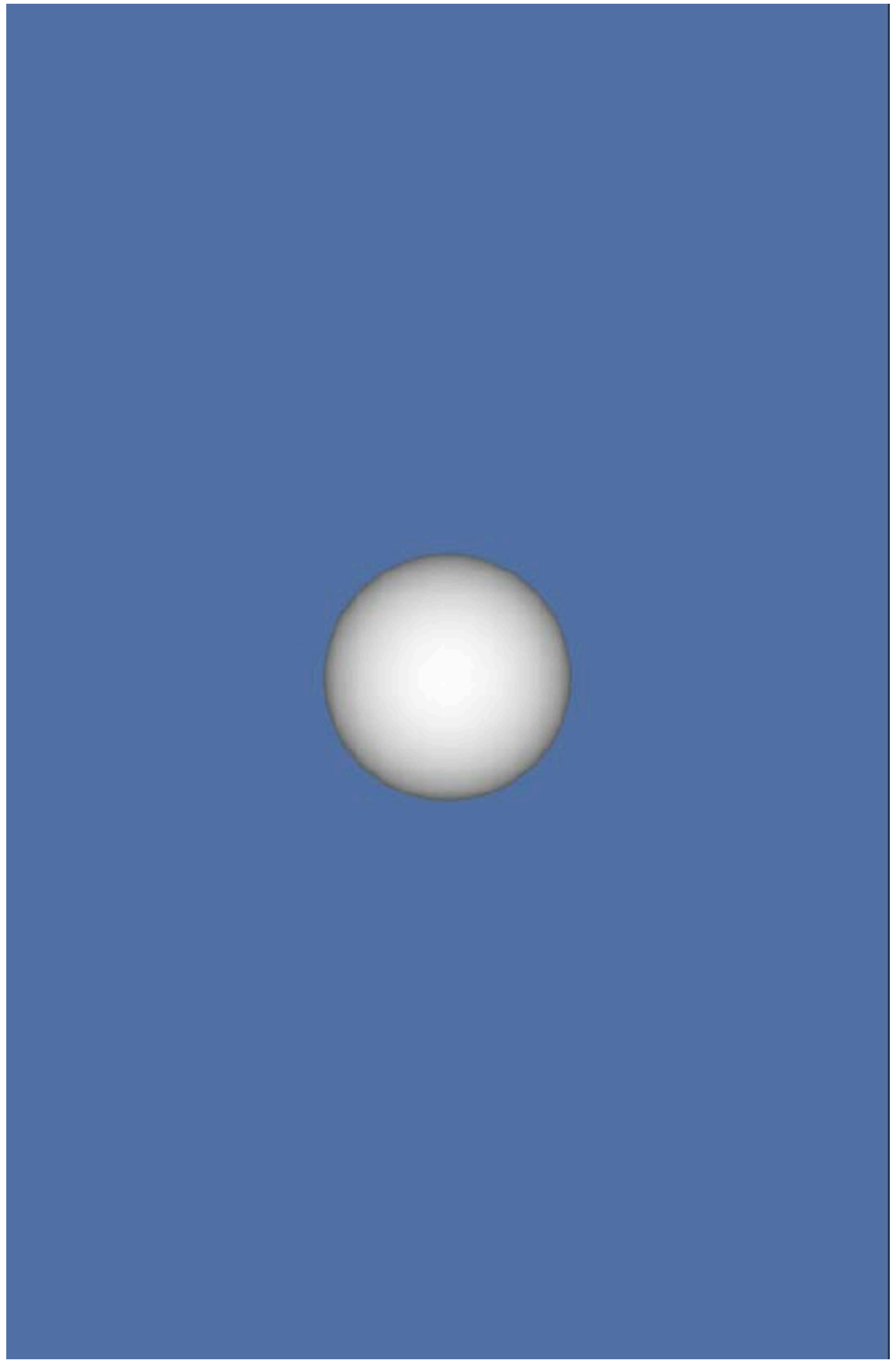




3D Viscoelasticity

3D Viscoelastic



3D Viscoelastic

$$Oh = \frac{\eta_s}{\sqrt{\rho\gamma\mathcal{L}}} = 0.05$$

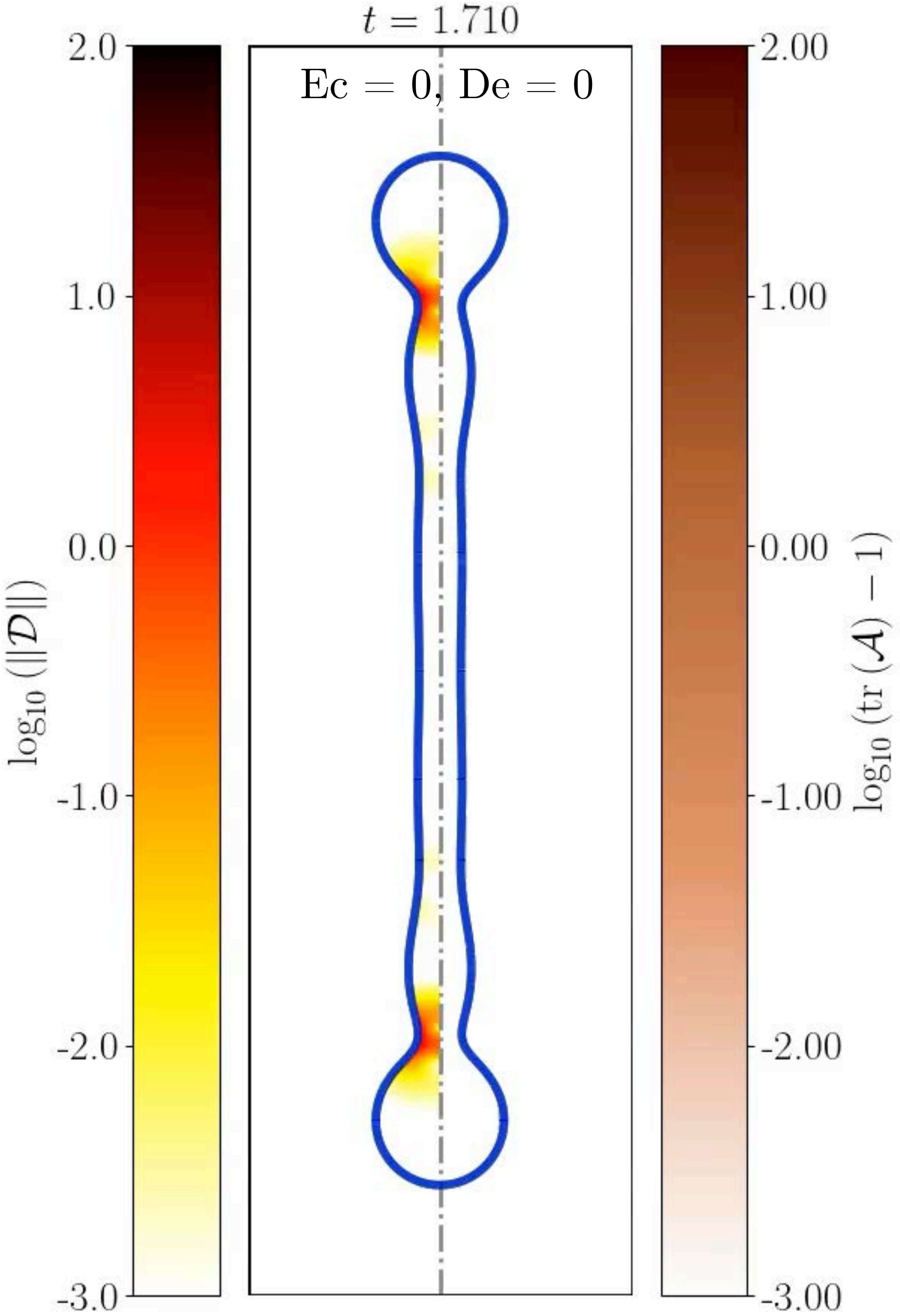
$$Ec = \frac{G\mathcal{L}}{\gamma}$$

$$De = \frac{\lambda}{\sqrt{\rho\mathcal{L}^3/\gamma}}$$

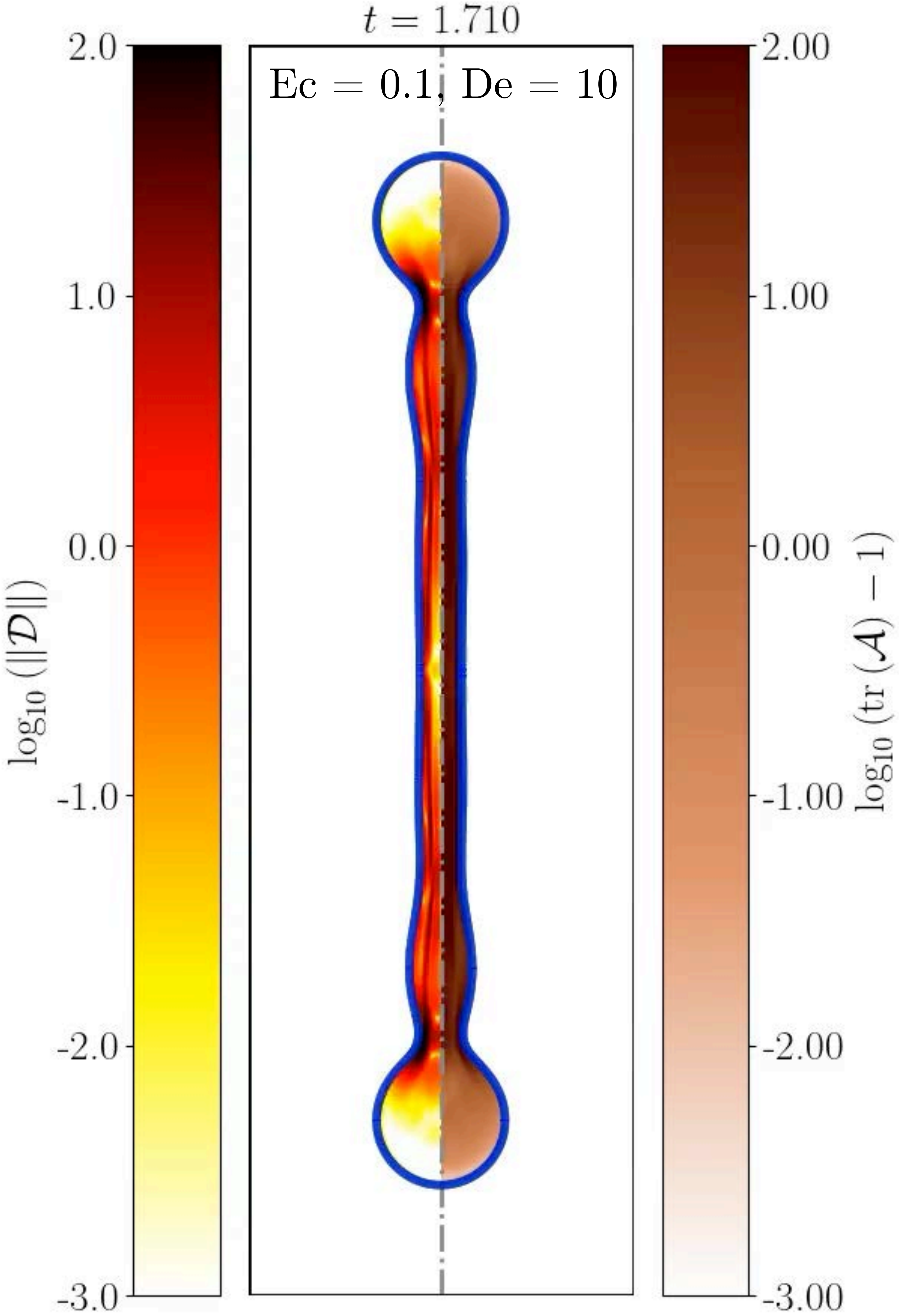
$$\frac{\eta_a}{\eta_s} = 0.01$$

$$\mathcal{L} = \left(\frac{3V}{4\pi}\right)^{1/3}$$

Newtonian filament



Viscoelastic filament



Acknowledgments & Collaborators

Physics of Fluids

- **Detlef Lohse**
- Ayush Dixit
- Aman Bhargava
- Jnandeep Talukdar
- **Saumili Jana**

- **Jacco Snoeijer**
- **Alex Oratis**
- **Vincent Bertin**
- Alvaro Marin
- Tommie Verouden
- ...

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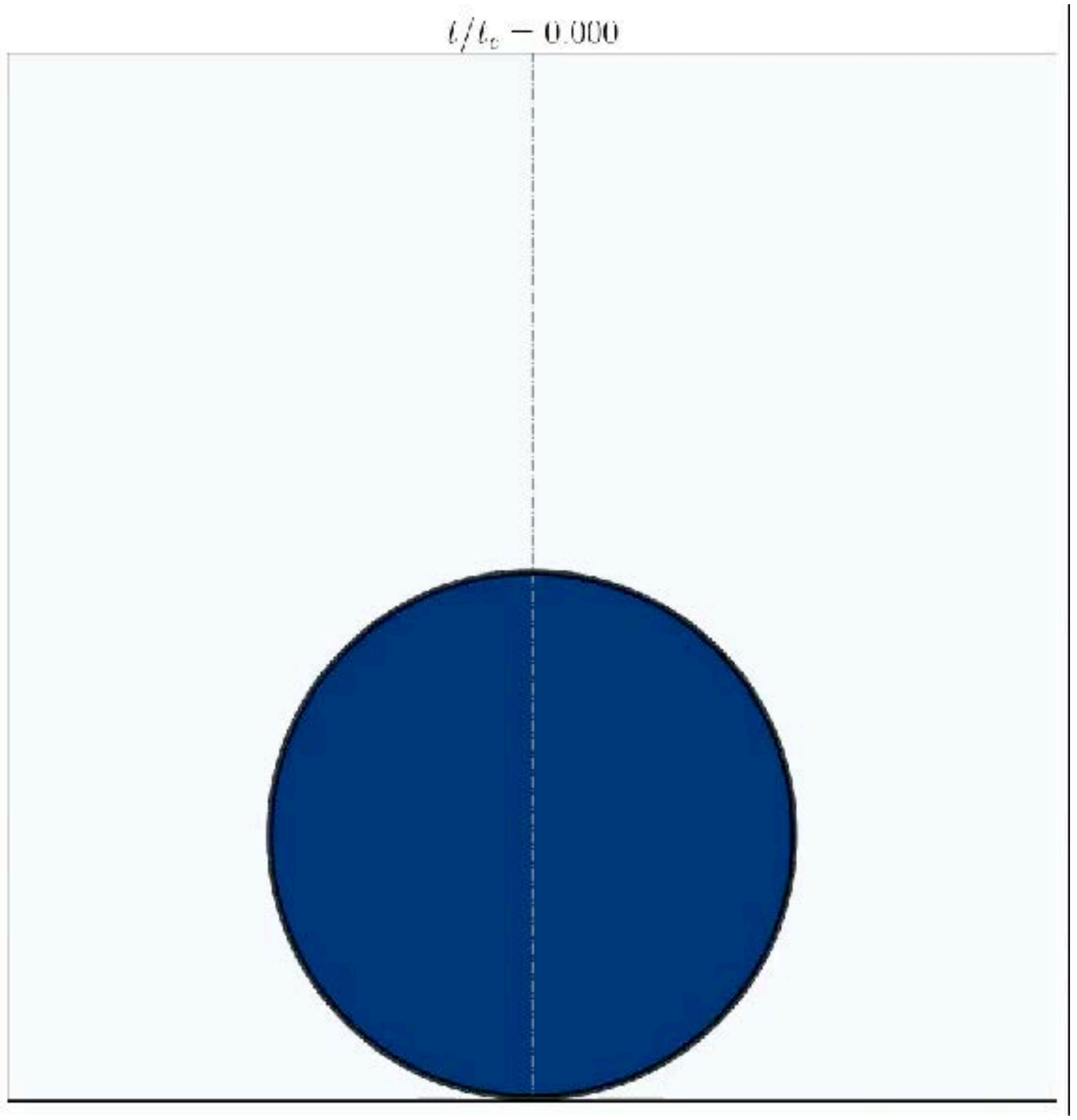
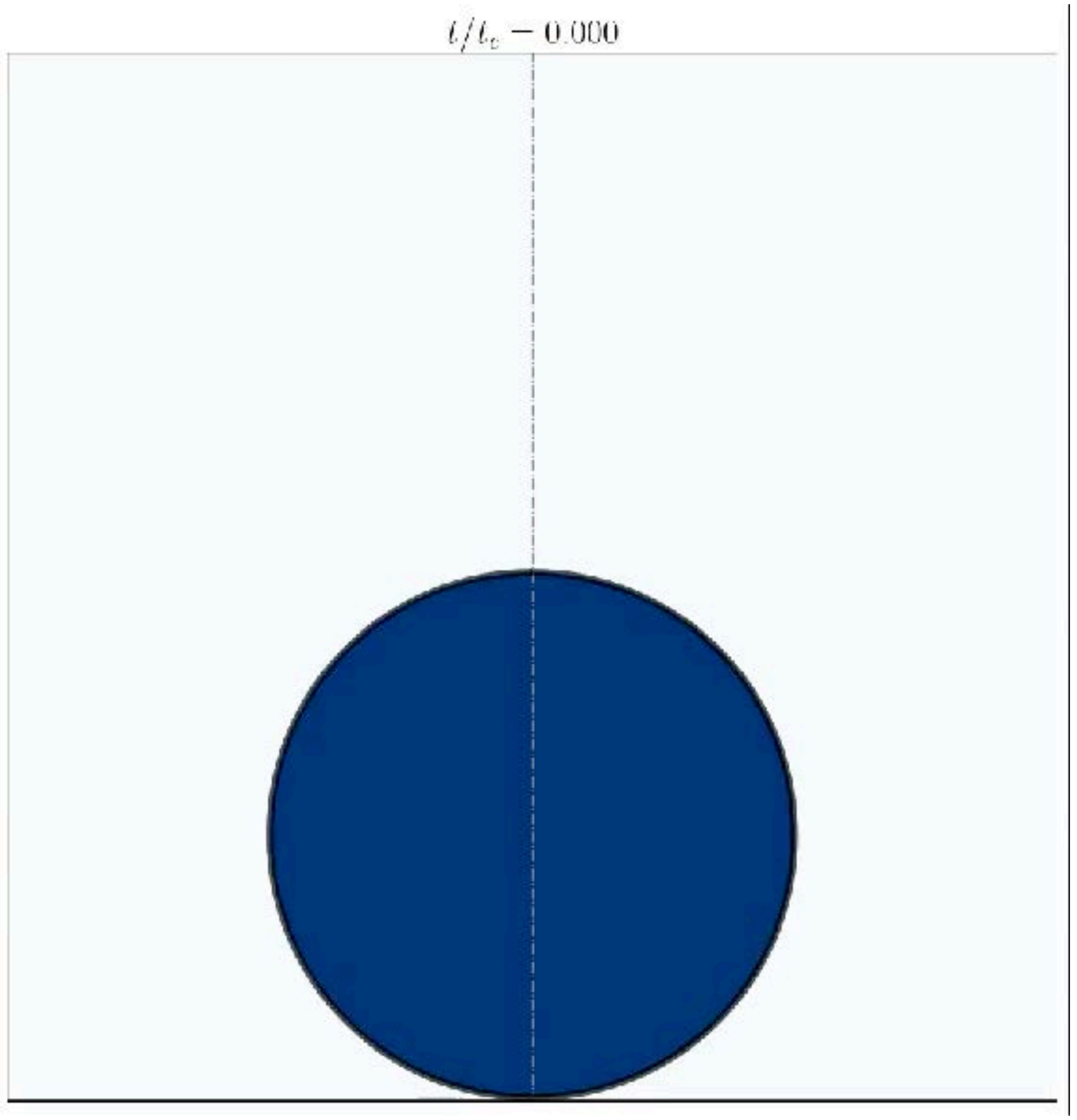
- Pierre Chantelot (Institut Langevin, ESPCI)
- Gareth McKinley (MIT, USA)
- Jie Feng (UIUC, USA)
- **John Kolinski (EPFL, Lausanne)**
- Mazi Jalaal (UvA, Amsterdam)
- Ari. Balasubramanian (KTH Sweden)
- Outi Tammisola (KTH Sweden)
- Konstantinos Zinelis (MIT, USA)
- Ricardo Constante Amores (UIUC, USA)
- ...

Thank you!



Liquid impacts



Solid impacts

Contact

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Durham University

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