Dancing droplets:
the physics of post-impact retraction dynamics

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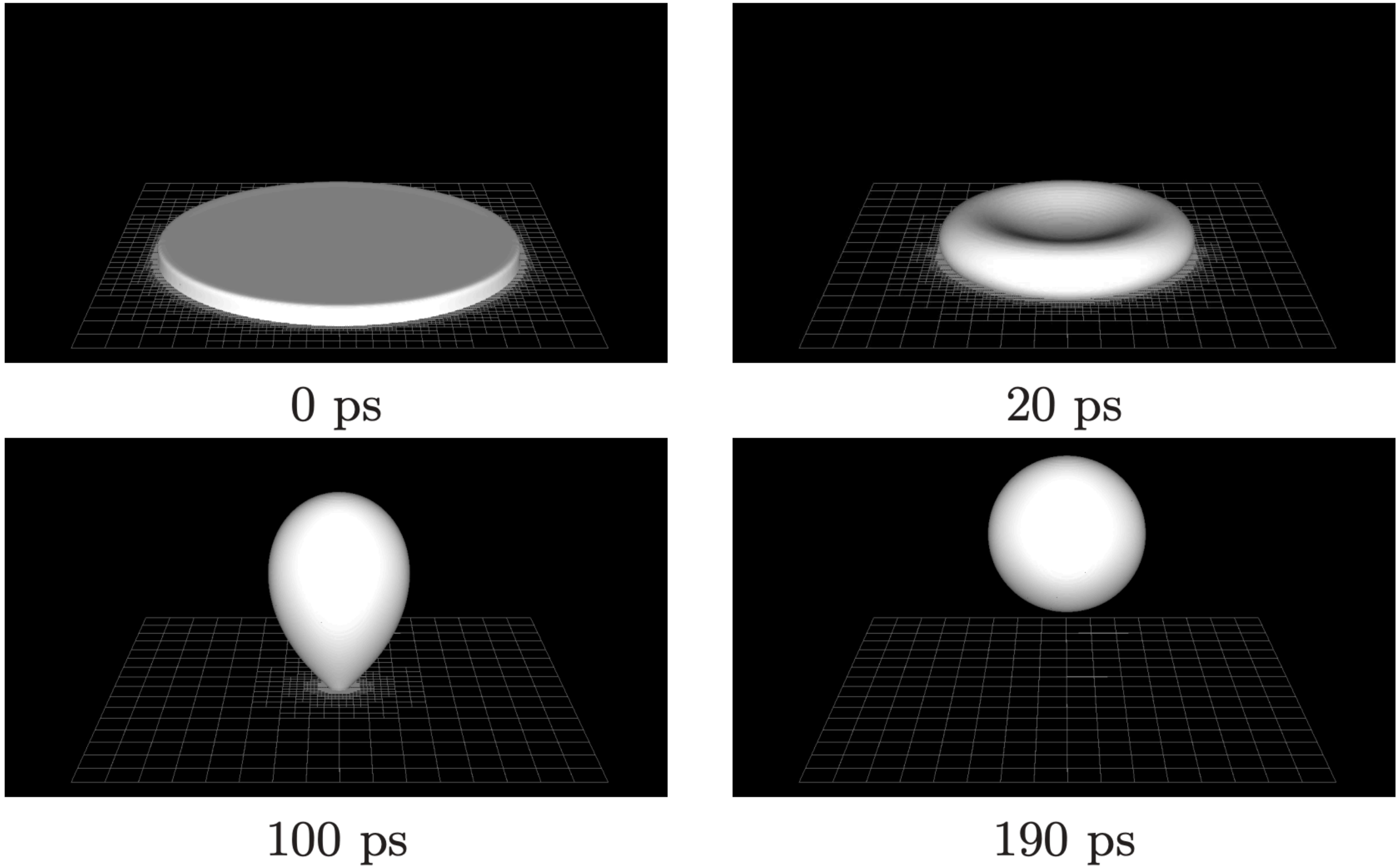
KRÜSS

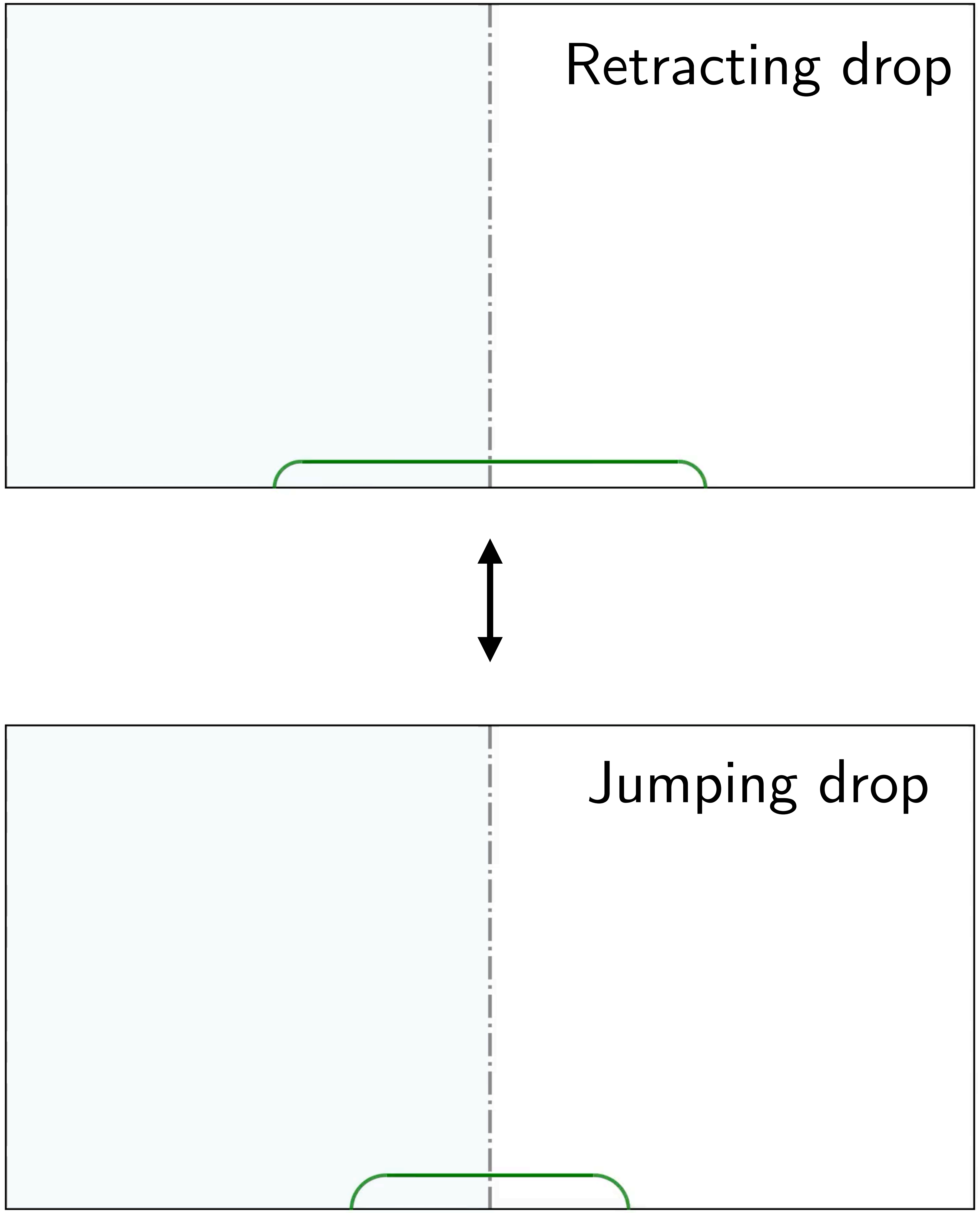
Liquid Ping-Pong in Space: https://www.youtube.com/watch?v=TLbhrMCM4_0

Jumping nanodroplets

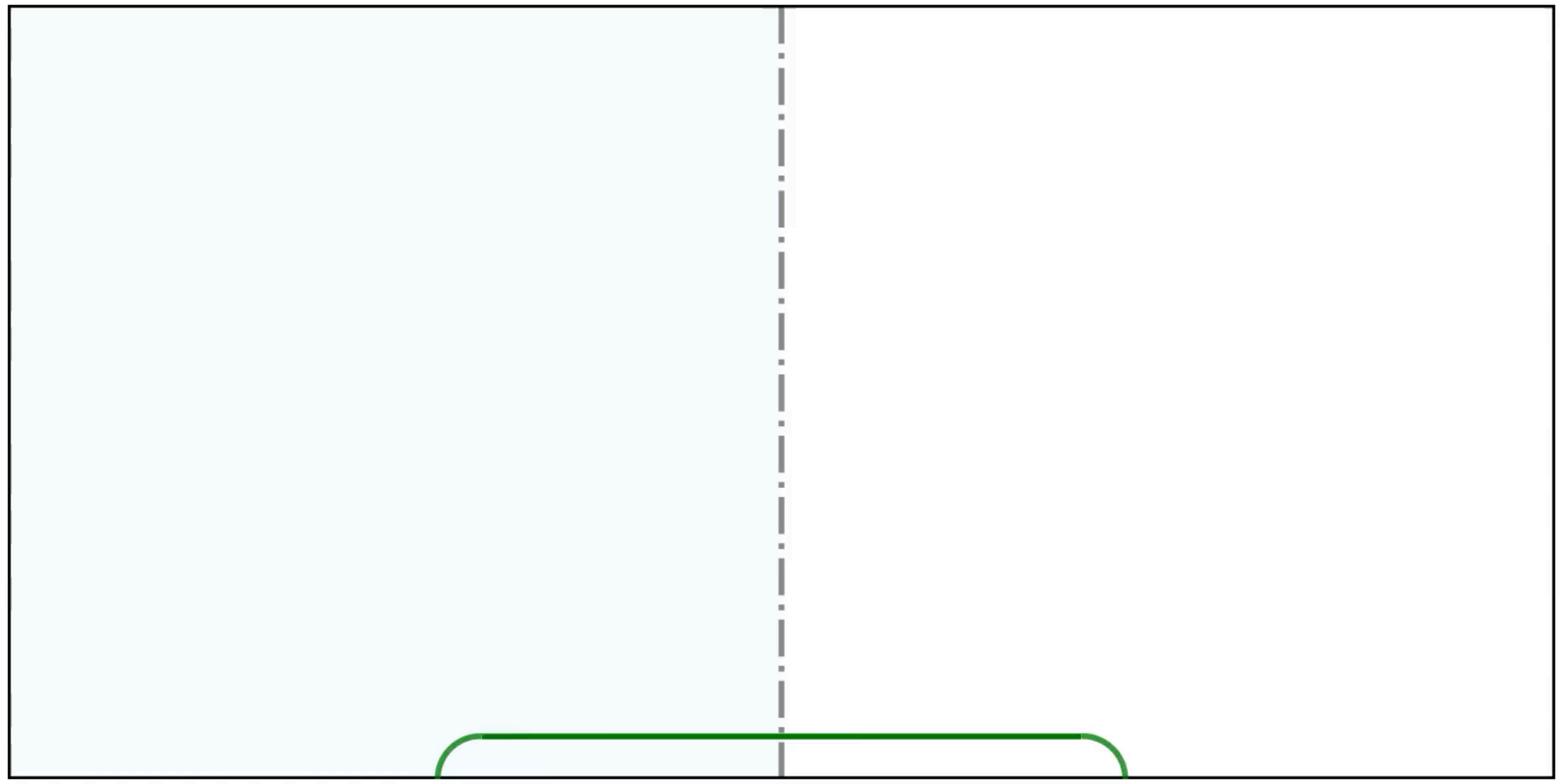


S. Afkhami and L. Kondic, PRL (2013)

Today's talking points



Bubble entrainment



Simulation details

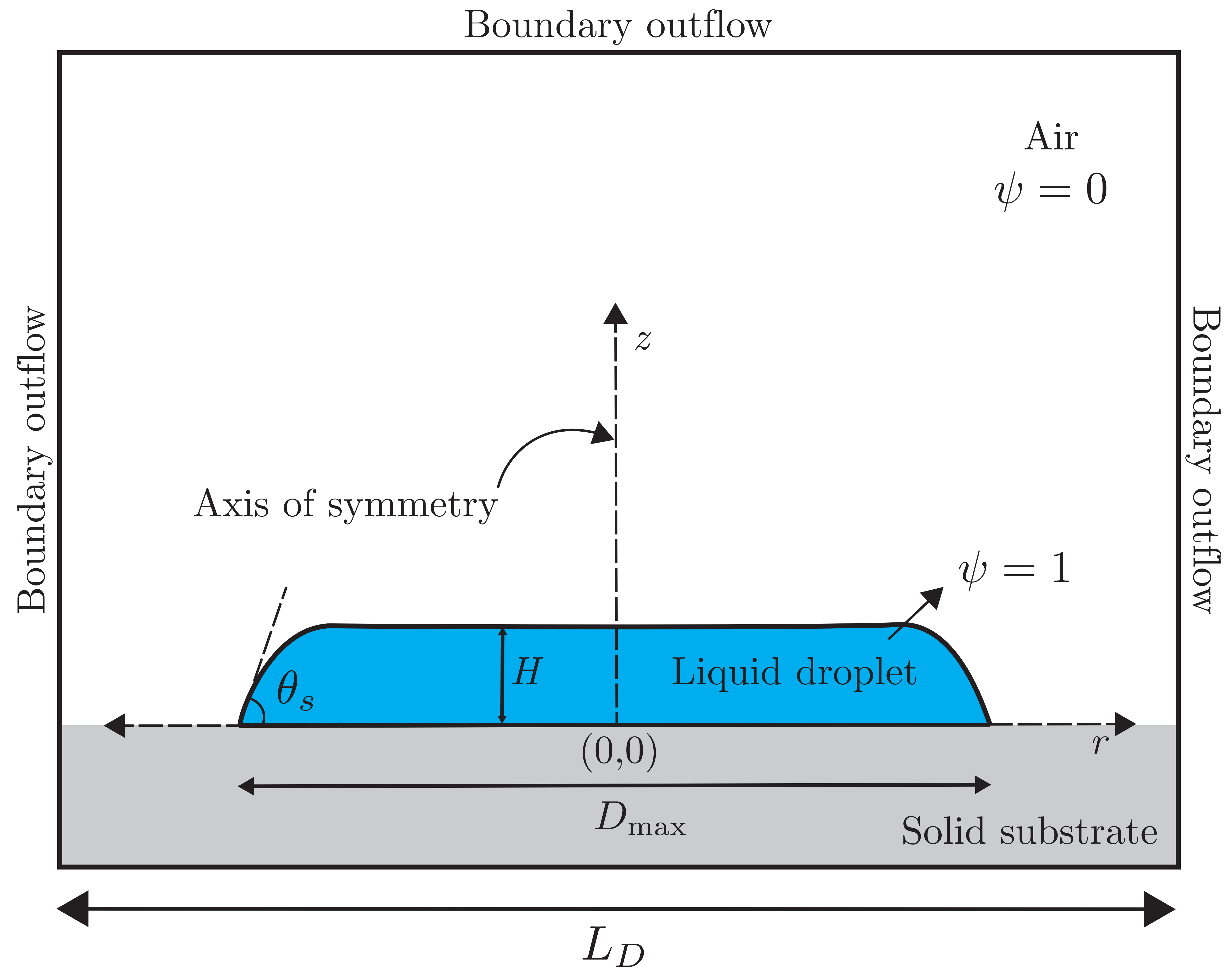
#ilovefs

Numerics: Basilisk C

Cauchy momentum + VoF

Stéphane Popinet & collaborators

Numerical Simulations setup



$$V = \frac{\pi H^3}{6}\left(\frac{3\Gamma^2}{4} + 1\right) \sim O(\mu L)$$

Control Parameters

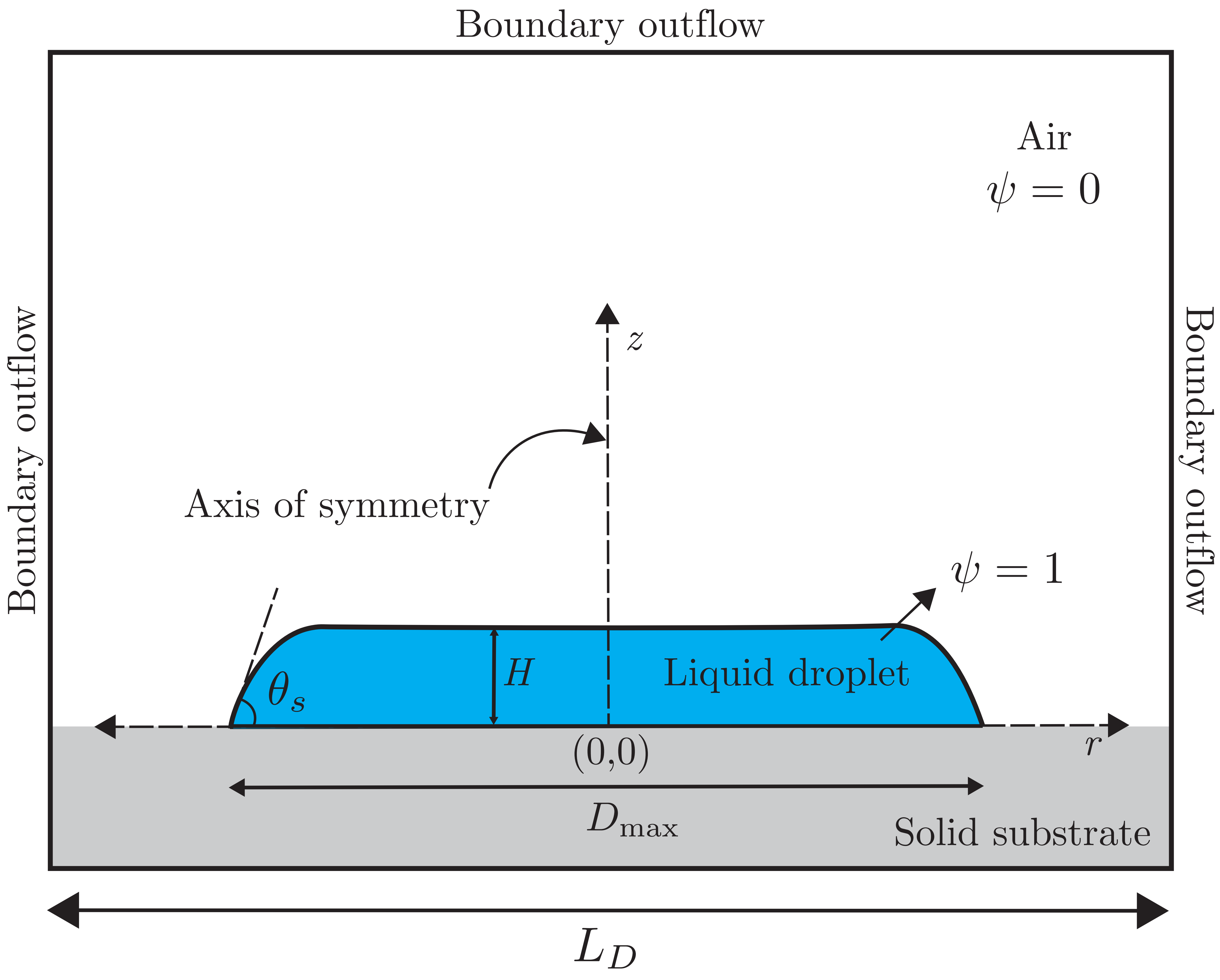
$$Oh_l = \frac{\mu_l}{\sqrt{\rho\gamma H}} \sim O(10^{-2} - 10^{0})$$

$$\Gamma = \frac{D}{H} \sim O(10)$$

θ_s = Static contact angle

$$Bo = \frac{\Delta\rho g H^2}{\gamma} \sim O(10^{-3})$$

Governing equations and boundary conditions



$$\left(\frac{\partial \tilde{\boldsymbol{v}}}{\partial \tilde{t}} + \tilde{\nabla} \cdot (\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}})\right) = -\tilde{\nabla}\tilde{p} + \tilde{\nabla} \cdot (2Oh\tilde{\boldsymbol{D}}) + \tilde{\boldsymbol{f}}_\gamma$$

$$\nabla \cdot \boldsymbol{v} = 0$$

$$t_\gamma = \frac{H}{v_\gamma} = \sqrt{\frac{\rho_f H^3}{\gamma_L}} \quad \Big| \quad v_\gamma = \sqrt{\frac{\gamma_L}{\rho_f H}} \quad \Big| \quad \tilde{p} = pH/\gamma_L$$

$$\tilde{\boldsymbol{D}} = (\tilde{\boldsymbol{\nabla}}\tilde{\boldsymbol{v}} + (\tilde{\boldsymbol{\nabla}}\tilde{\boldsymbol{v}})^T)/2$$

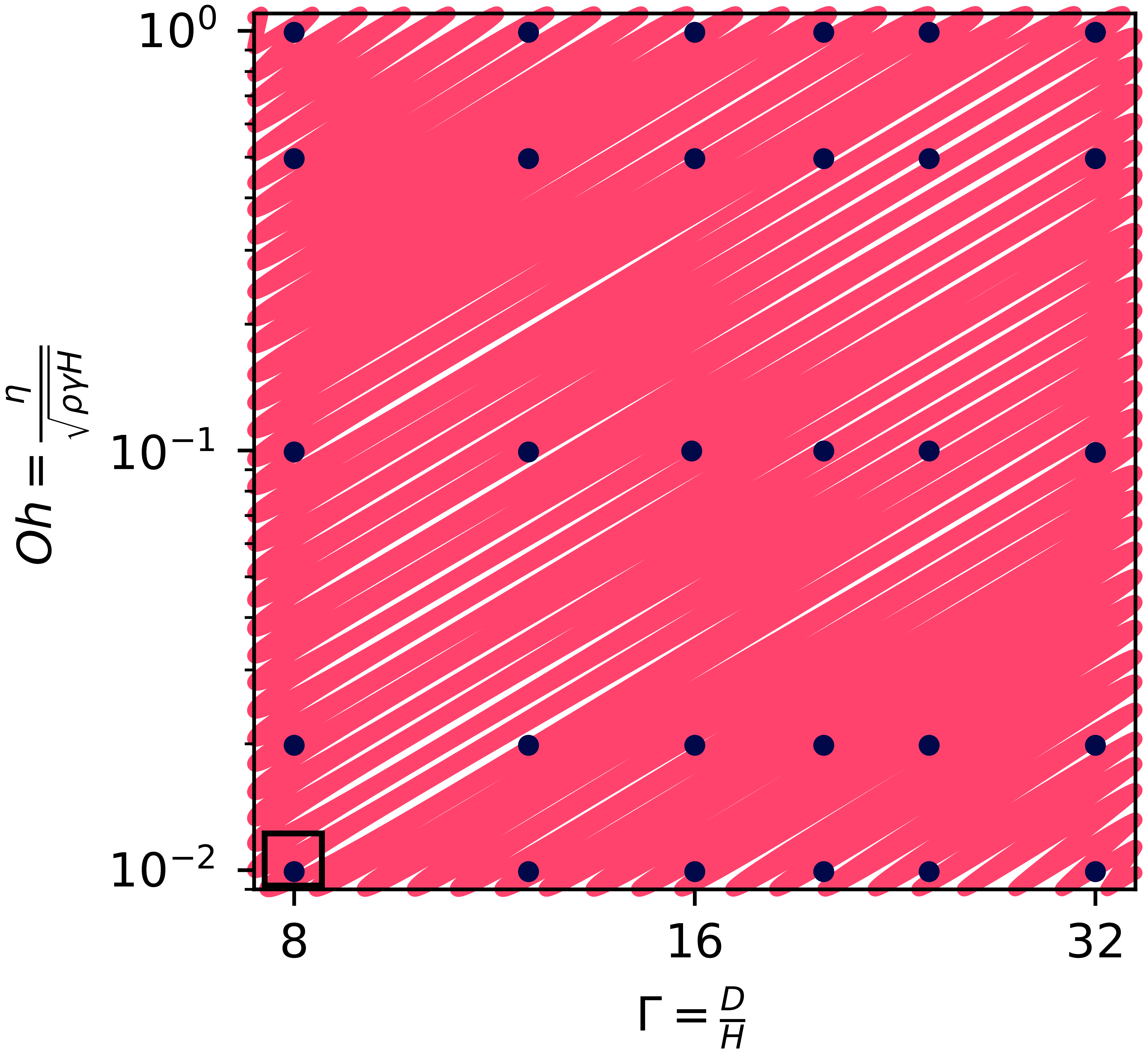
Symmetric part of the velocity gradient tensor

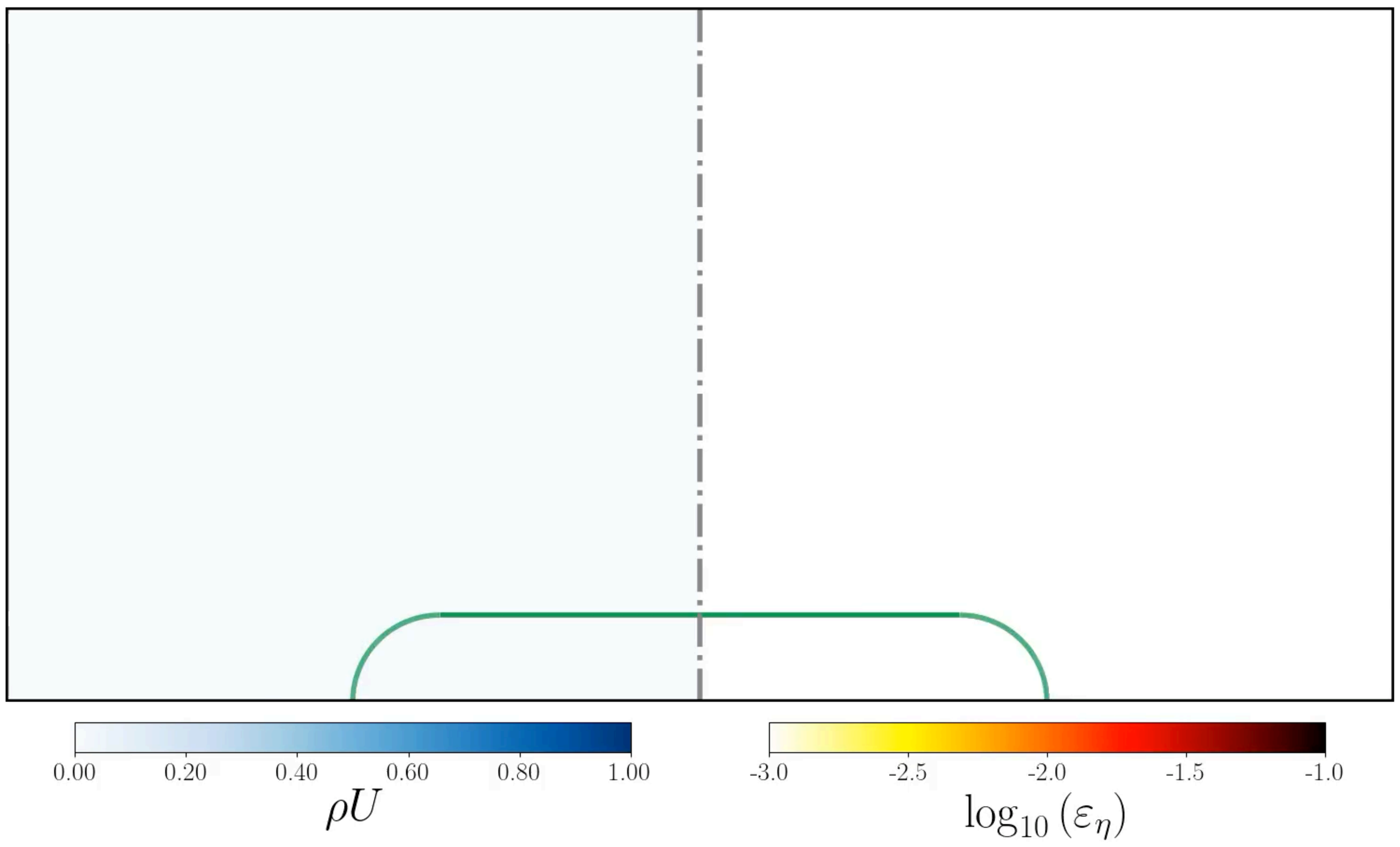
$$\tilde{\boldsymbol{f}}_\gamma \approx \tilde{\kappa}\tilde{\nabla}\psi$$

Dimensionless surface tension force

Hydrophilic substrates

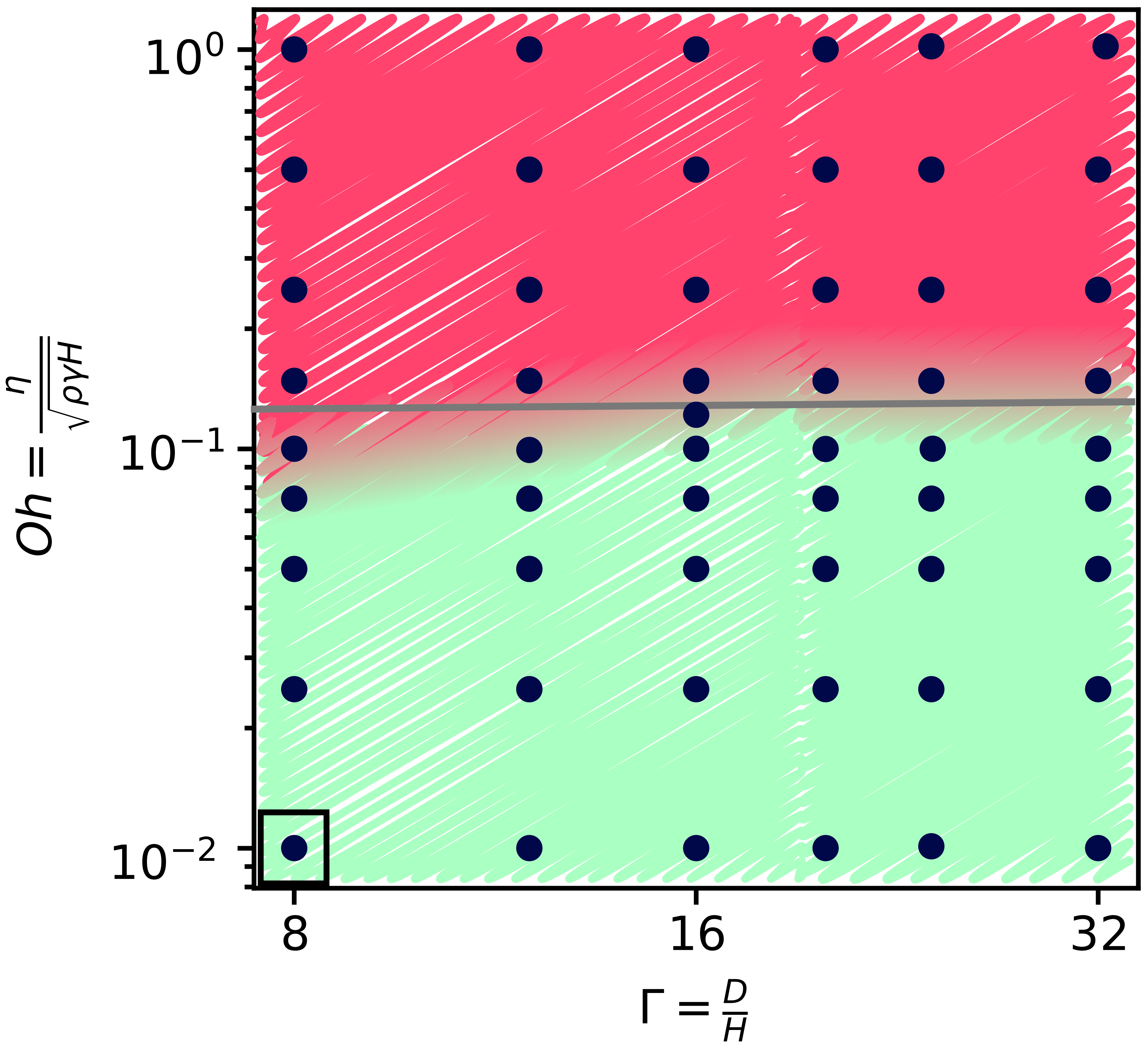
$\theta = 60^\circ, Bo = 0$

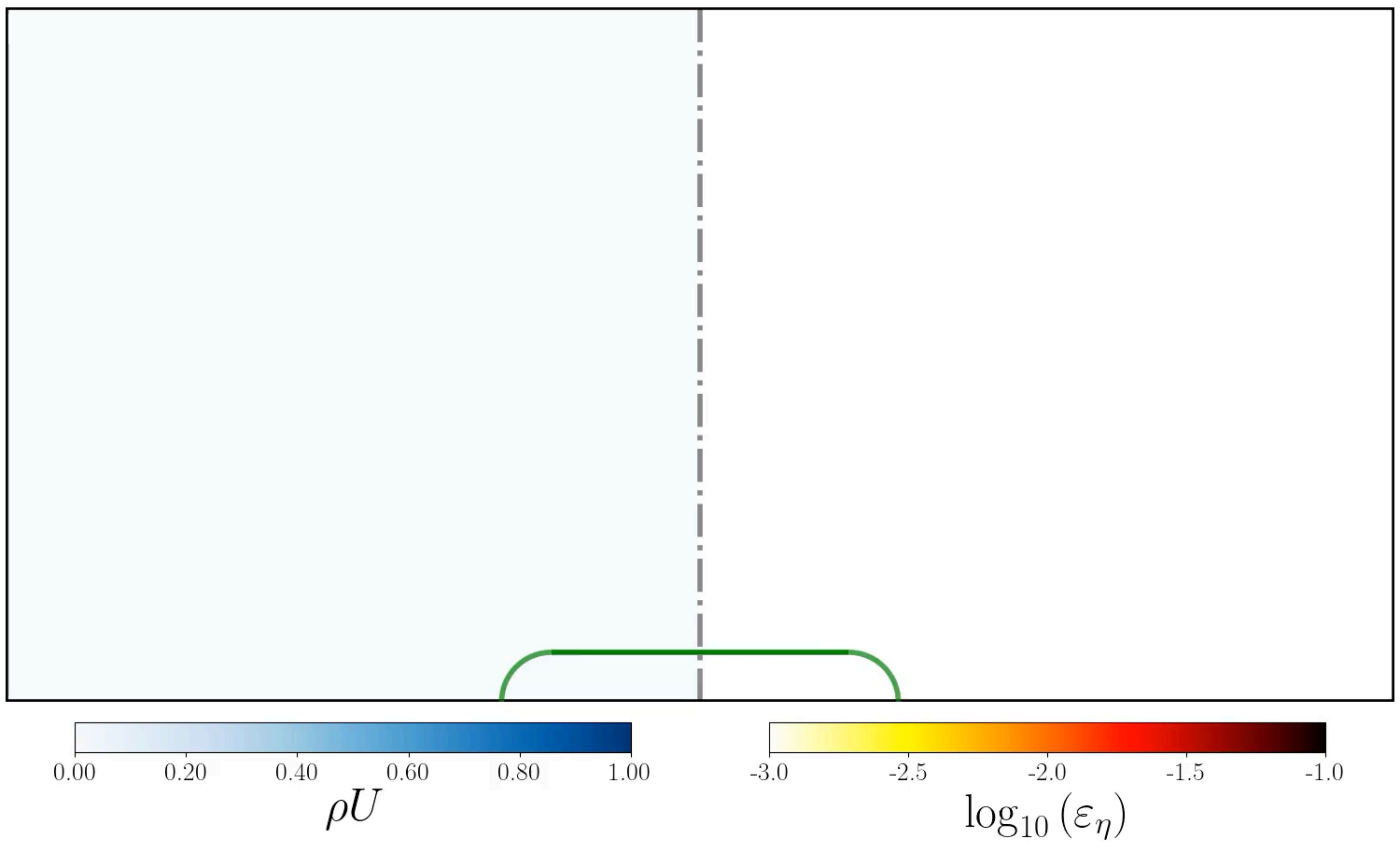




Hydrophobic substrates

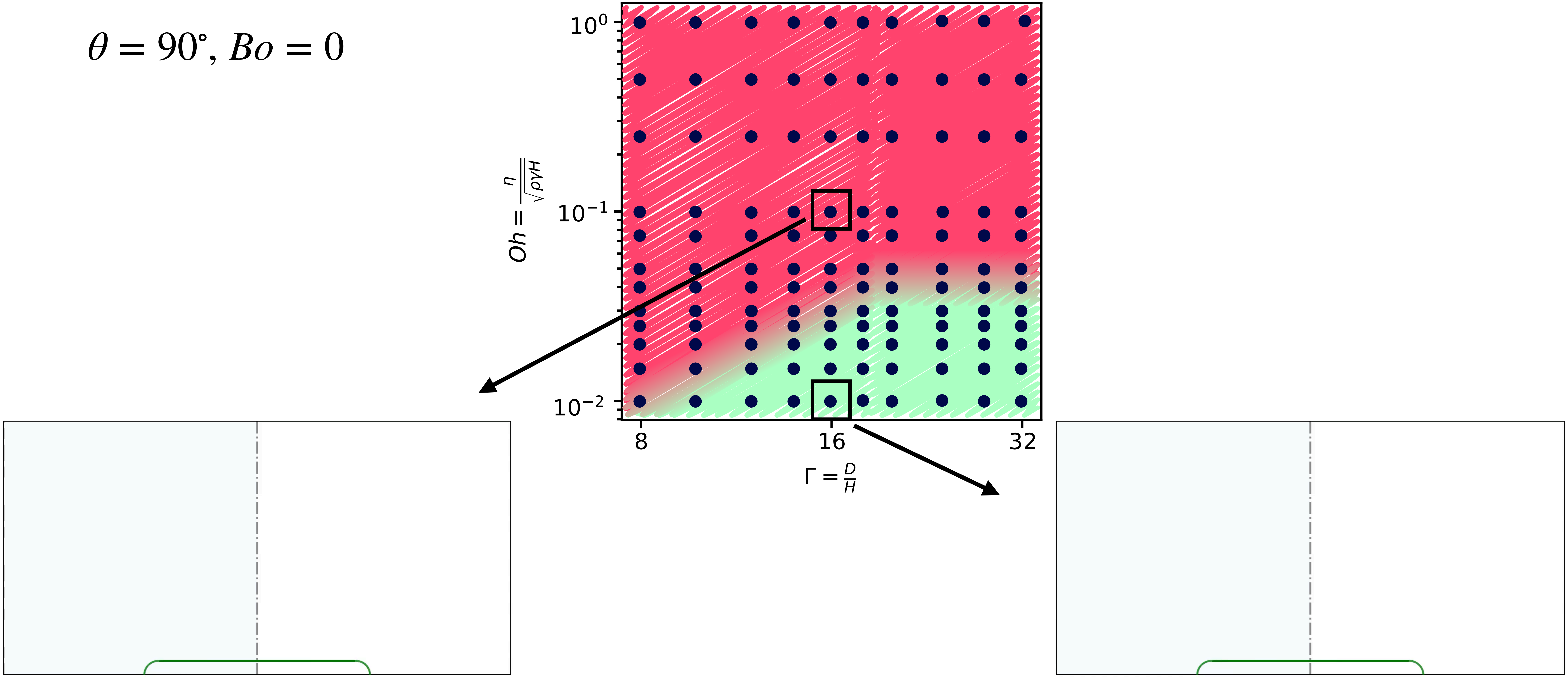
$\theta = 120^\circ, Bo = 0$





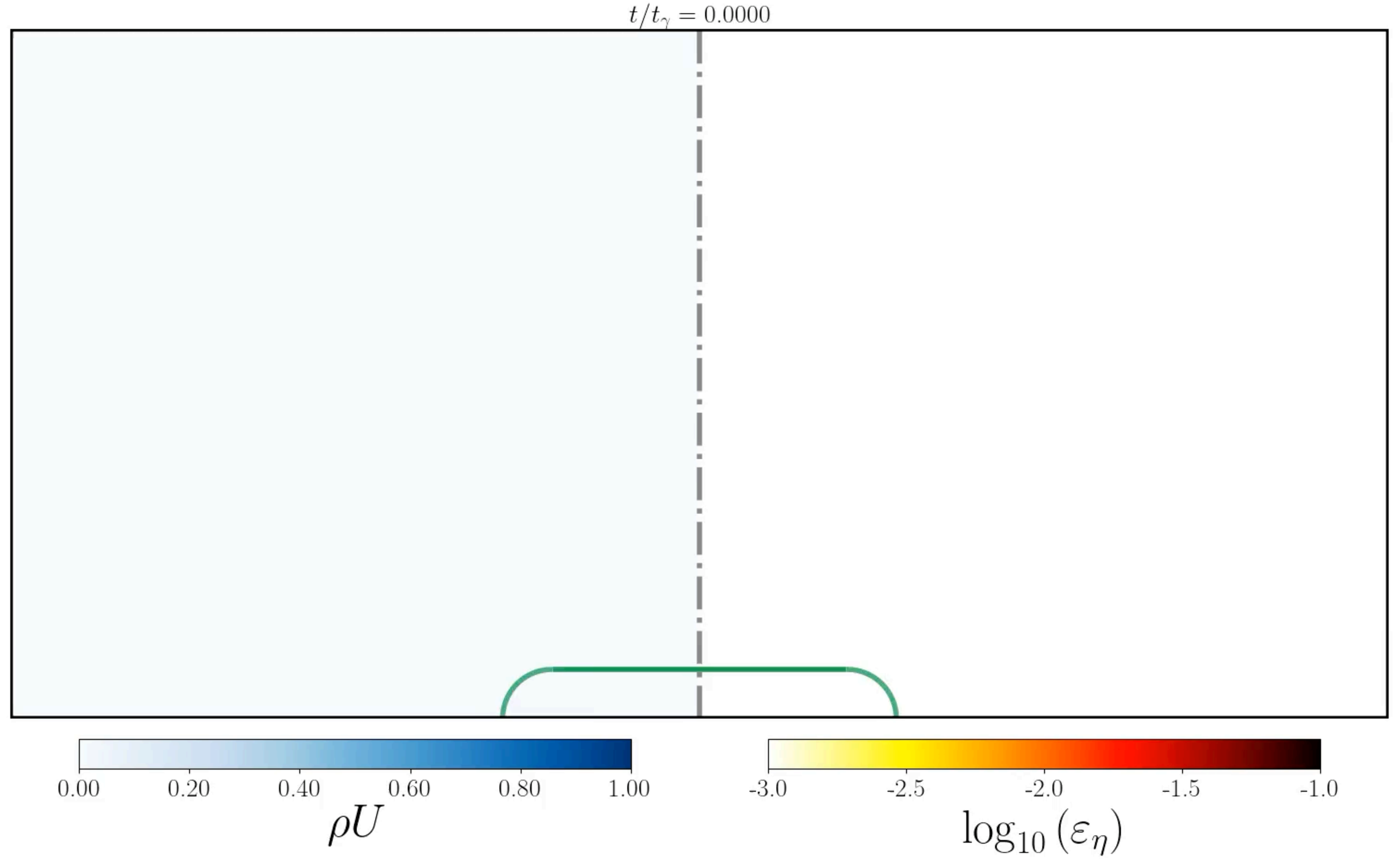
What happens in moderate hydrophilicity?

$\theta = 90^{\circ}$, $Bo = 0$



How do we estimate when a droplet takes off?

$$Bo = 0 \quad \Gamma = 8 \quad \theta_s = 150^\circ \quad Oh = 0.1$$

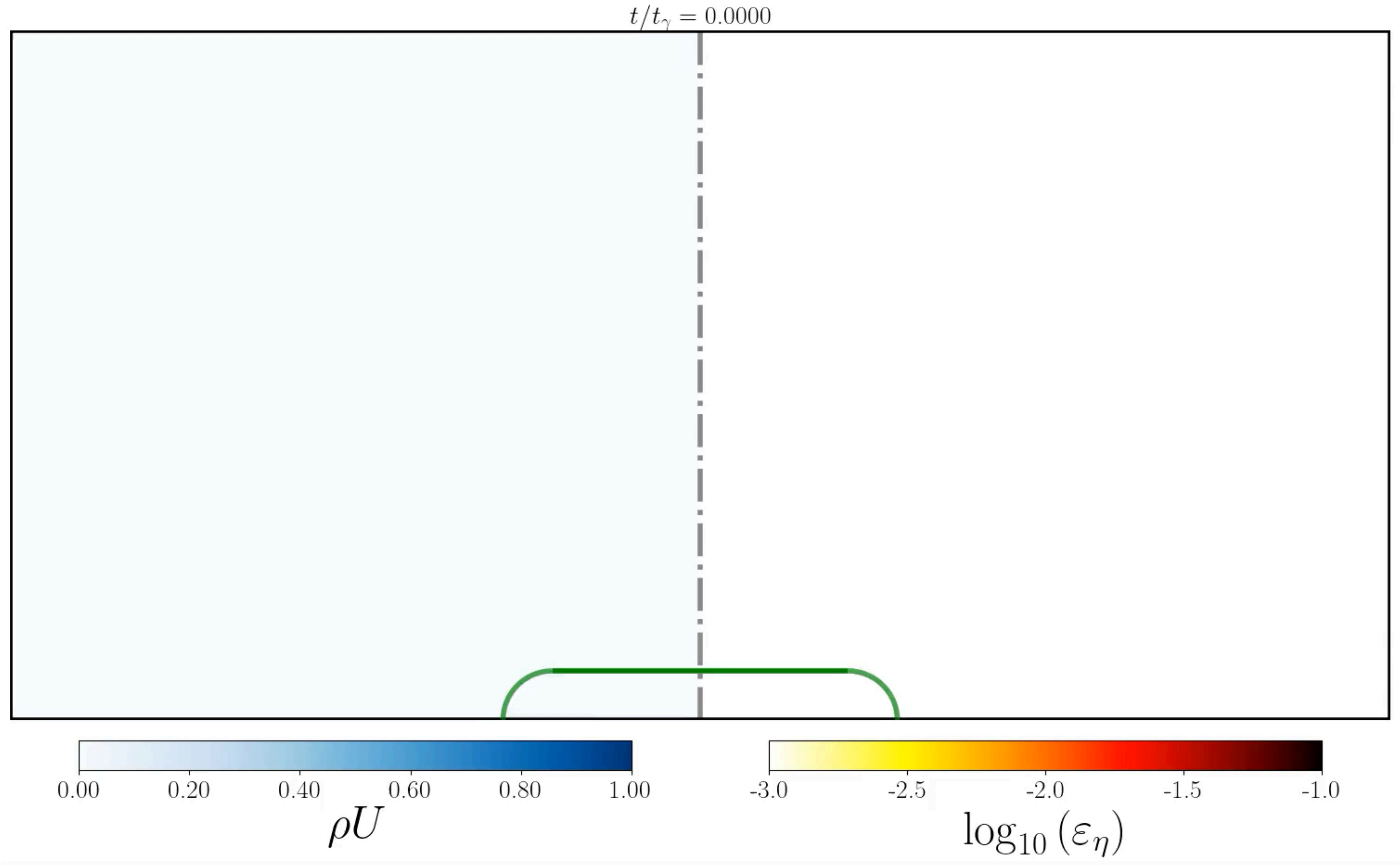


Factors affecting takeoff:

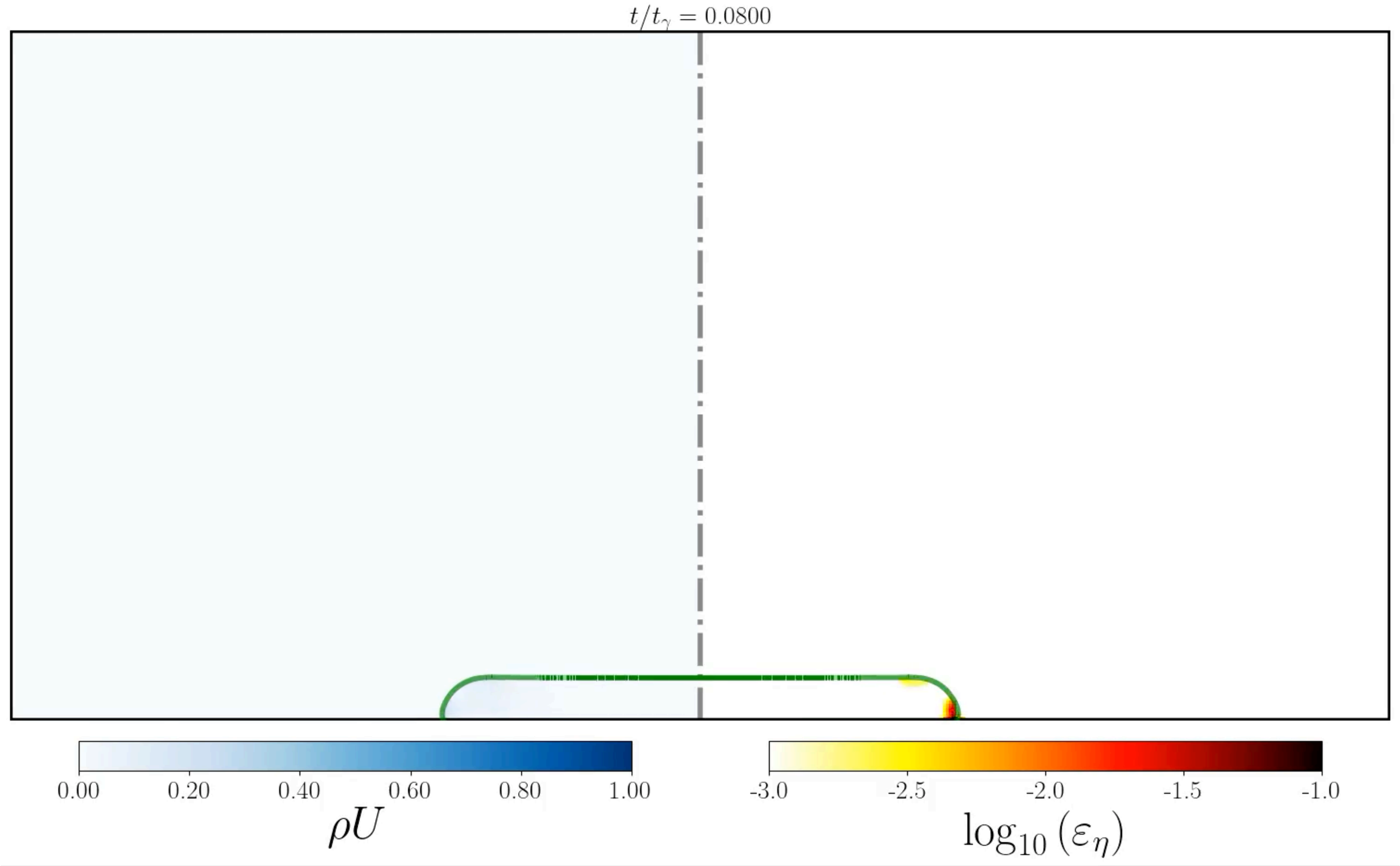
- Ohnesorge number

$$Oh_l = \frac{\eta}{\sqrt{\rho\gamma H}} \qquad \Gamma = \frac{D}{H} \qquad Bo = \frac{\Delta\rho g H^2}{\gamma}$$

Aspect ratio dependence



$\Gamma = 8$



$\Gamma = 12$

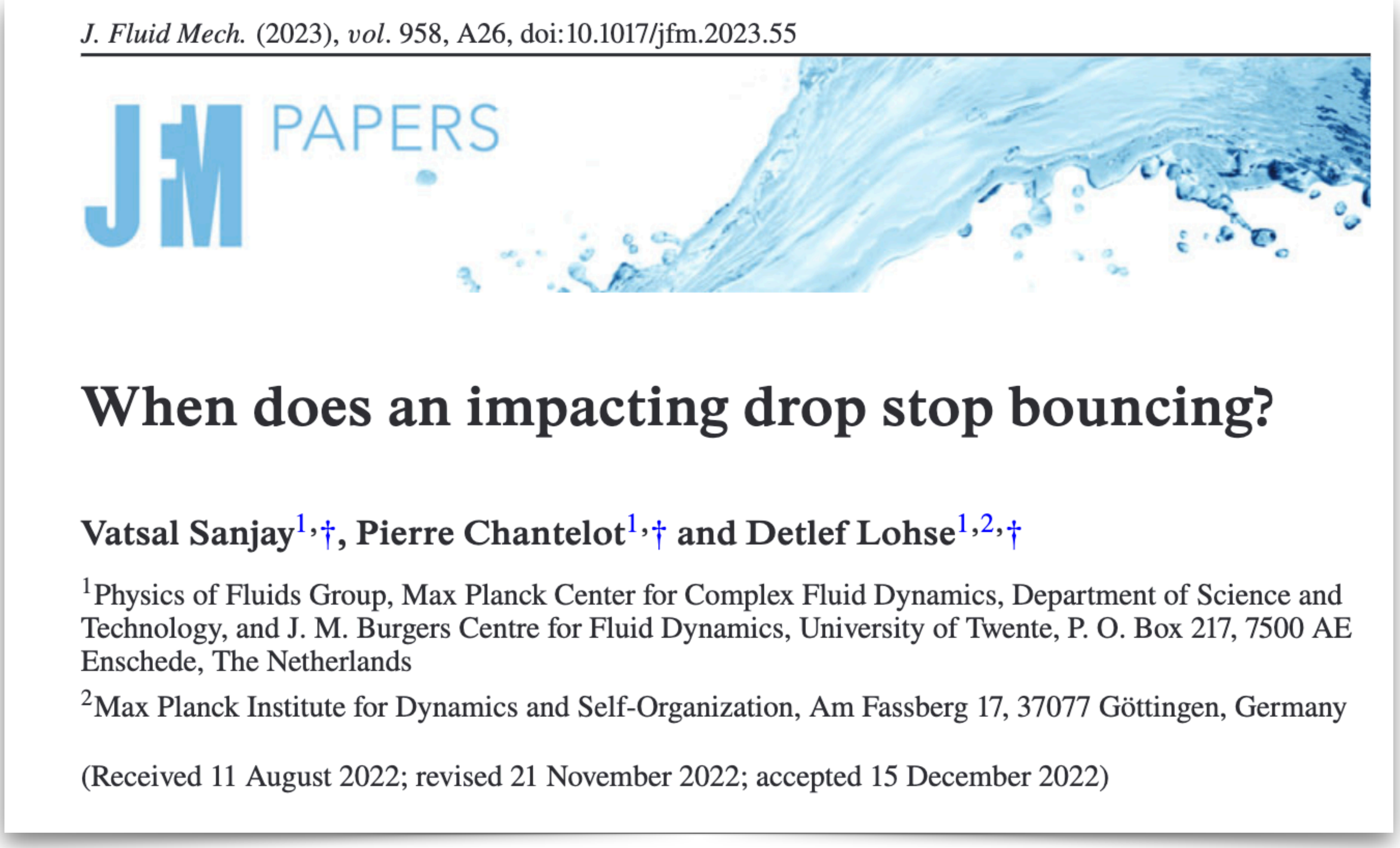
Factors affecting takeoff:

- Ohnesorge number
- Aspect ratio

$$Oh_l = \frac{\eta}{\sqrt{\rho \gamma H}} = 0.1 \qquad Bo = \frac{\Delta \rho g H^2}{\gamma} = 0$$

$$\Gamma = \frac{D}{H} \qquad \theta_s = 120^\circ$$

Viscous inhibition: is it enough?


J. Fluid Mech. (2023), vol. 958, A26, doi:10.1017/jfm.2023.55

JFM PAPERS

When does an impacting drop stop bouncing?

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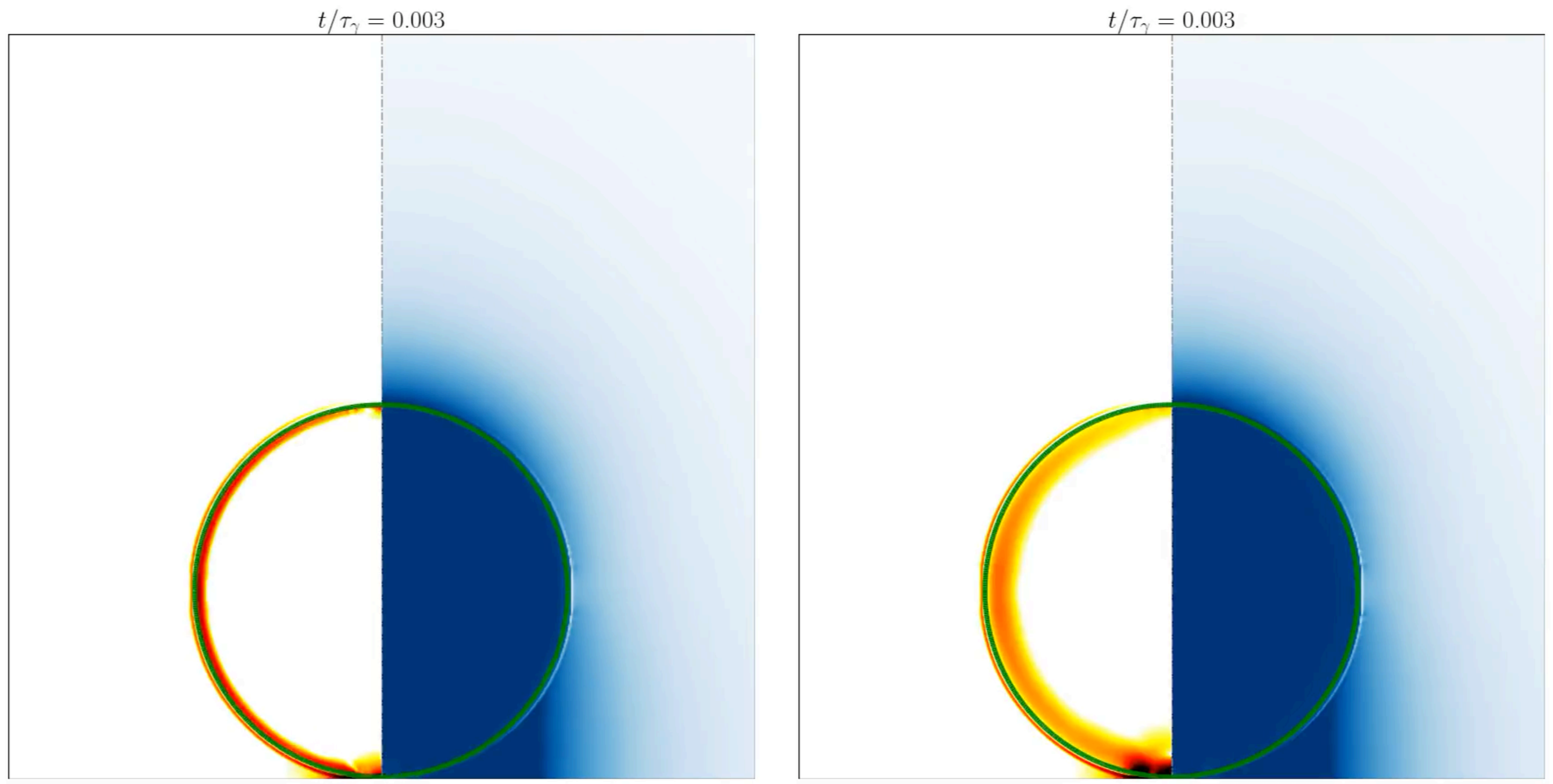
$$We = \frac{Drop\ \ inertia}{Capillary\ \ pressure}$$

$$Weber\ number \leftrightarrow Aspect\ ratio$$

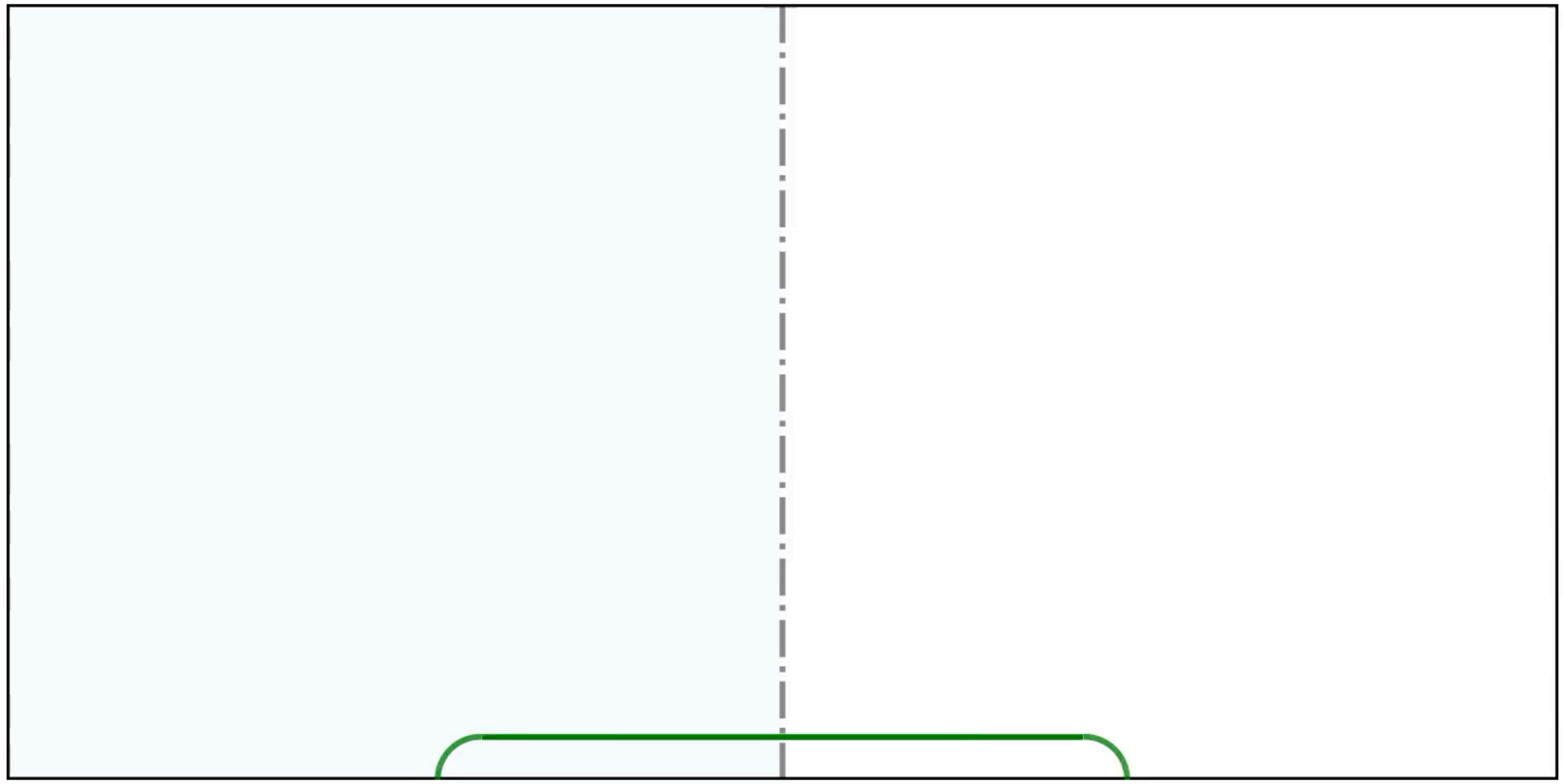
Superhydrophobic substrate

- No We dependence
- $Oh_c + Bo_c \sim 1$ scaling



$Oh + Bo = 0.1$ $\qquad$ $Oh + Bo = 1$

What happens if the substrate is hydrophilic?



$Oh = 0.01$, $Bo = 0$, $\theta_s = 30^\circ, \Gamma = 16$

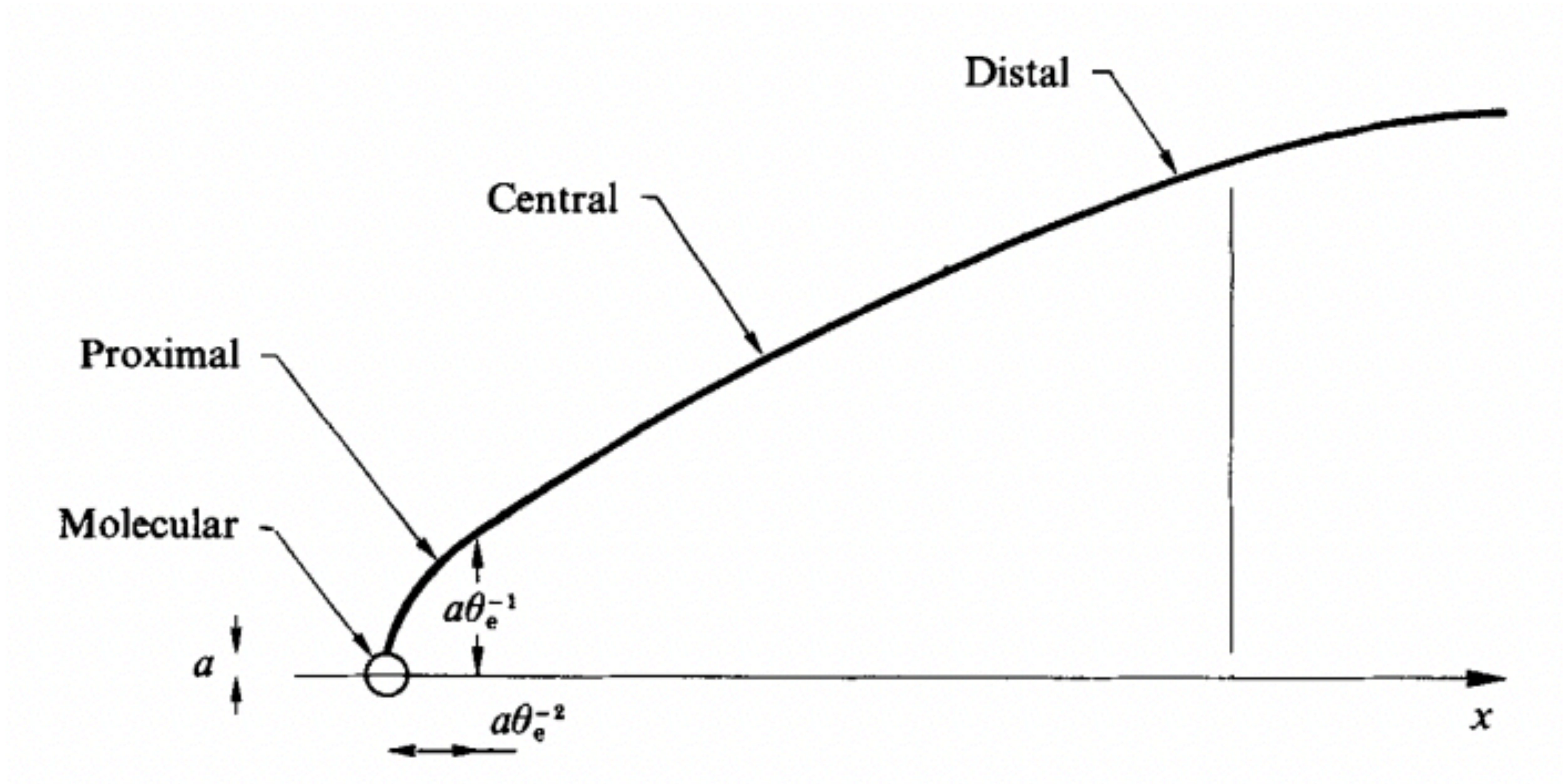
Hydrophilic substrate

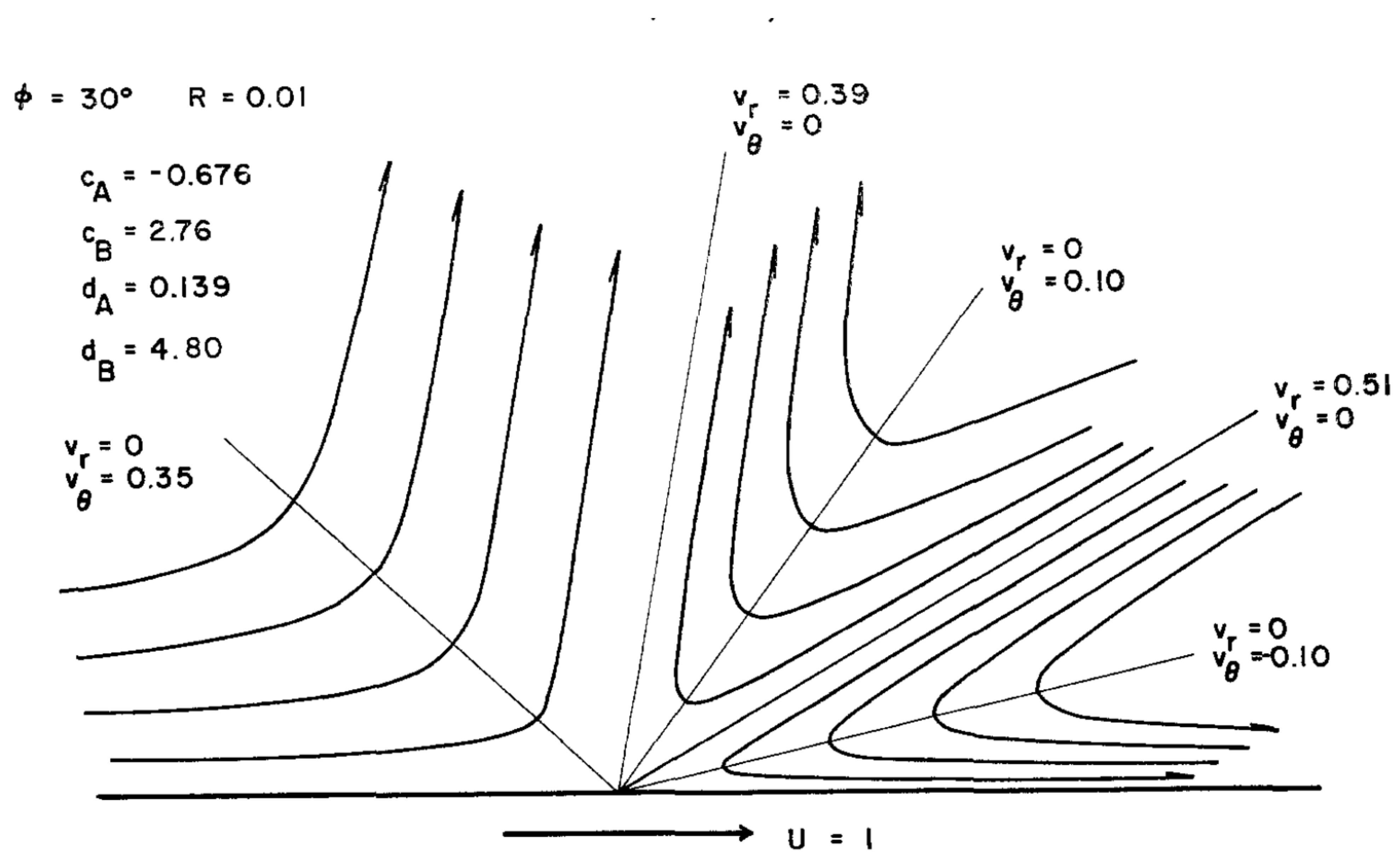
- Γ dependence is observed
- No jumping is observed for $Oh_c + Bo_c \ll 1$

Superhydrophobic substrate

- No We dependence
- $Oh_c + Bo_c \sim 1$ scaling

Contact angle effect on dissipation





$$\boldsymbol{D} = \frac{3\eta U^2}{\theta_{eq}} ln(x)$$

Smaller contact angles result in larger dissipation

C. Huh and L. Scriven, JCIS (1970)

P.G. De Gennes, X. Hua and P. Levinson, JFM (1990)

Scaling to determine Oh_c

Surface energy ~ Bulk viscous dissipation + Contact line dissipation

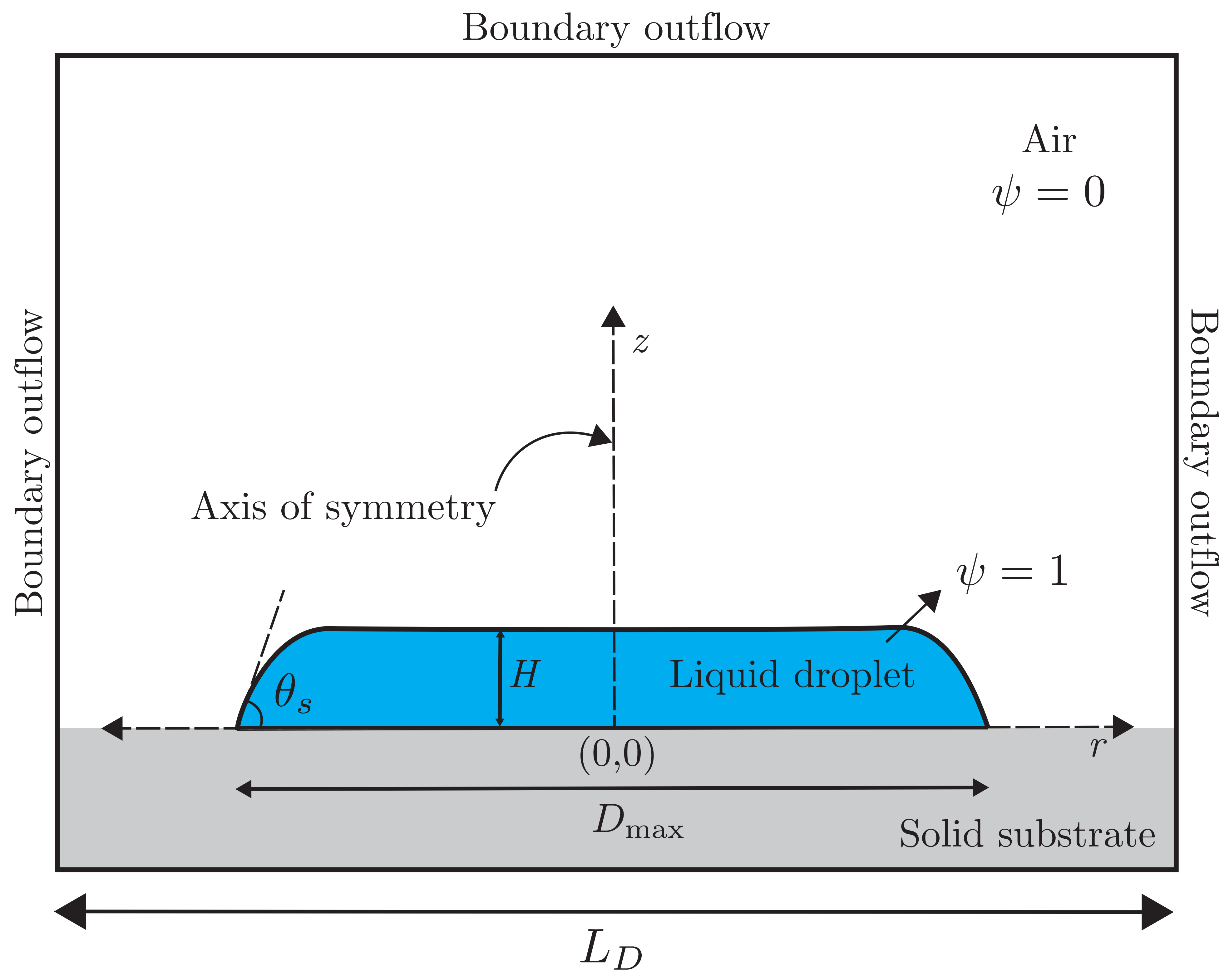
$$E_{surf} \sim \gamma_{LG} H^2 \Gamma^2$$

$$E_{diss} \sim \eta \left(\frac{V_\gamma}{\lambda}\right)^2 t_\gamma \Omega \sim \eta V_\gamma H \left(\frac{\Omega}{\lambda^2}\right)$$

V_γ, t_γ : Visco-capillary velocity and time scales

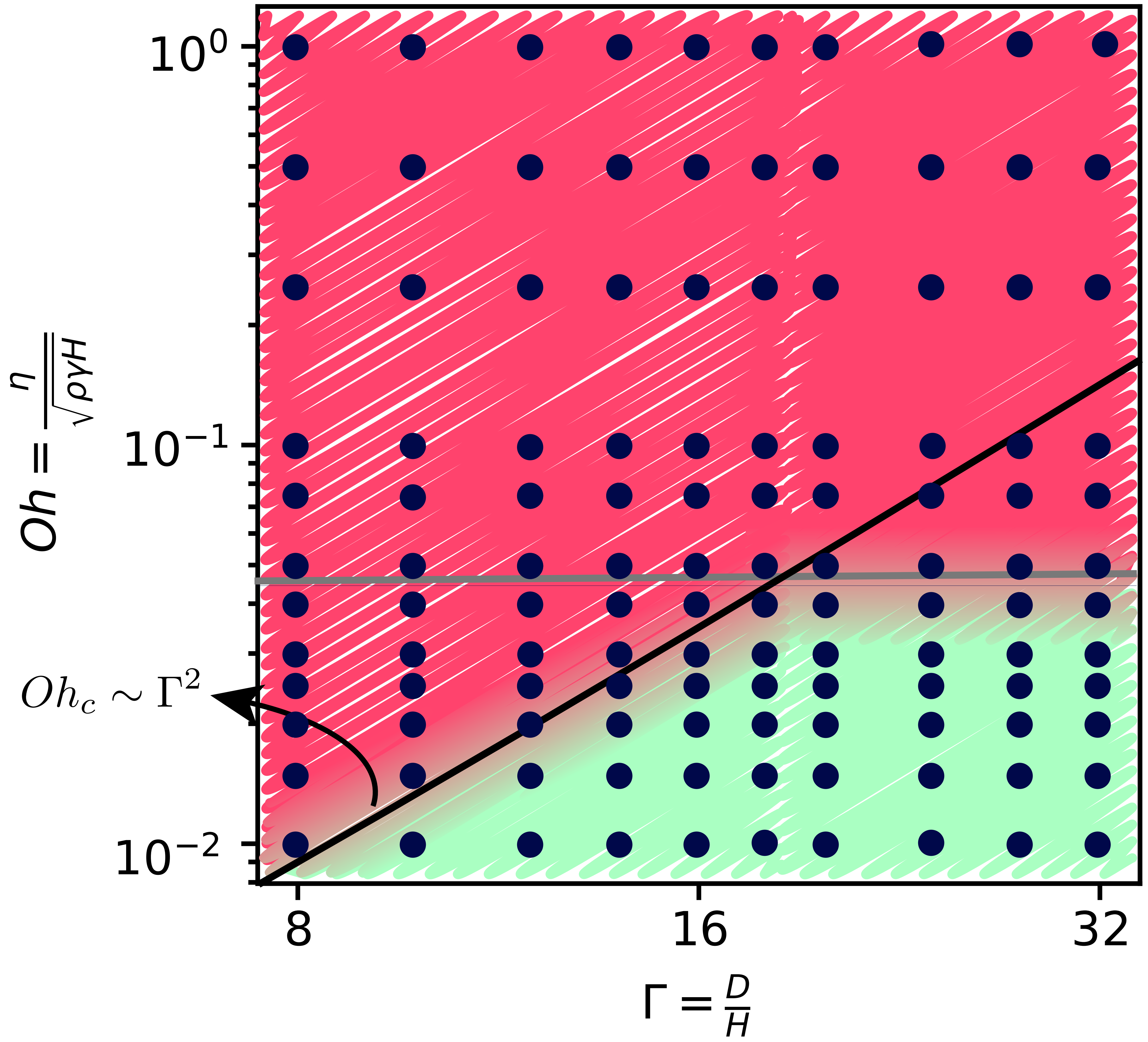
Ω : Volume over which dissipation occurs

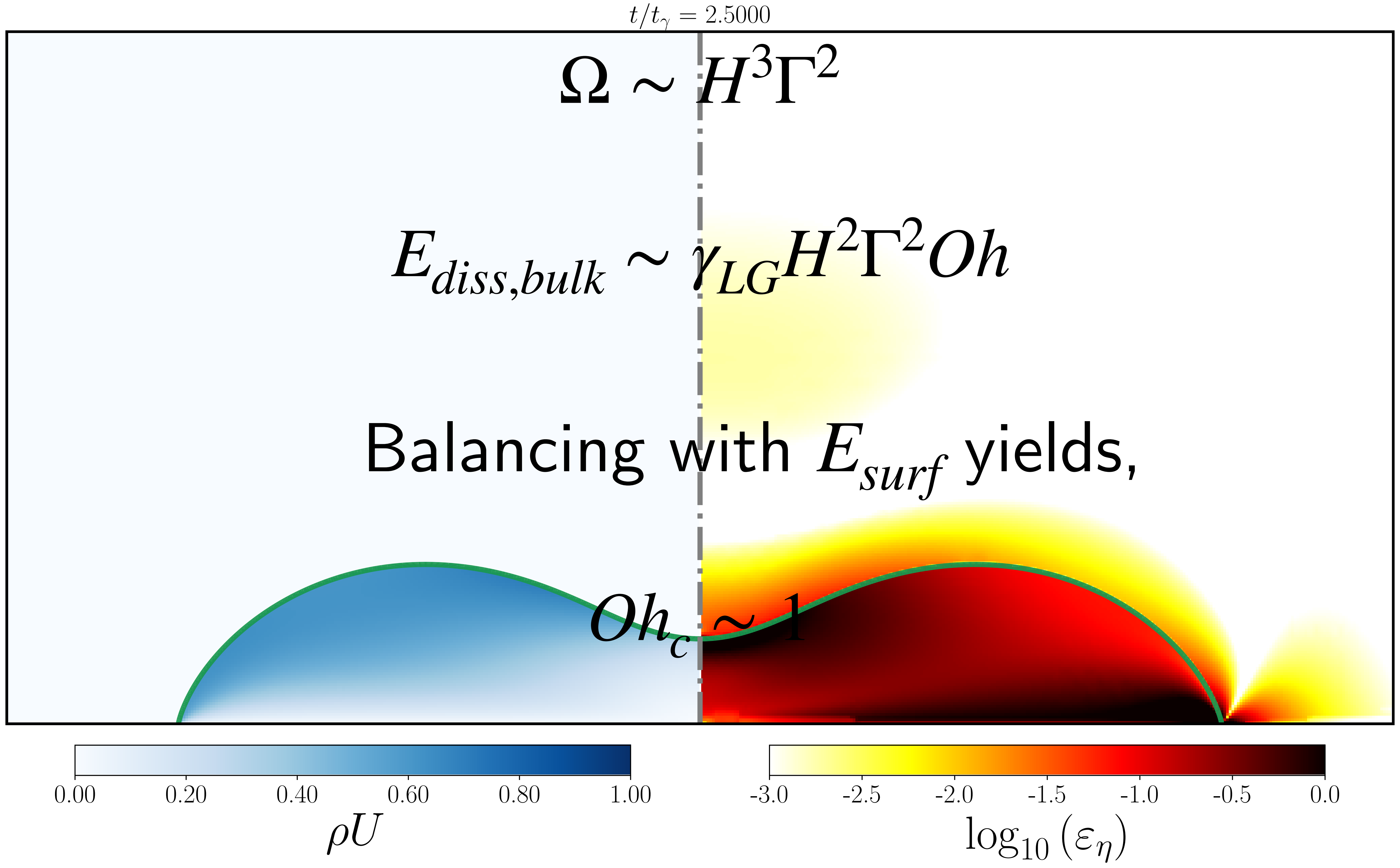
λ : Lengthscale over which velocity gradients develop



Large Oh regime

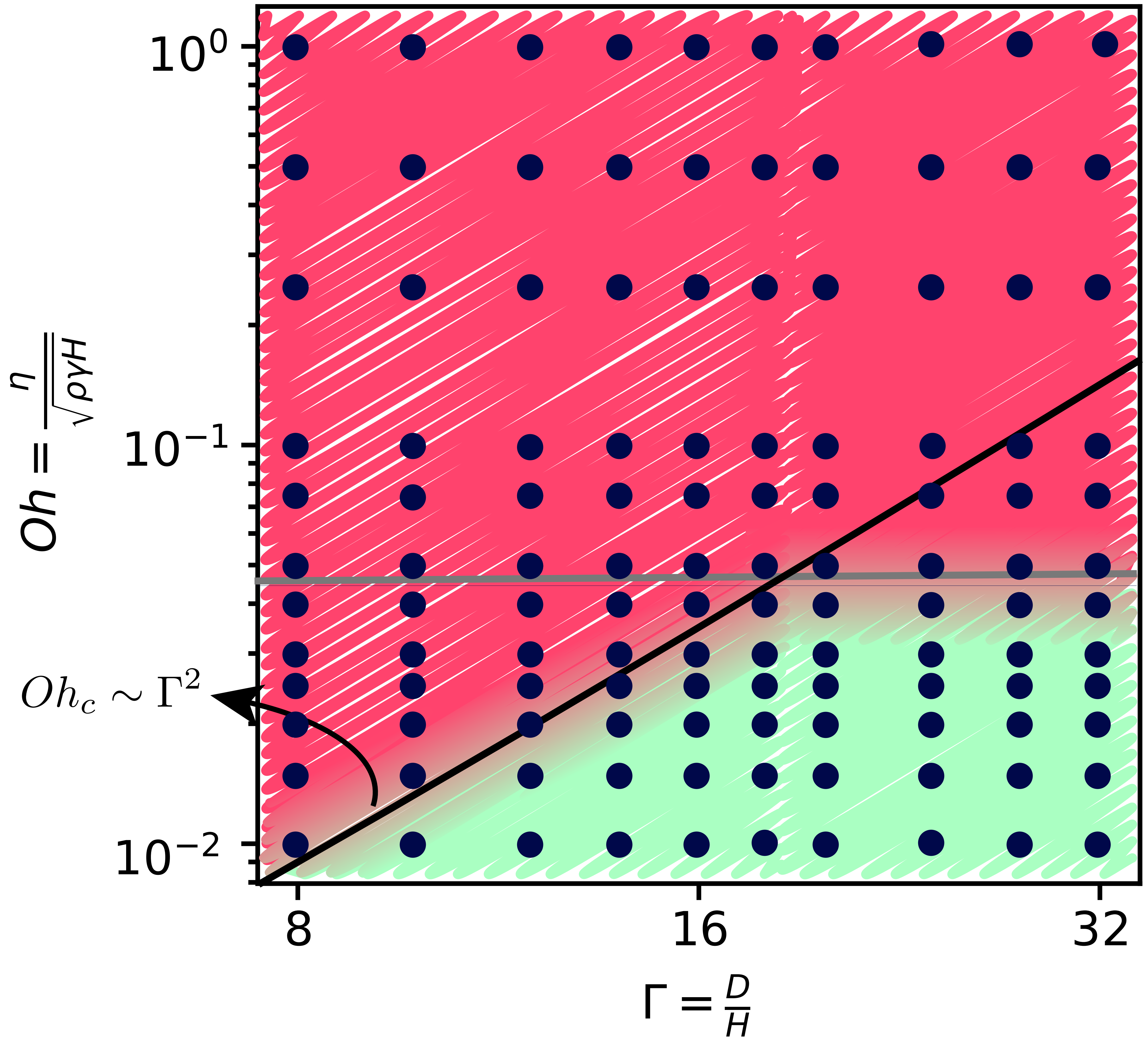
At large Oh, velocity gradients develop immediately throughout the bulk.

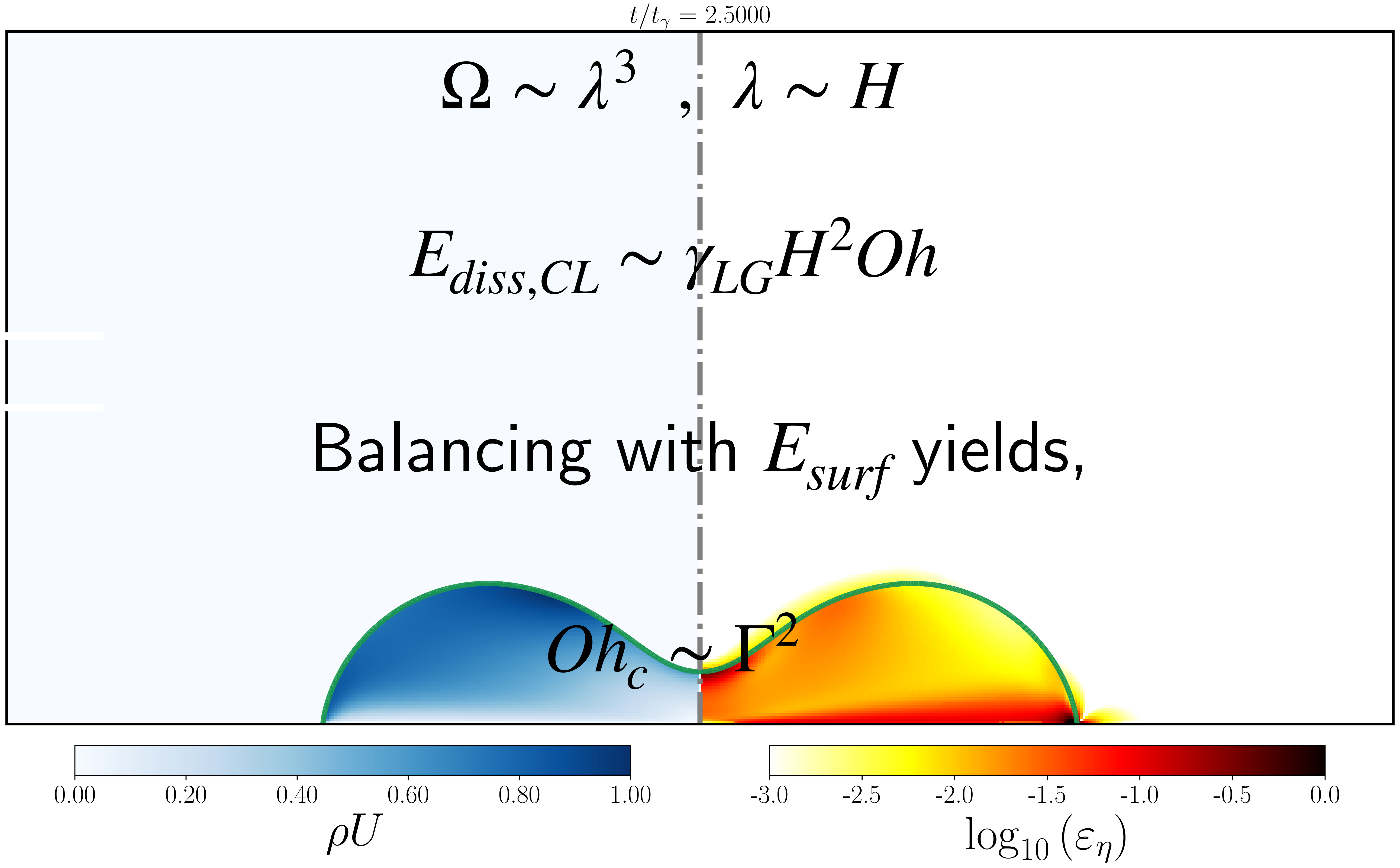




Small Oh regime

At small Oh, viscous effects remain localized near the contact line.





$$\Omega \sim \lambda^3 \quad , \quad \lambda \sim H$$

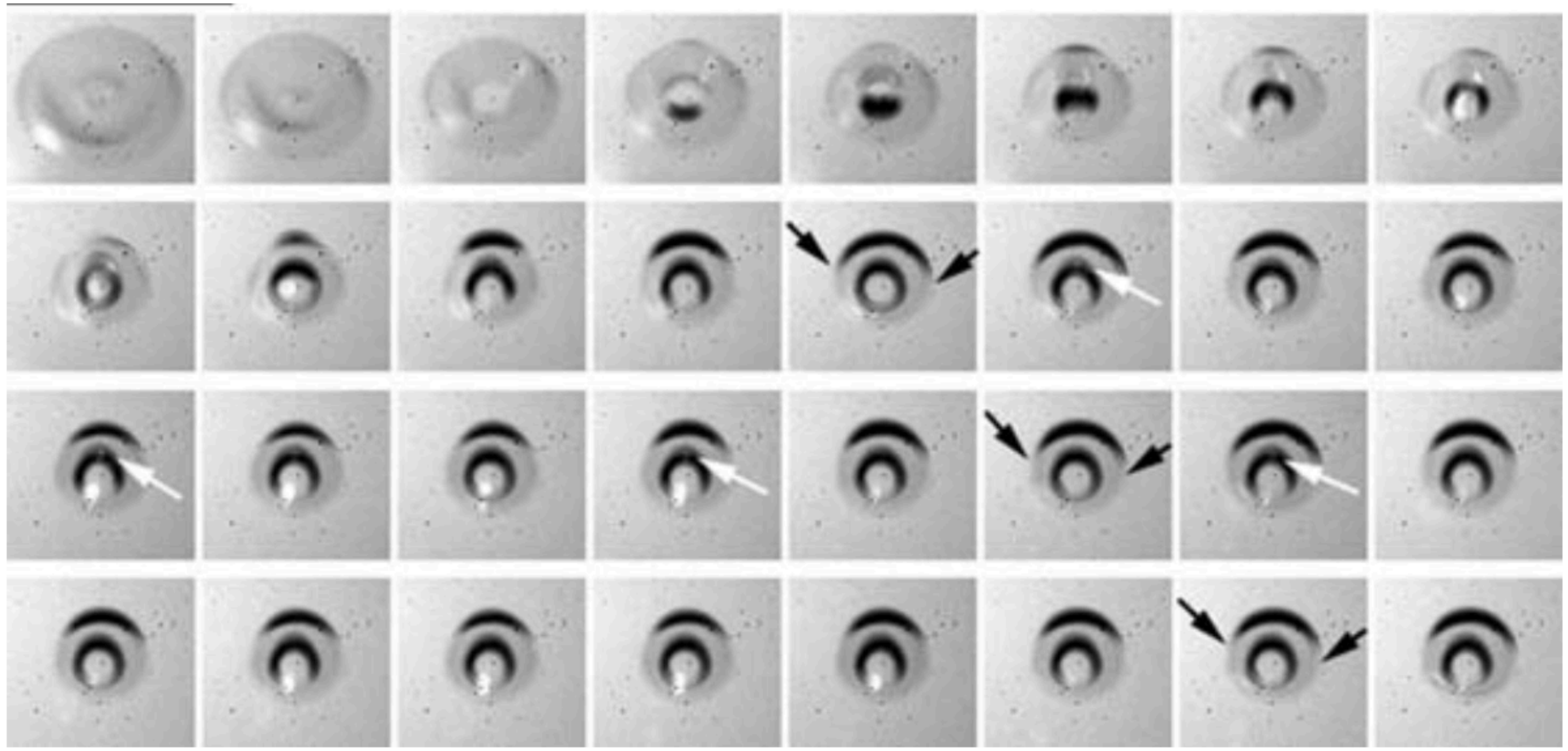
$$E_{diss,CL} \sim \gamma_{LG} H^2 Oh$$

Balancing with E_{surf} yields,

$$Oh_c \sim \Gamma^2$$

Bubble entrainment

Air bubble entrapped under an impacting drop on a solid surface



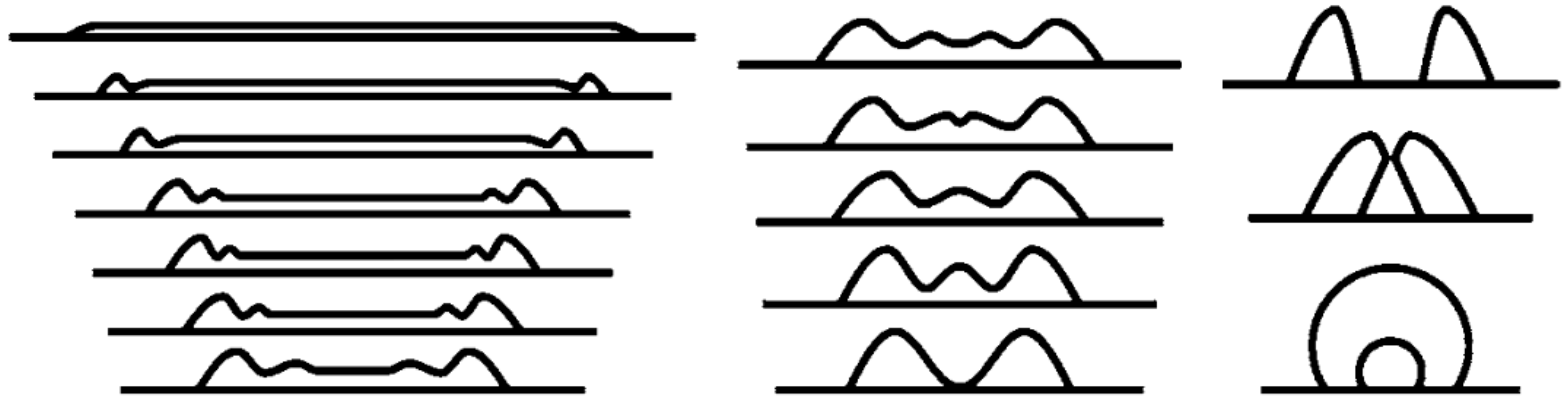
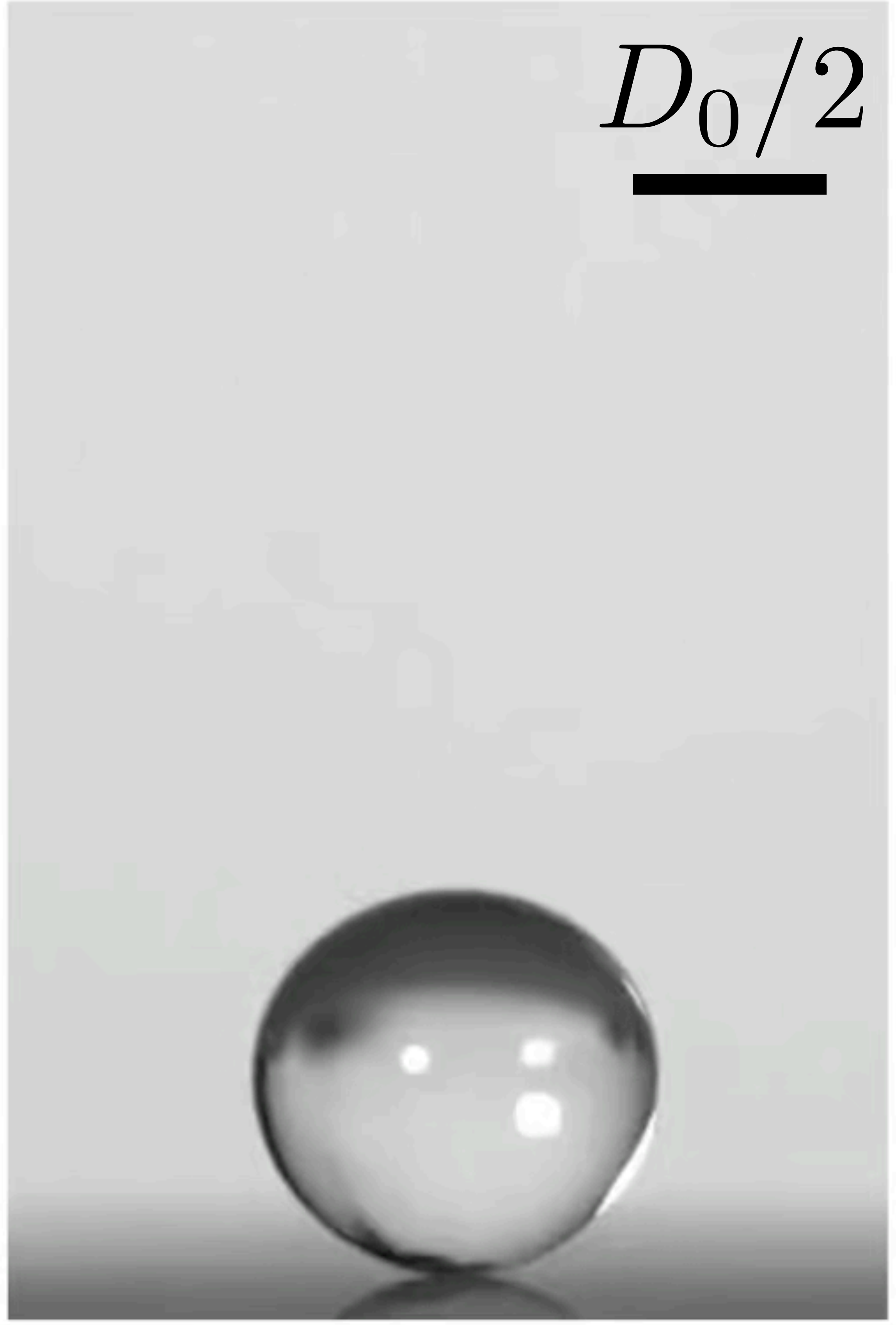
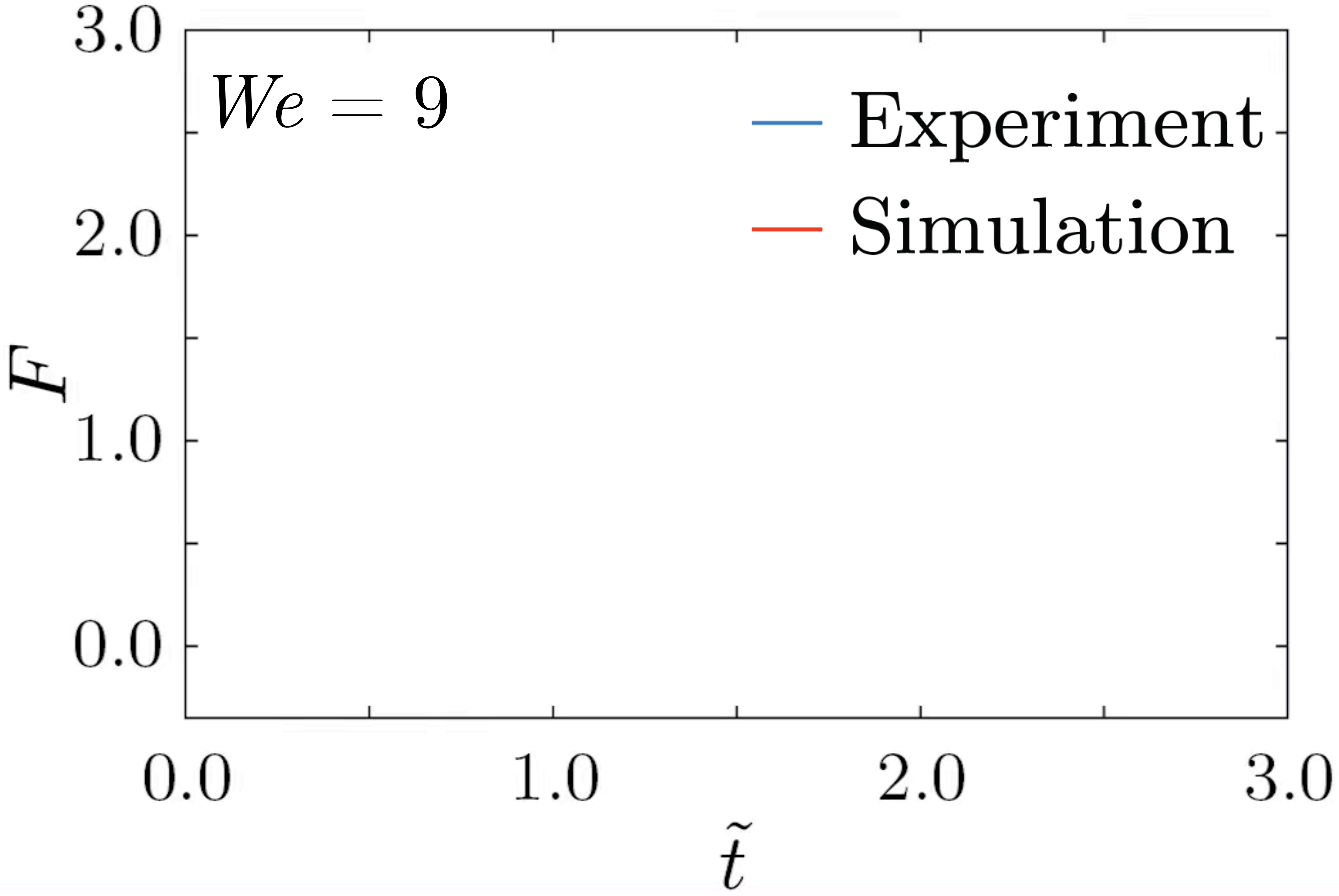


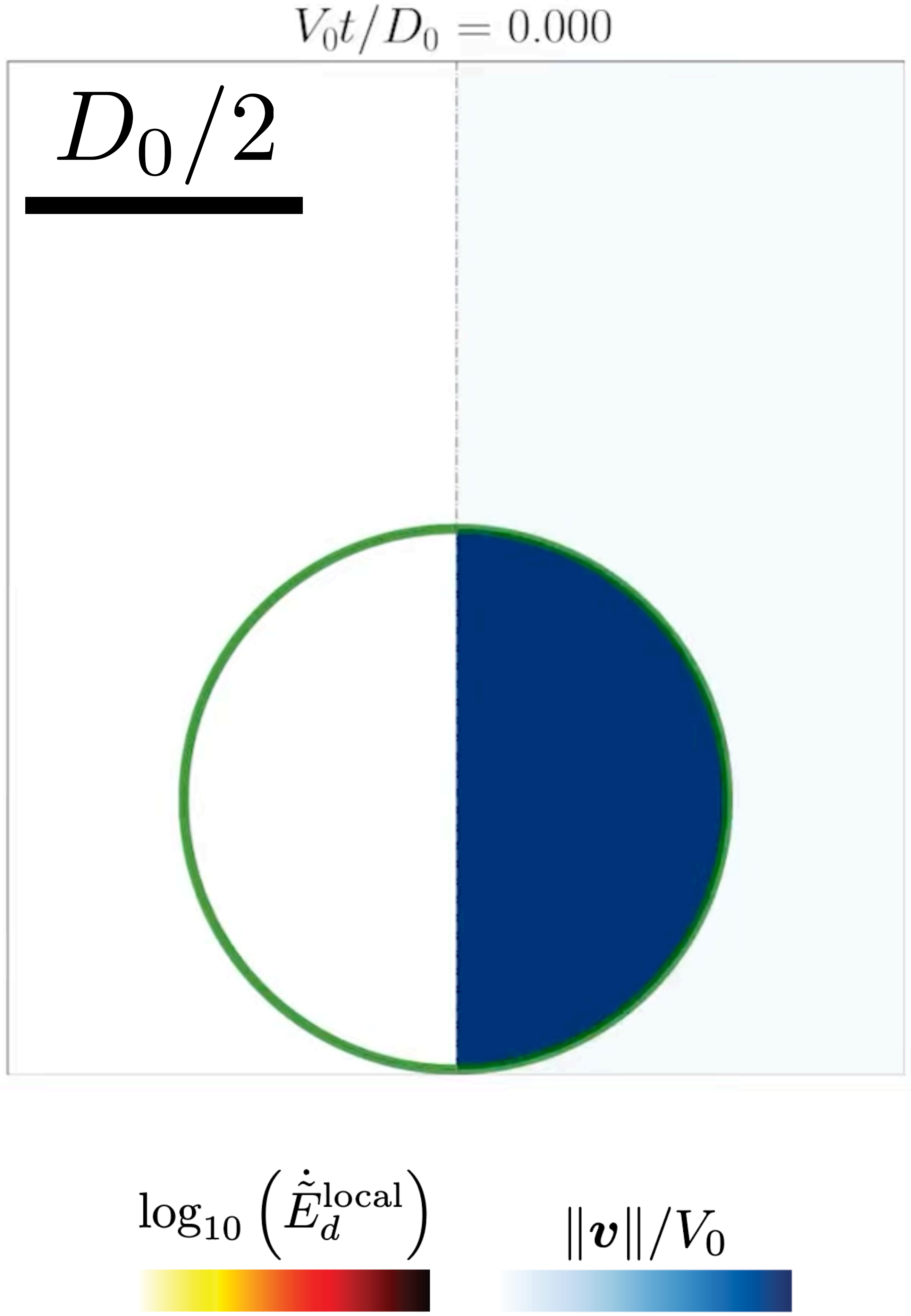
FIGURE 8. Sketch (based on figure 7) of the proposed pinch-off of a droplet inside the bubble, with arbitrary, but greatly exaggerated, vertical scale.

S. T. Thoroddsen, K. Takehara, H. D. Nguyen & T. G. Etoh, J. Fluid Mech., 848, R3 (2018)

Singular jet & bubble during drop impact







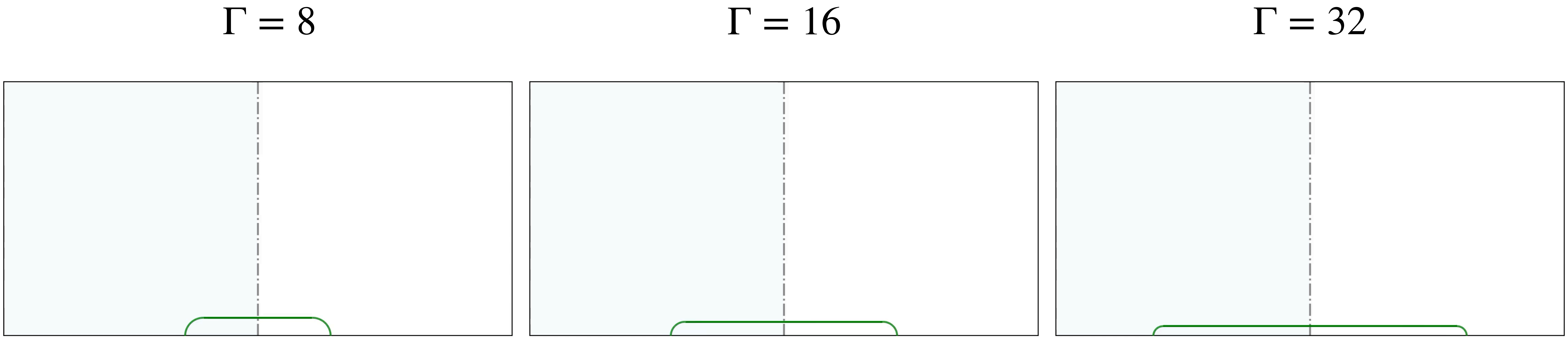
$$We = \frac{\rho_d V_0^2 D_0}{\gamma}$$

Zhang, Sanjay *et al.*, Phys. Rev. Lett., 129, 104501 (2022)

Bubble entrainment

For low Oh, bubbles formation suppressed at low as well as large aspect ratios

Singular bubble entrainment?



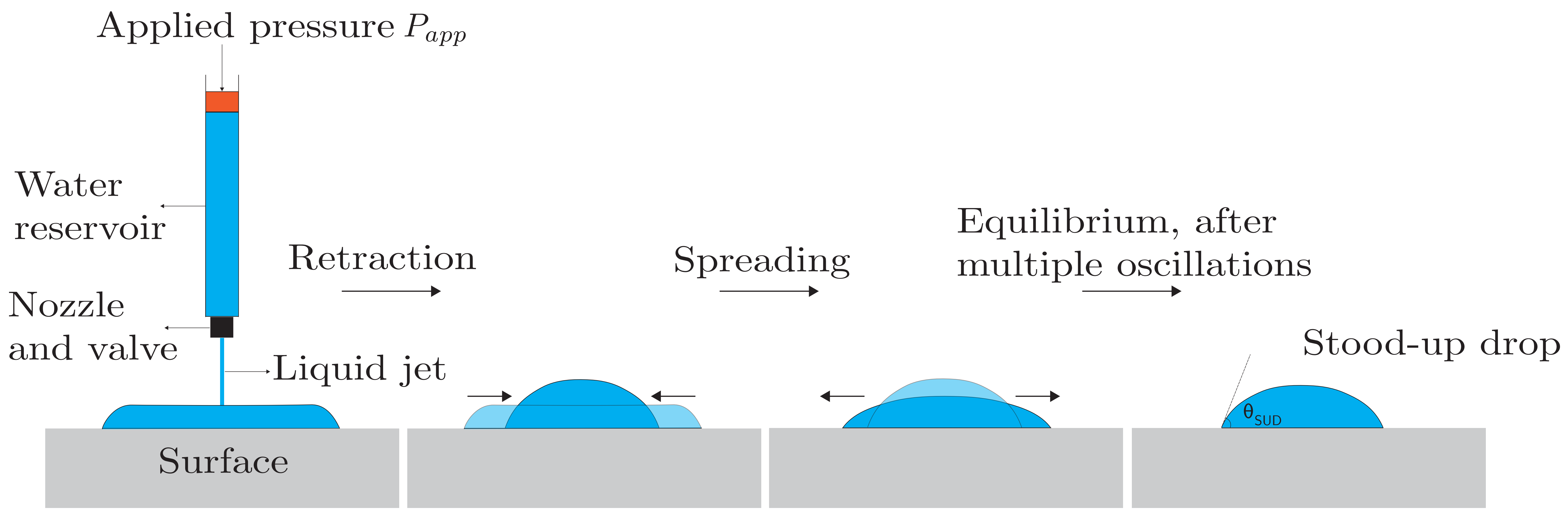
- Capillary waves get damped more at large Oh
- "Sweet-spot" region at moderate Γ, allowing formation of bubbles

$$Oh_l = \frac{\eta}{\sqrt{\rho\gamma H}} \qquad Bo = \frac{\Delta\rho g H^2}{\gamma}$$

$$\Gamma = \frac{D}{H}$$

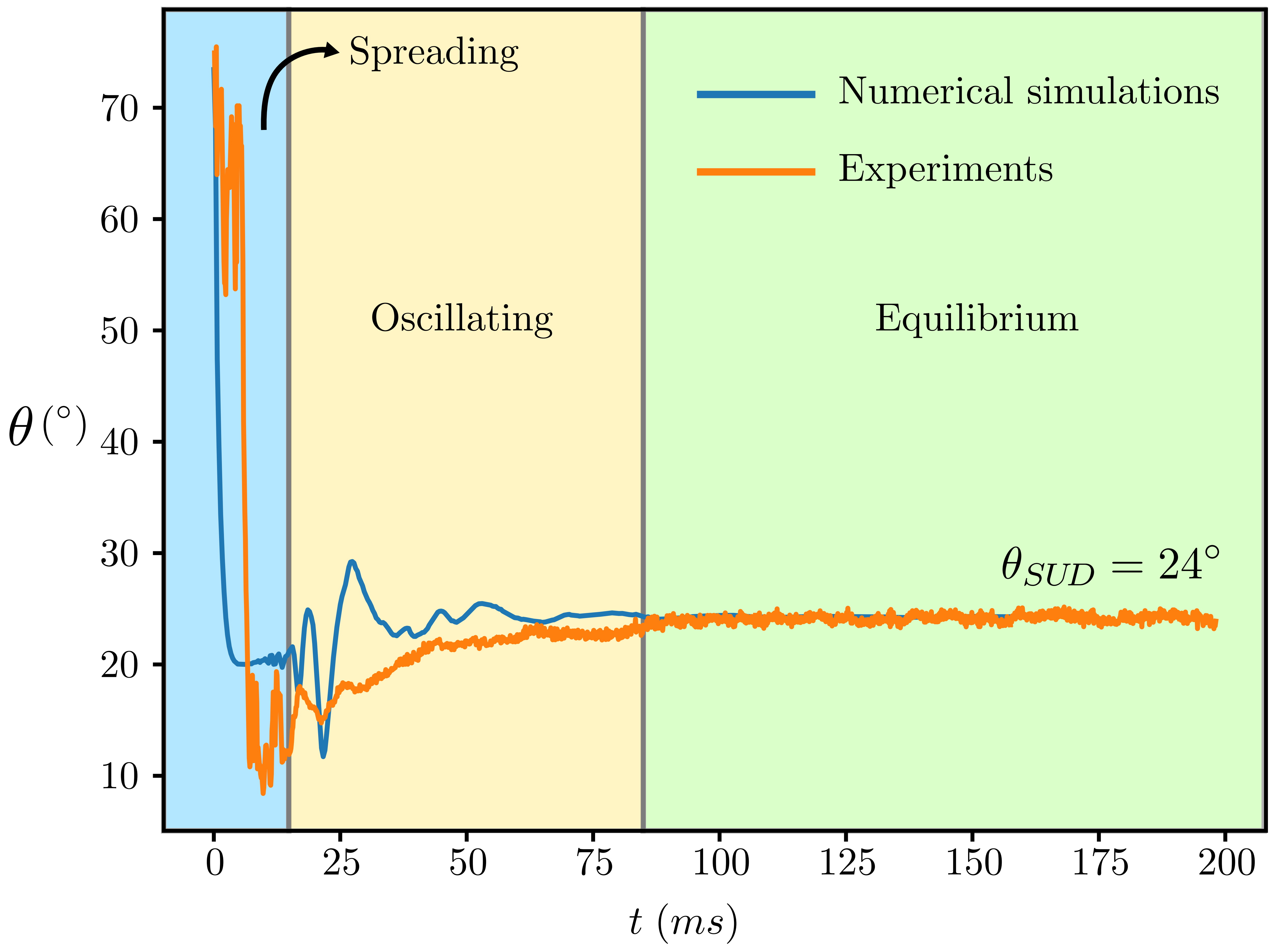
Stood-Up Droplet Technique (SUD)

Doris Vollmer Diego Díaz Thomas Willers

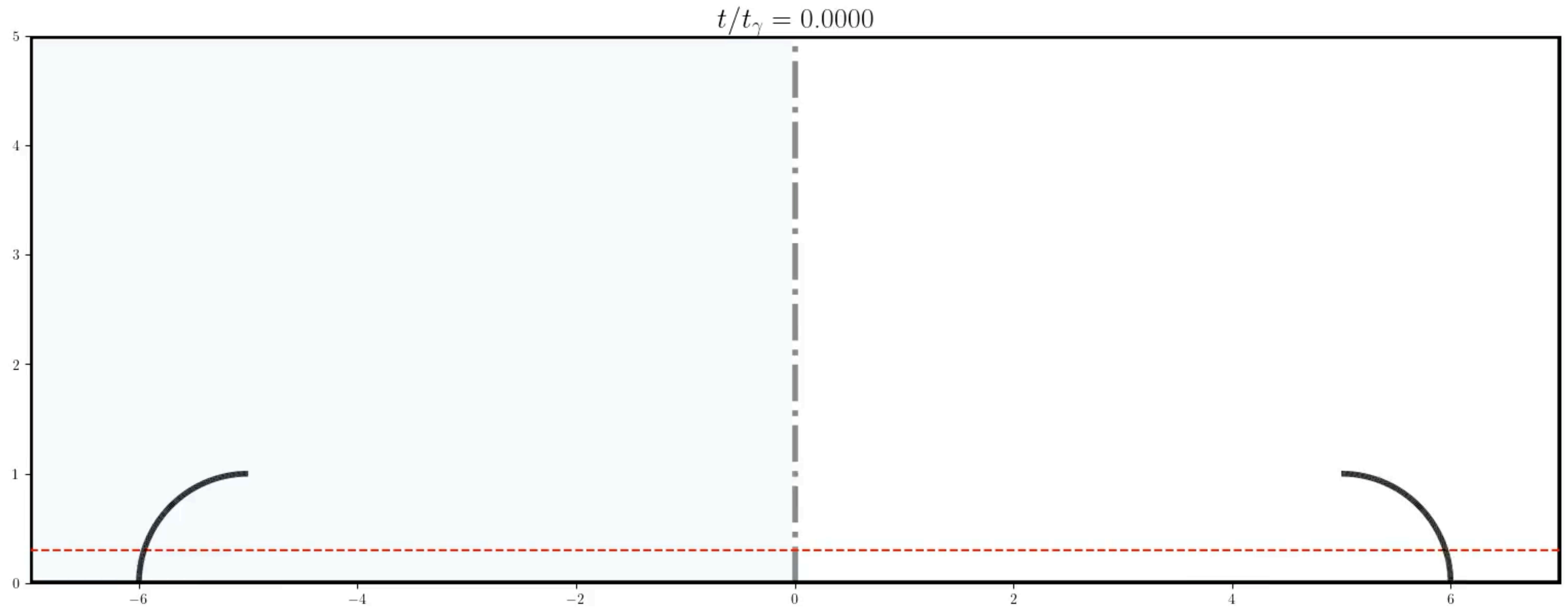


Kinetic energy $\leftrightarrow$ Surface energy

Comparison to experiments



Water on Si wafer

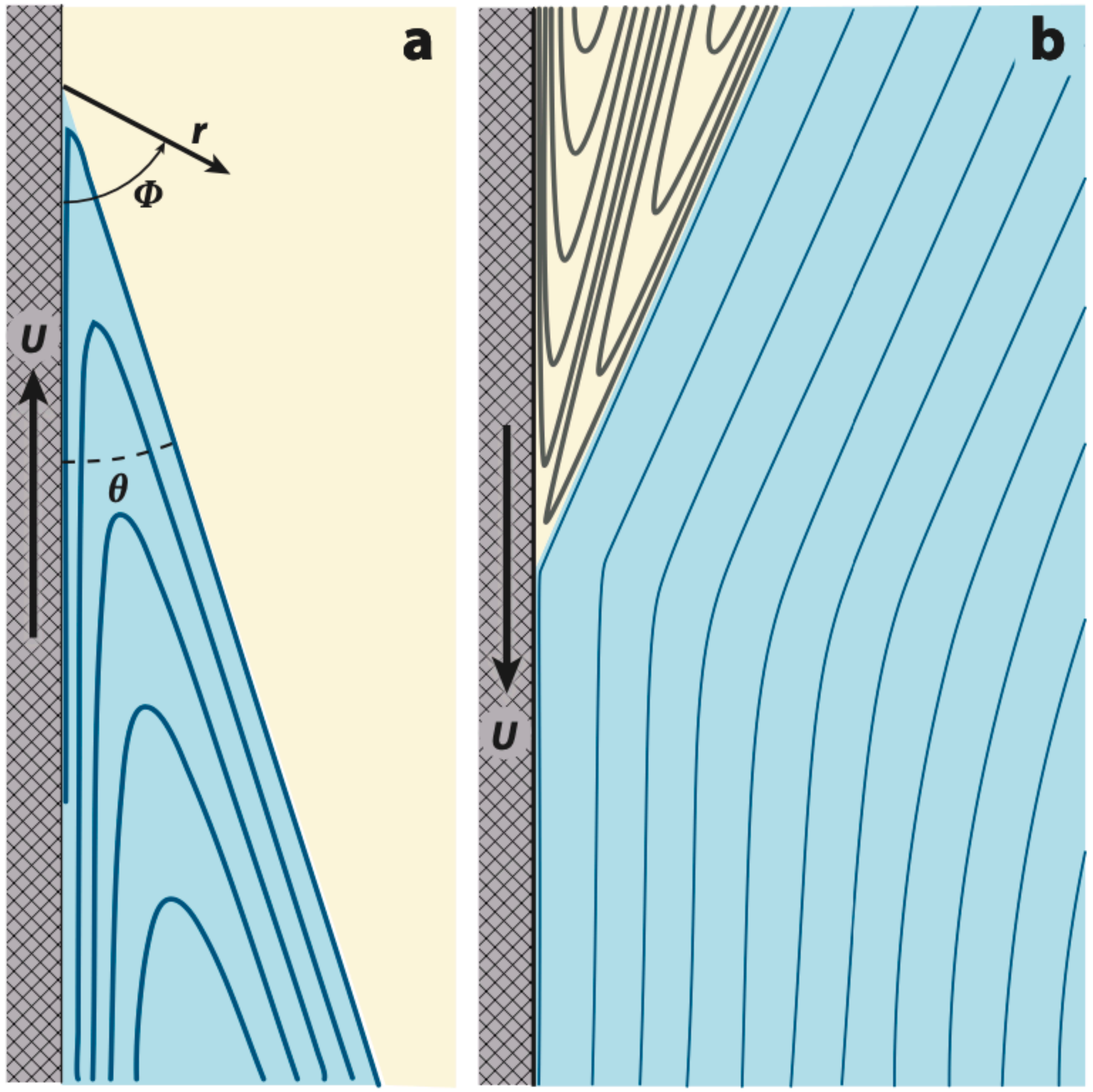


$Bo = 7.6 * 10^{-3}$ $Oh = 5.7 * 10^{-3}$

$\Gamma = 12$ $\theta_s = 27°$

Contact line singularity

Non-integrable energy dissipation



Shear stress : $\varepsilon \sim \frac{\eta U}{r}$ Diverges at $r \to 0$

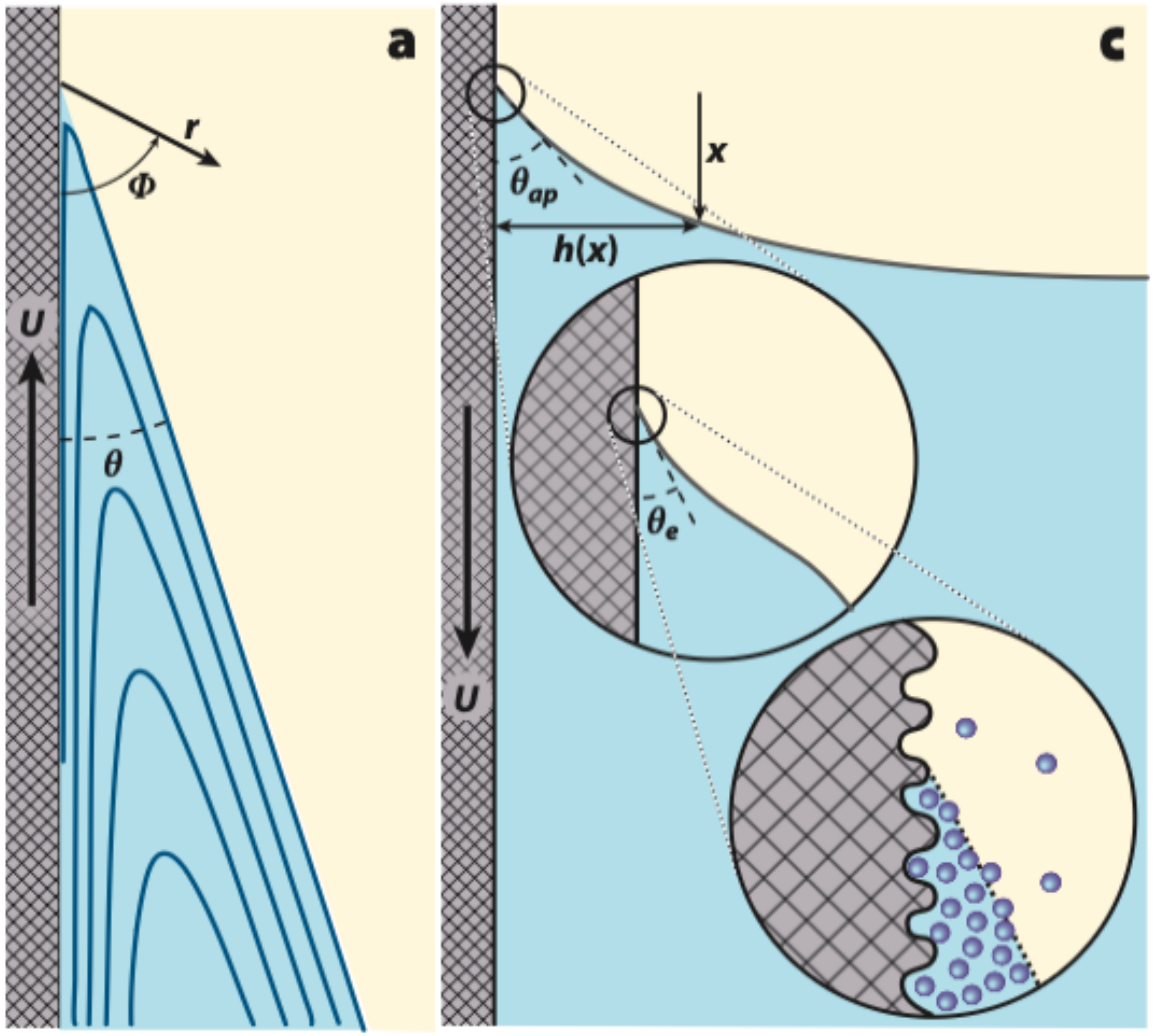
Rate of energy dissipation : $d\dot{E} \sim \eta U^2 \frac{dr}{r} \sim \eta U^2 (d\ ln\ r)$

Not integrable at $r \to 0$ and $r \to \infty$

Each decade in r contributes comparably

de Gennes PG, Rev. Mod. Phys. 57:827–63 (1985)

Need for a better contact line model

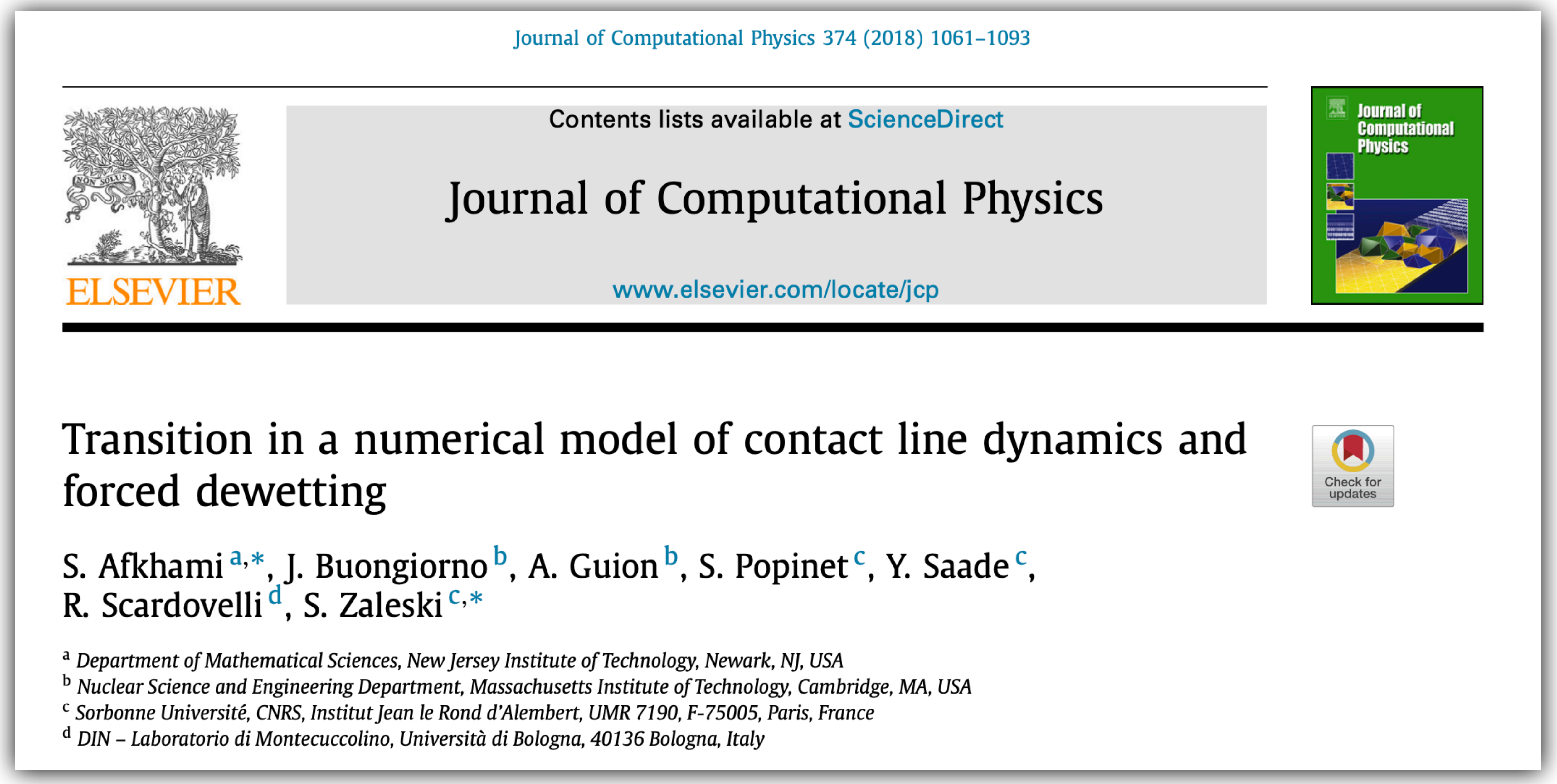


Numerical slip: $\lambda \sim \Delta/2$

For accurate contact line velocities, we need the smallest grid cells to be order of the physical slip length

Not always possible with multiscale problems

Grid dependence in contact line simulations


Journal of Computational Physics 374 (2018) 1061–1093

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Transition in a numerical model of contact line dynamics and forced dewetting

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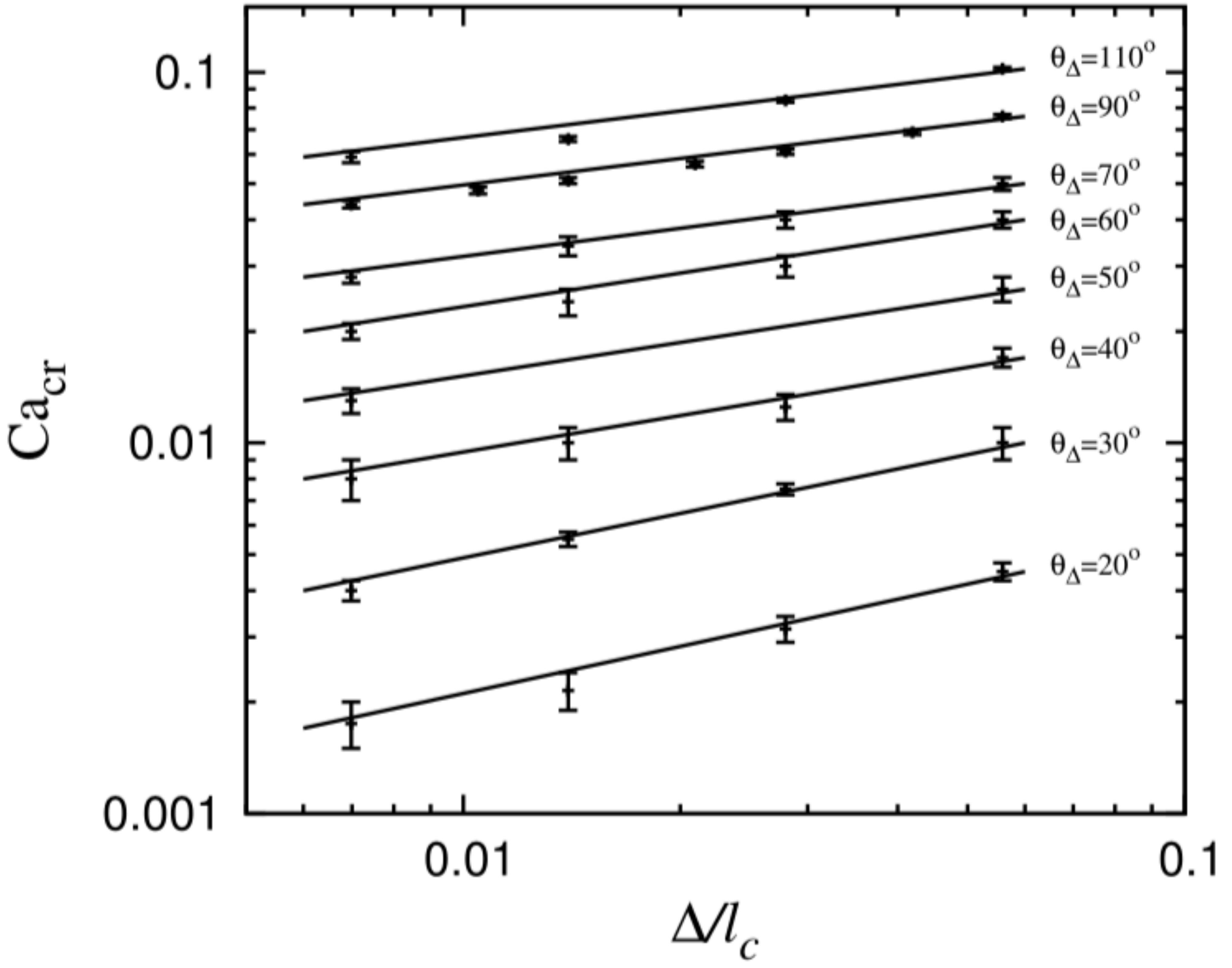
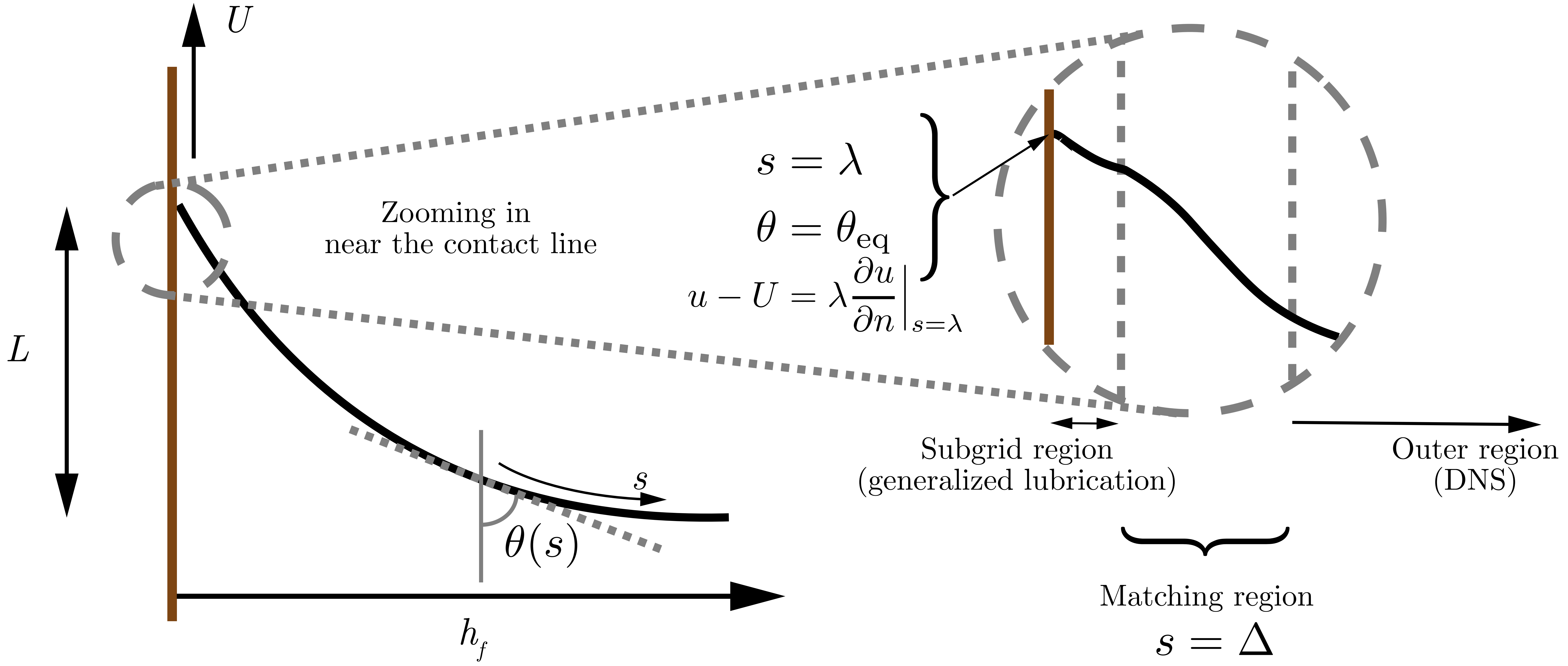


$$Ca_{\text{critical}}\left(l_r, \tilde{\lambda}, \tilde{\Delta}\right) \qquad \tilde{\lambda} = \tilde{\Delta}/2$$

Our subgrid modeling schematic



Summary

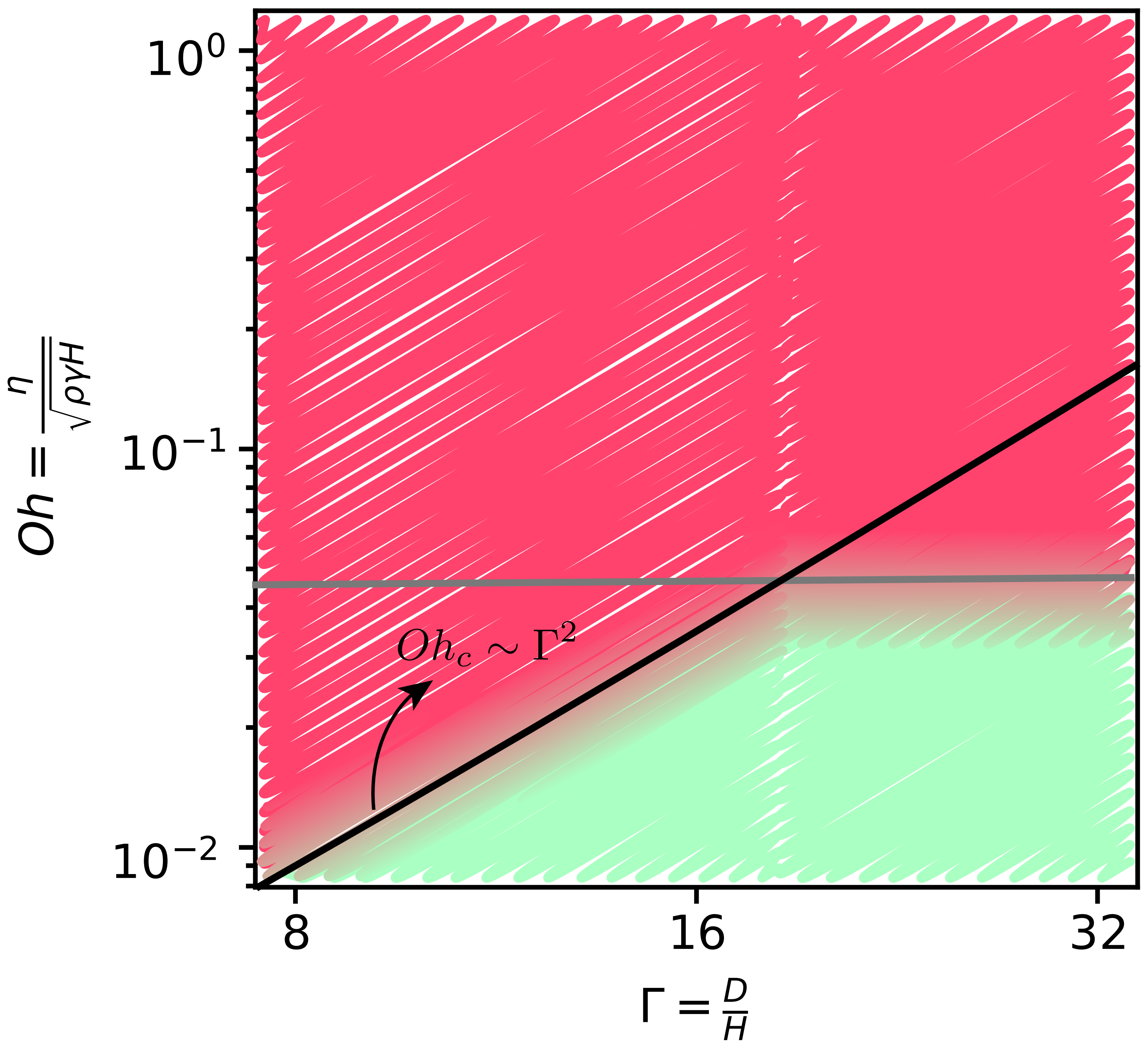
- 2 regimes for transition from surface oscillations to jumping:

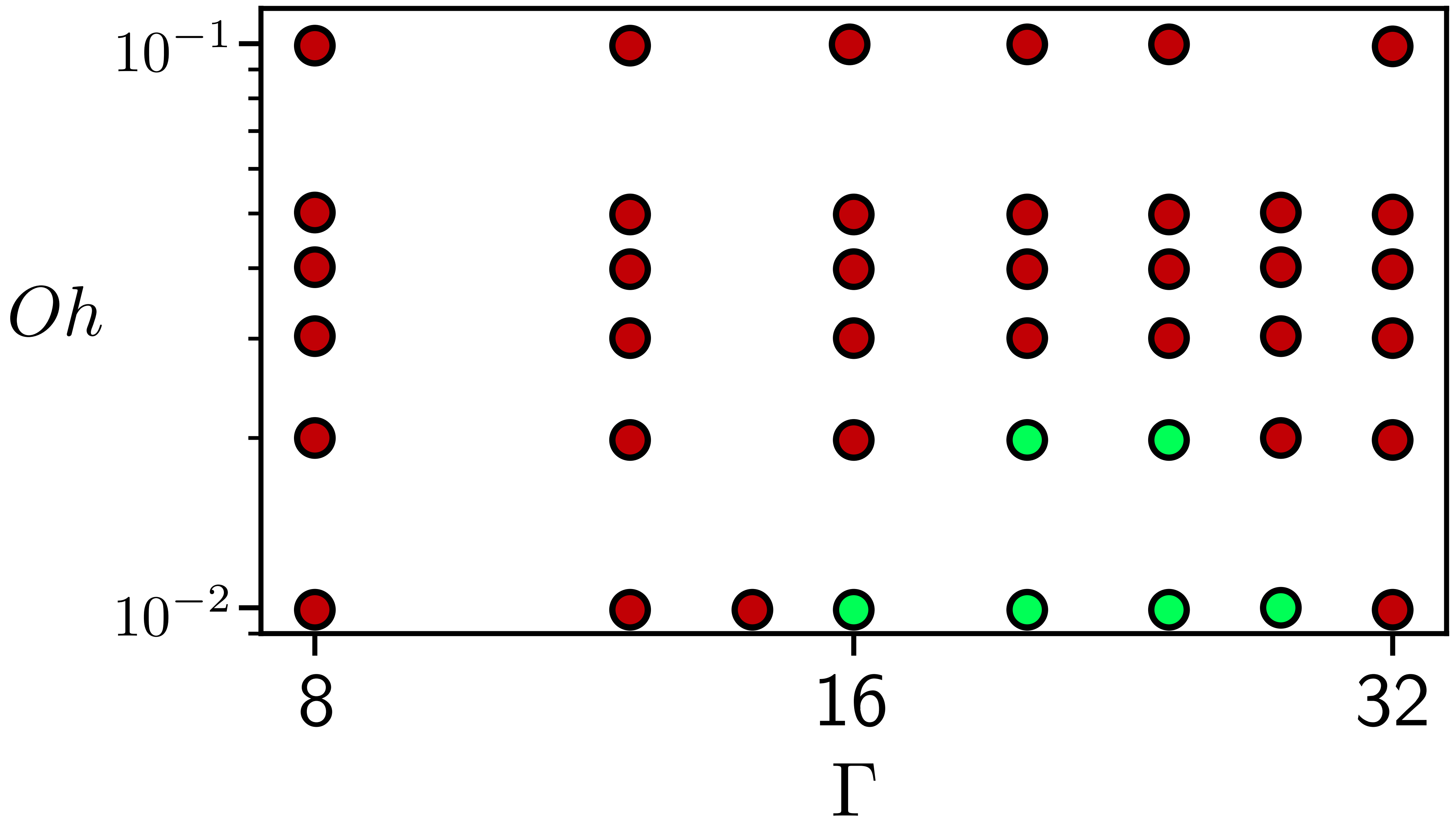
  Small aspect ratio: $Oh_c \sim \Gamma^2$ Large aspect ratio: $Oh_c \sim 1$

- Retracting droplets entrain air bubbles in a “sweet spot” range of moderate Γ

- Developing a mesoscale contact line subgrid model in basilisk

Thank you!


10^0
10^{-1}
10^{-2}
$Oh = \frac{\eta}{\sqrt{\rho\gamma H}}$
$Oh_c \sim \Gamma^2$
8
16
32
$\Gamma = \frac{D}{H}$


10^{-1}
10^{-2}
Oh
8
16
32
Γ