Vorticity and Interfaces

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Effect of vorticity generated at an interface

via surface tension ?

via density difference ?

Gerris/Basilisk

Axisymmetric or 2D flow
Immersed boundary
Two-phase flow
Surface tension
Adaptative mesh refinement

Part I

Interface and generation of vorticity
in Two-dimensional Flows

Part II

Two flow examples on the role of
Vorticity generated at an interface

Lagrangian Domain A bounded by a Closed Material Line C in a Monophasic Case

$$\frac{D\omega}{Dt} = [\partial_t + \mathbf{U}.\nabla]\omega = \nu \boldsymbol{\Delta}\omega$$

Lagrangian Transport

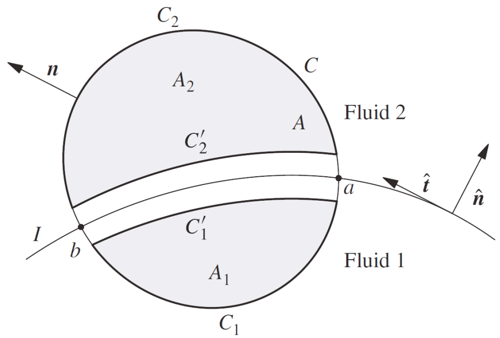
Viscous Diffusion

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\int_A \omega \; \mathrm{d}x\mathrm{d}y\right) = -\int_C J_j \, n_j \, \mathrm{d}s$$

production of vorticity $J_j \equiv -\nu \, \partial_j \, \omega$

Lagrangian Domain A in a Two-phase Flow

Lagrangian domain with an interface separating two phases



Brons et al (JFM 2014)

$$[[Q]] \equiv Q_1 - Q_2$$

Across an interface separating two phases

The no-slip case $[[\vec{u}]] = 0$

Laplace law $[[t_j \mu e_{ij} n_i]] = 0 \qquad e_{ij} \equiv \frac{1}{2}(\partial_j u_i + \partial_i u_j)$

$$p^{(2)} - p^{(1)} + 2[[\mu]](n_i \partial_i u_j) n_j = \frac{\sigma}{R}$$

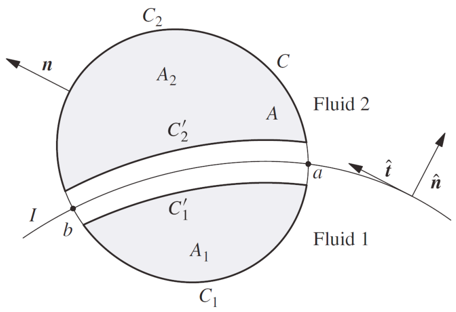
$$n_i(\partial_i [[u_j]]) n_j = 0$$

$$[[\omega]] = [[\frac{1}{\mu}]] \quad t_j \tau_{ij} n_i$$

Free slip $\omega = -2 t_k \partial_k (u_j n_j) - 2\kappa t_j u_j$

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\int_{A_p}\omega\,\mathrm{d}x\mathrm{d}y\right) = -\int_{(c_p U c'_p)} J_j\, n_j\, \mathrm{d}s, \qquad p=1 \quad \text{or} \quad p=2$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\int_{A}\omega\;\mathrm{d}x\mathrm{d}y\right) = -\int_{C} \underbrace{J_j}_{\text{vorticity diffusion}}\, n_j\, \mathrm{d}s + \int_I \underbrace{\Sigma}_{\text{vorticity source}}\,\mathrm{d}s$$



$$\Sigma = \Sigma_1 + \Sigma_2$$

$$\Sigma_1 \equiv -\vec{n}^{1\to 2}\cdot\vec{J}^{(1)}$$

$$\Sigma_2 \equiv -\vec{n}^{2\to 1}\cdot\vec{J}^{(2)}$$

$$\Sigma \equiv -\vec{n}^{1\to 2}\cdot[[\vec{J}]]$$

The no-slip case $[[\vec{u}]] = 0$

Navier-Stokes equation

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p - \vec{e}_z \times \vec{J} + \vec{F}$$

Viscous Diffusion

Projection along the tangential direction on the interface

$$\vec{t}\cdot\frac{D\vec{u}}{Dt} + \frac{1}{\rho}\vec{t}\cdot\vec{\nabla}p - \vec{F}\cdot\vec{t} = -(\vec{t}\times\vec{e}_z)\cdot\vec{J} = -\vec{n}^{1\to 2}\cdot\vec{J}$$

$$\boxed{\Sigma = [[\frac{1}{\rho}\vec{t}\cdot\vec{\nabla}p]] = \frac{\partial}{\partial s}[[\frac{p}{\rho}]]}$$

vorticity generation by tangential pressure gradient

To use the jump in pressure, we rewrite the source as

$$\Sigma = \frac{1}{\rho_m}\frac{\partial}{\partial s}[[p]] + [[\tfrac{1}{\rho}]]\frac{\partial p_m}{\partial s}$$

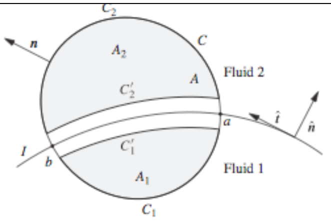
$$\rho_m \equiv \frac{2\rho_2\rho_1}{\rho_1+\rho_2}, \quad p_m \equiv \frac{p_1+p_2}{2}$$

Laplace equation on normal stress and continuity equation

$$[[p]] = -2\,[[\mu]]\,t_i\partial_i(u_j)t_j - \sigma\kappa$$

Interface curvature κ

$$t_i\partial_i(u_j)t_j = \frac{\partial[u_j t_j]}{\partial s} - \frac{\partial t_j}{\partial s}u_j = \frac{\partial[u_j t_j]}{\partial s} - \kappa n_j^{1\to 2}u_j$$



$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\int_A \omega \; \mathrm{d}x\mathrm{d}y\right) = -\int_C J_j \, n_j \, \mathrm{d}s + \int_I \Sigma \, \mathrm{d}s$$

$$\Sigma = -\frac{1}{\rho_m}\frac{\partial(\sigma\kappa)}{\partial s} + [[\tfrac{1}{\rho}]]\frac{\partial p_m}{\partial s} - \frac{2\,[[\mu]]}{\rho_m}\frac{\partial^2[u_j t_j]}{\partial s^2} + \frac{2\,[[\mu]]}{\rho_m}\frac{\partial(\kappa n_j u_j)}{\partial s}$$

extra vorticity source

surface tension

density difference

viscosity difference

viscosity difference

$$\rho_m \equiv \frac{2\rho_2\rho_1}{\rho_1+\rho_2}, \quad p_m \equiv \frac{p_1+p_2}{2}$$

When the interface is a loop, i.e. phase 1 included in phase 2

$\int_I \Sigma \, \mathrm{d}s = 0$ since the vorticity source is a gradient

Equal production of positive or negative vorticity

Σ is the total production but not
the production in each respective phase

$$\Sigma_1 - \Sigma_2 = \frac{1}{2}[[\frac{1}{\rho}]] t_i \, \partial_i [[p]] + 2 \frac{t_i}{\rho_m} \, \partial_i p_m - t_j \frac{\partial (\vec{u} \cdot \vec{n})^2}{\partial x_j} - 2\kappa (\vec{u} \cdot \vec{t})(\vec{u} \cdot \vec{n}) + 2 \frac{D}{Dt} [\vec{u} \cdot \vec{t}]$$

Two-phase Flow Problems

In the sequel we set $[[\mu]] = 0$

$$\Sigma = -\underbrace{\frac{\sigma}{\rho_m}}_{\text{surface tension}}\frac{\partial \kappa}{\partial s} + \underbrace{[[\frac{1}{\rho}]]}_{\text{density difference}}\frac{\partial p_m}{\partial s}$$

Continuity of velocity gradients at interface

$$[[\omega]] = [[e_{ij}]] = 0$$

A Single Vortex with an Interface separating two phases

Fluid 1 is on the left of the «interface» and contains the vortex

$$u_\theta = \Gamma \frac{1 - \exp(-(r/a_0)^2)}{2\pi r}, \quad \omega_z = \frac{\Gamma}{\pi a_0^2} \exp\left(-(r/a_0)^2\right)$$

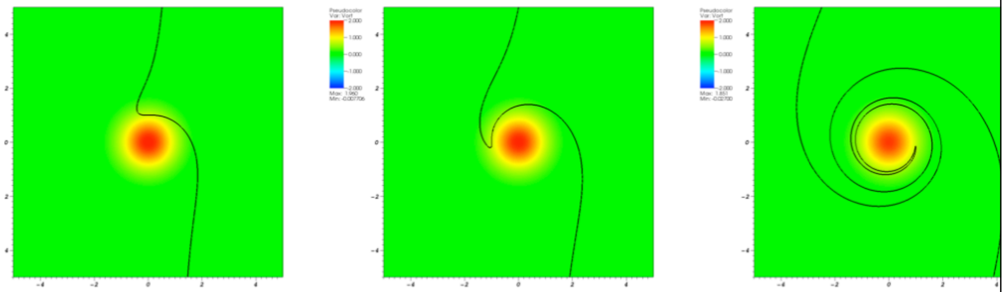
Interface with a surface tension σ initially located at $x = a_0$

Fluid 2 is on the right of the «interface»

Dimensionless Numbers

$$We = \frac{\rho_1 \Gamma^2}{(2\pi)^2 a_0 \sigma} \qquad r_\rho \equiv \frac{\rho_2}{\rho_1} \qquad Re = \frac{\Gamma}{2\pi \nu_1}$$

$$We = \infty \qquad r_\rho \equiv \frac{\rho_2}{\rho_1} = 1$$



$$\theta = \frac{1}{r^2}[1 - \exp(-\frac{r^2}{r_1^2(t)})]t + \phi_0, \qquad r = \frac{1}{\mid \cos\phi_0 \mid} \quad \text{with} \;\; \phi_0 \in]-\pi/2, \pi/2[$$

Roll-up

Width $h \sim \dfrac{1}{t^3}$

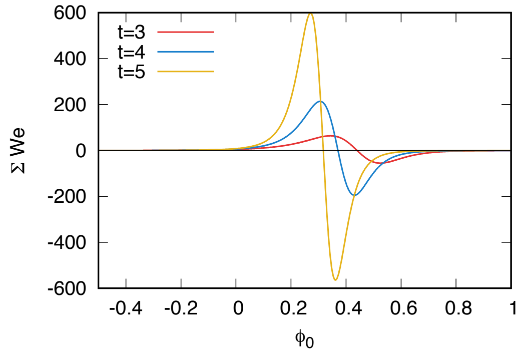
«Two-phase» flows

with surface tension only

$$We \neq \infty$$

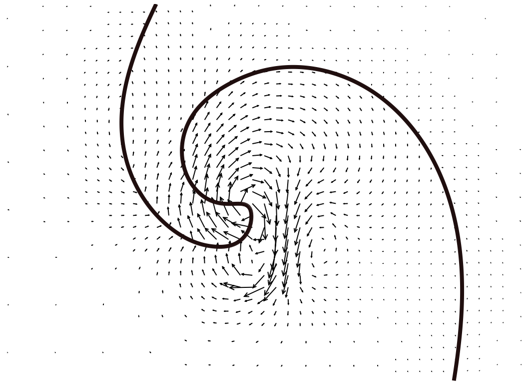
$$r_\rho \equiv \frac{\rho_2}{\rho_1} = 1$$

$$\Sigma = -\frac{1}{r_\rho We}\frac{\partial \kappa}{\partial s}$$



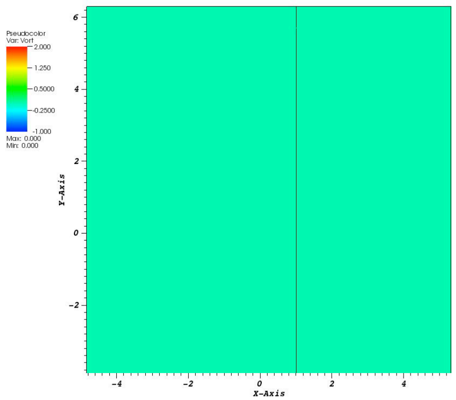
Vorticity stronger around the tip of the interface

Velocity generated by the extra vorticity



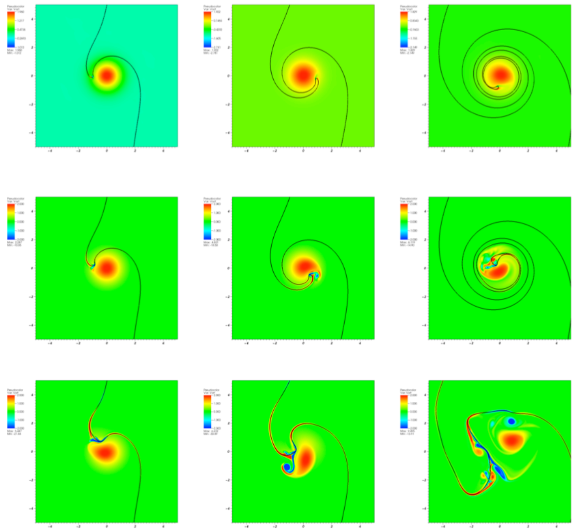
vorticity stronger on the interface than in the vortex

$$We = 100$$



Formation of a rim around the tip

Shedding of vorticity near the tip

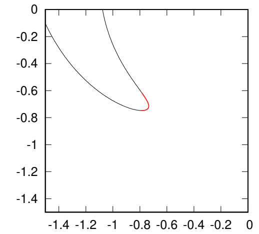


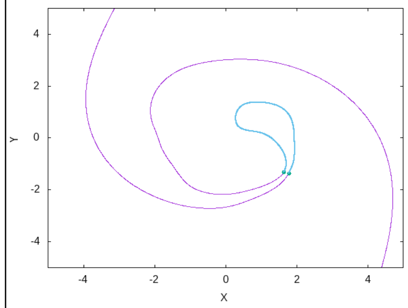
$We = 10^3$

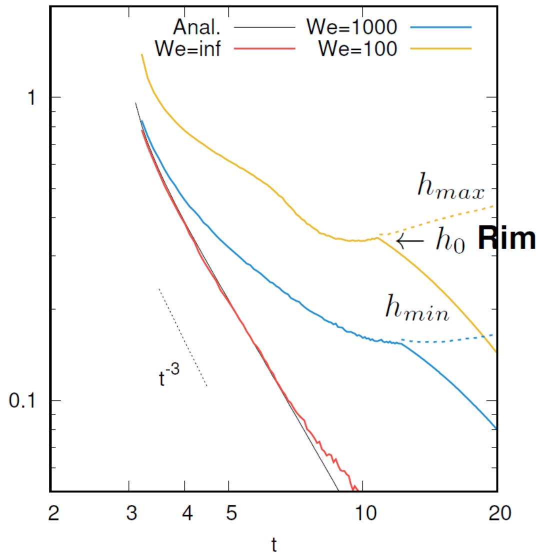
$We = 10^2$

$We = 10$

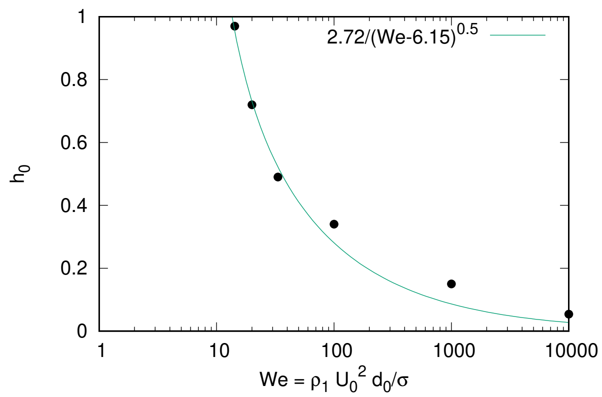
Rim larger with decreasing Weber







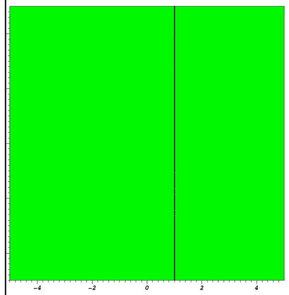
computation of the rim



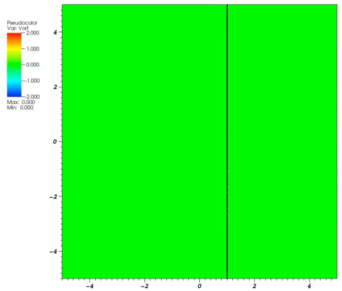
Thickness of the rim as a function of We

$$We_c \sim 14$$

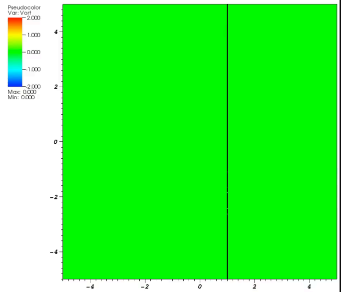
Transition = Roll-up versus No roll-up



$We \sim 14$



$We = 10$



$We = 5$

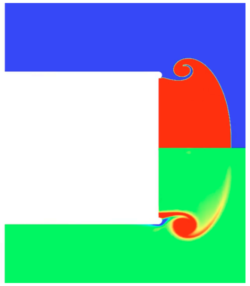
creation of a dipole

Two-phase Flow Axisymmetric Nozzle Problem

Fluid ejected in another immiscible fluid

Axisymmetric nozzle of radius R

Tube thickness $2b_0$



Fluid 1 fluid ejected ρ_1 ν_1

Fluid 2 ambient fluid ρ_2 ν_2

Surface tension σ

At the nozzle exit, a velocity field is imposed

$$u_x(r,t) = U_0\,\mathrm{erf}(t/T)\,\mathrm{erf}(\eta) \quad \text{with} \quad \eta = \frac{y}{\sqrt{2}\,\Delta_0}, \qquad \text{and } y \equiv R - r \geq 0$$

Thickness of the boundary layer Δ_0

Dimensionless Numbers

$$We = \frac{\rho_1 U_0^2 R}{\sigma} \qquad r_\rho \equiv \frac{\rho_2}{\rho_1}$$

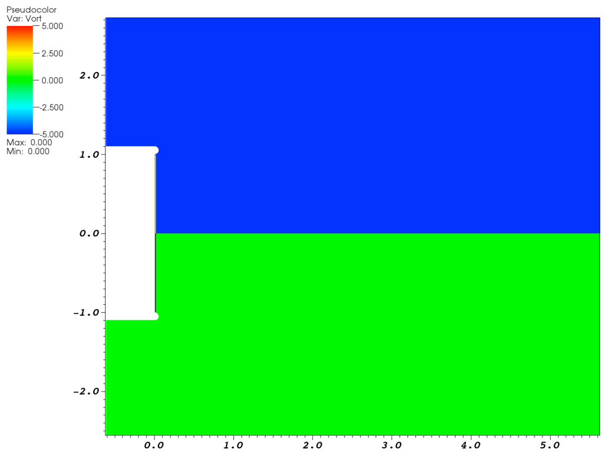
$$Re = U_0 R/\nu_1 = 1000 \qquad r_\mu \equiv \frac{\mu_2}{\mu_1} = 1$$

Three aspect ratios

$$b_0/R = 0.05 \qquad \Delta_0/R = 0.05 \qquad U_0 T/R << 1$$

Axisymmetric Nozzle Problem

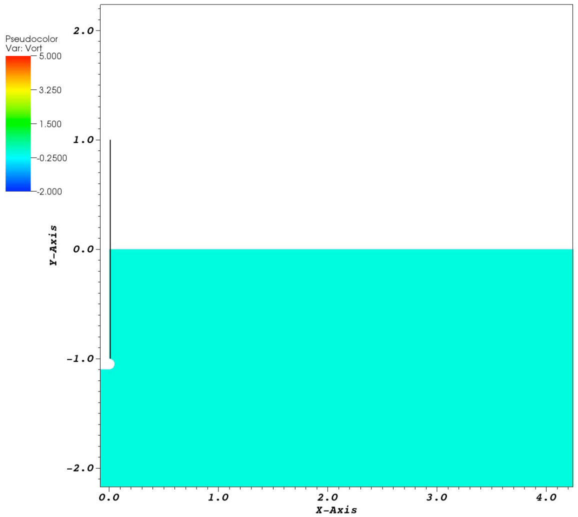
$$Re = U_0 R/\nu = 1000$$



Passive scalar field

Vorticity field

Vorticity Roll-up and Entrainment

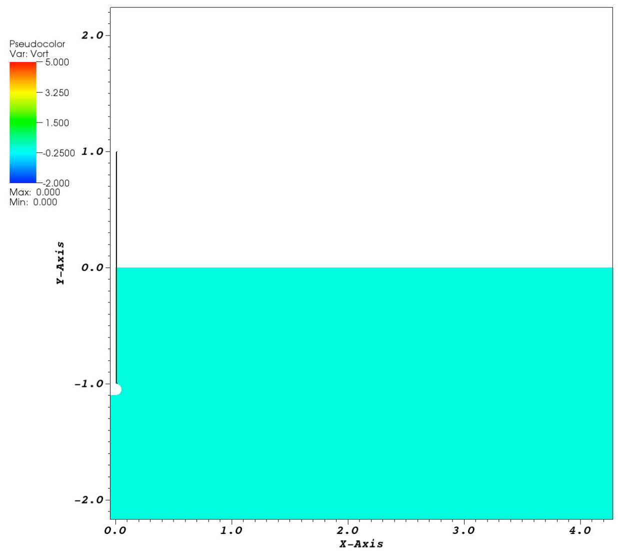


$We = 1000$

orange line:
classical case

Roll-up and Formation of a Rim

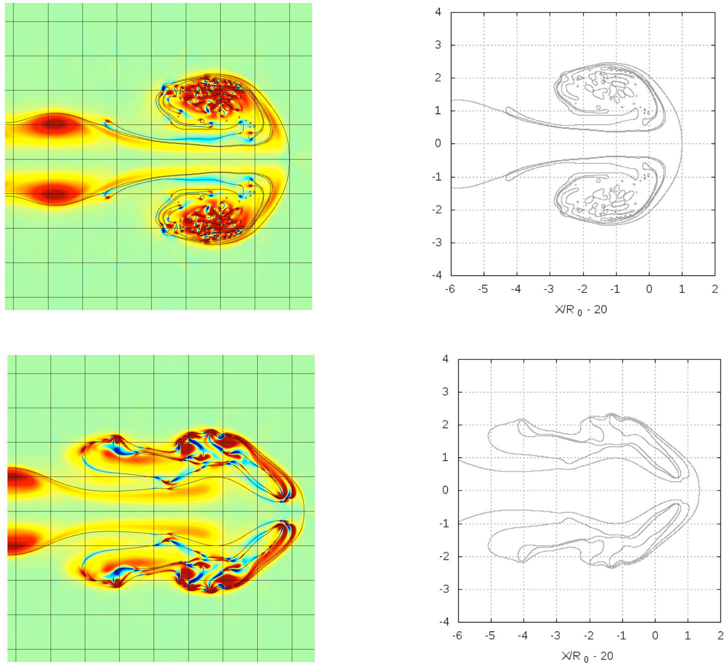
Vortex Slightly Modified



$We = 100$

Roll-up and Formation of a Larger Rim
Vorticity Field Modified

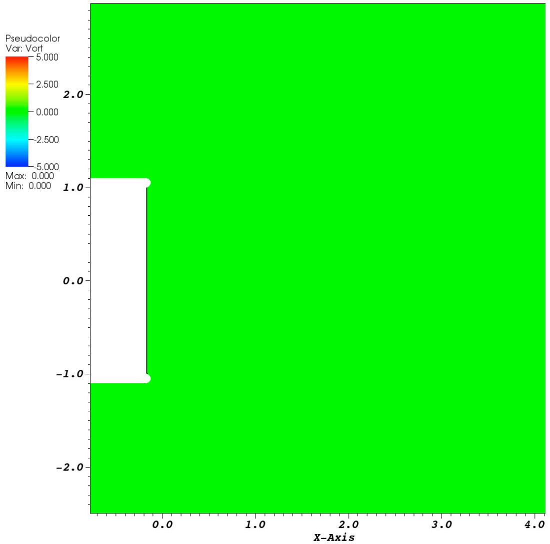
Fragmentation (t=40): Vorticity and Interface



$We = 1000$

$We = 100$

Transition for two-phase flow with surface tension



$We = 100$

Roll-up Regime

Entrainment

$We = 10$

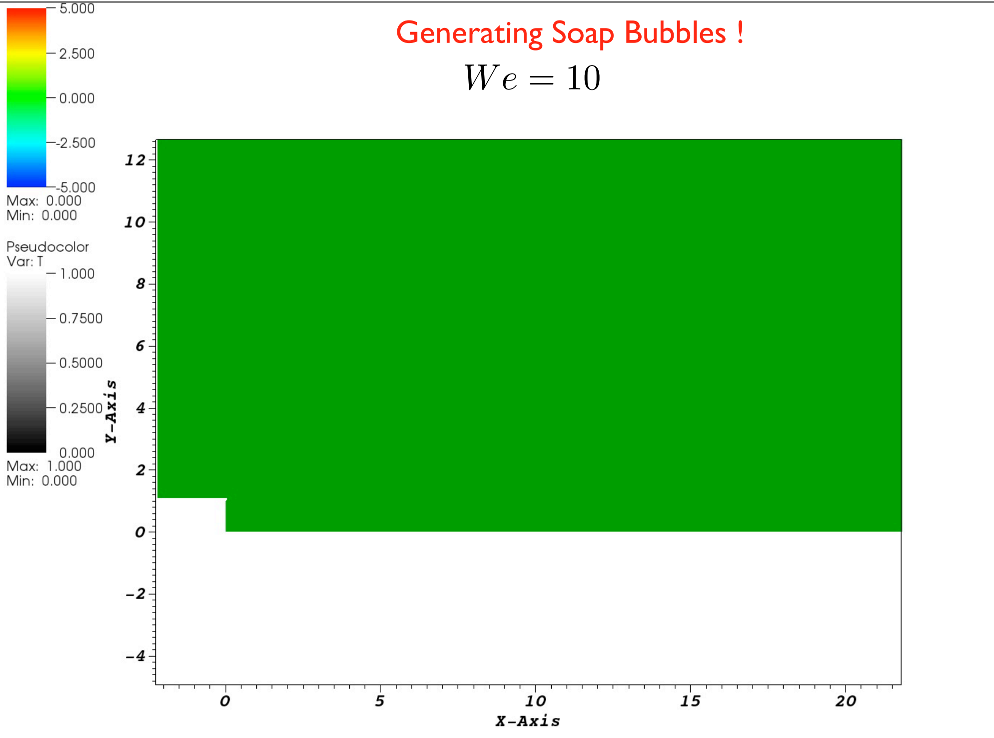
Bag Regime

No entrainment

Vortex roll-up inside ejected fluid

Generating Soap Bubbles !

$We = 10$



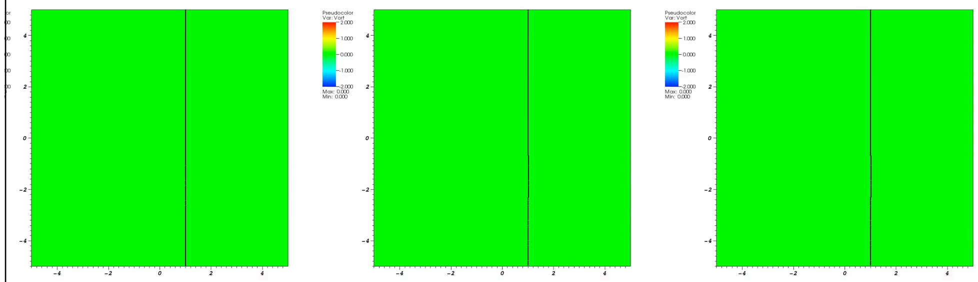
Two-phase flows

with density difference only

$$r_\rho \equiv \frac{\rho_2}{\rho_1} \neq 1$$

$$We = \infty$$

Interface Roll-up and Entrainment Modified



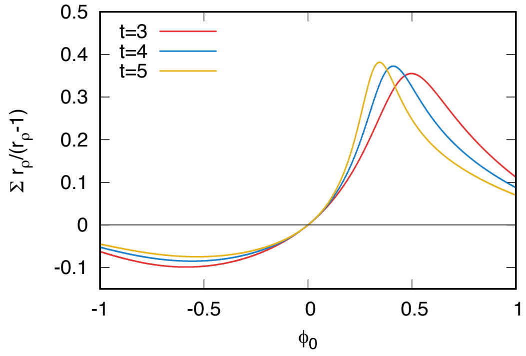
$We = \infty$

$\rho_2/\rho_1 = 1$ $\rho_2/\rho_1 = 0.83$ $\rho_2/\rho_1 = 1.2$

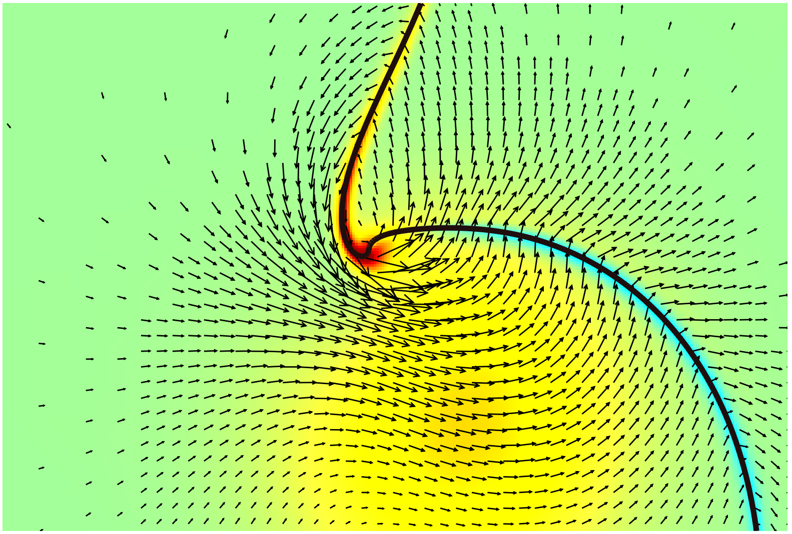
Sharp point on the interface at short times

Instability

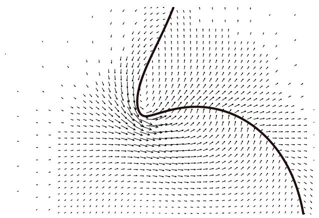
$$\Sigma = \frac{r_\rho - 1}{r_\rho}\frac{\partial p_m}{\partial s} \qquad \frac{\partial p_m}{\partial s} = \frac{\partial p_m}{\partial r}\frac{\partial r}{\partial s} = \frac{u_\theta^2}{r}\frac{\partial r}{\partial \phi_0}\frac{1}{\frac{\partial s}{\partial \phi_0}}$$



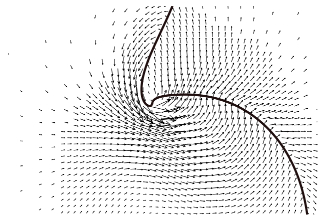
Velocity generated by the extra vorticity



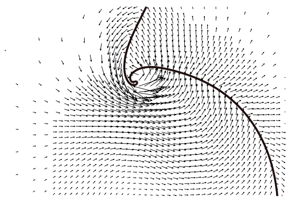
Trace of Moore Singularity ?



$$\frac{\rho_2}{\rho_1} = 1.2$$



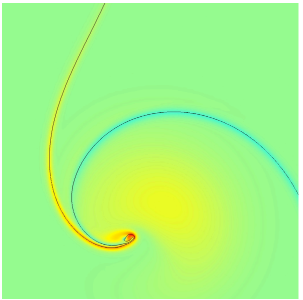
$$\frac{\rho_2}{\rho_1} = 2$$

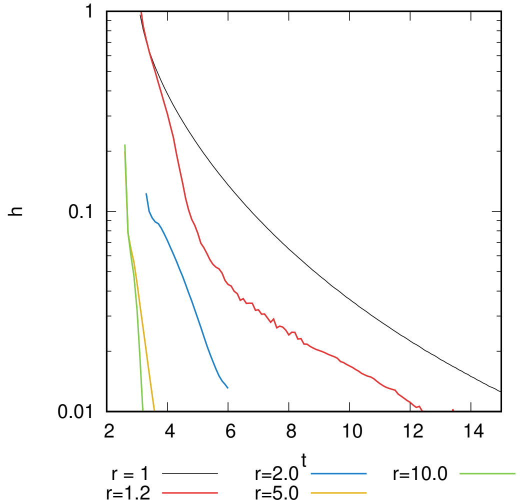


$$\frac{\rho_2}{\rho_1} = 5$$

Creation of smaller structures at the tip as density difference increases

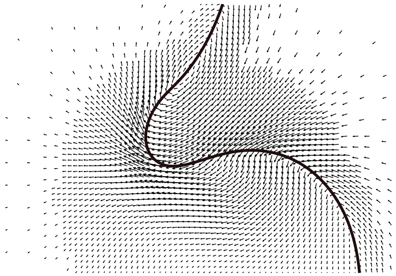
Thinner ligament and rim



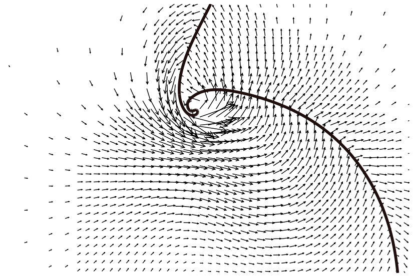


Faster (exponential ?) decrease of h

Faster break-up



$$\frac{\rho_2}{\rho_1} = 0.2$$



$$\frac{\rho_2}{\rho_1} = 5$$

extra vorticity changes sign

$$\Sigma = \frac{r_\rho - 1}{r_\rho} \frac{\partial p_m}{\partial s}$$

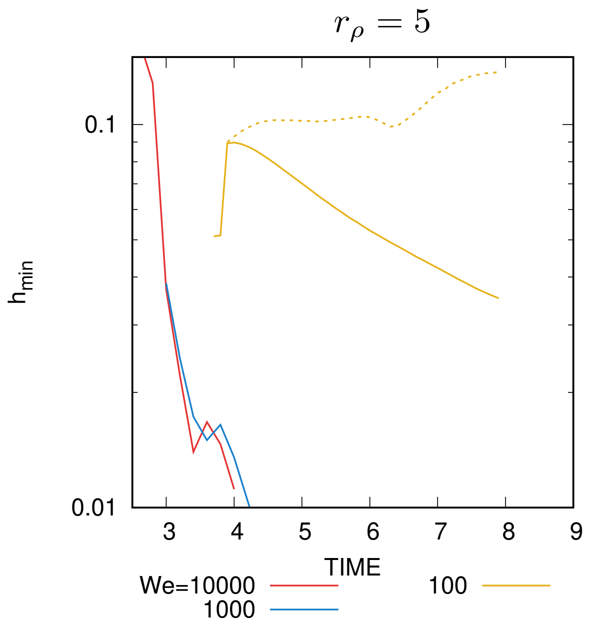
Gas-jets large bubbles

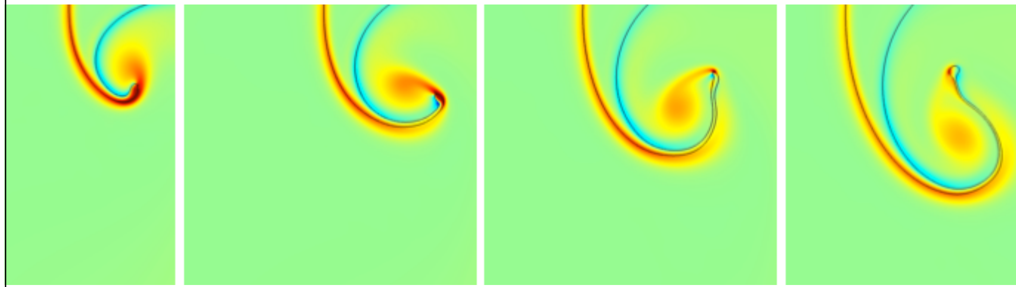
liquid-jets small drops

«Two-phase» flows

with density difference and

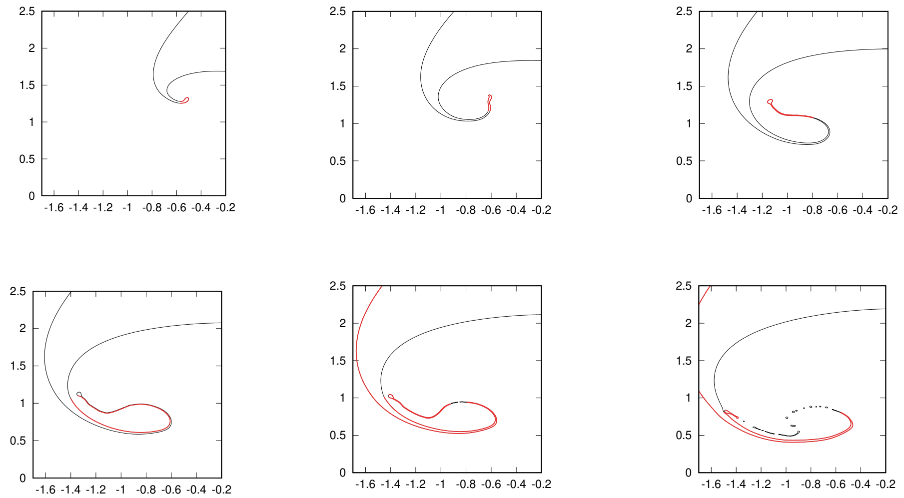
with   surface tension


$r_\rho = 5$
0.1
0.01
h_{min}
3
4
5
6
7
8
9
TIME
We=10000
1000
100



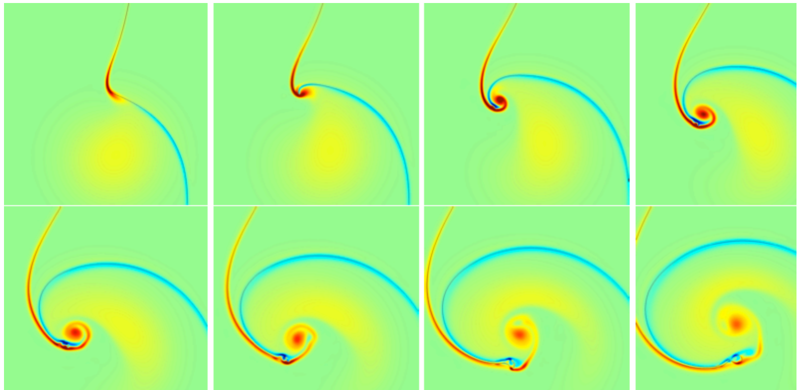
$r_\rho = 5$ $We = 1000$

Similar dynamics than for $We = \infty$



$r_\rho = 5 \qquad We = 1000$

Vorticity quickly breaks the ligament and surface tension plays a minor role



$$r_\rho = 5 \qquad We = 100 < We_{2c}$$

Surface tension becomes dominant
Saturation before the end of sharpening dynamics

Conclusions

Vorticity production at the interface by

surface tension linked to the gradient of curvature

density difference linked to pressure gradient along interface

Extra-vorticity changes the interface dynamics

Emergence of small structures caused by the interaction vorticity -- interface