Viscous drop retraction

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Context : liquid ligaments in a gas

- Oceam spumes
- Inkjet printing
- Atomization in fuel injectors
- Settling of a particle through a liquid-liquid interface

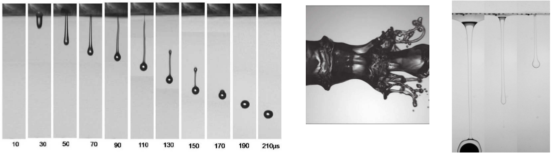
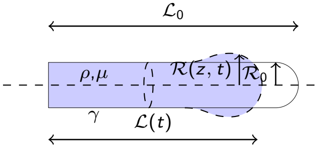


Figure : Left : sequence of drop formation during inkjet printing (Jang et al. 2009). Middle : breakup of a slow dense liquid jet by a fast light coaxial stream (Marmottant & Villermaux (2004). Right : settling of a sphere through a liquid-liquid interface (Pierson 2015).

Of particular interest in those applications is to predict if the ligament will break in several droplets or will retract. Such information can be obtain by equating $T_{retraction}$ to $T_{breakup}$ (Driessen et al. 2013). Since $T_{retraction} \sim L_{drop}/U_{retraction}$, how to estimate $U_{retraction}$?

Bibliography



2 dimensionless parameters :
$Oh = \mu/(\rho \mathcal{R}_0 \gamma)^{1/2}$, $\mathcal{L}_0/\mathcal{R}_0$

Authors	Geometry	Oh	$\mathcal{L}_0/\mathcal{R}_0$	Results
Taylor (1959) & Culick (1960)	2D	$\ll 1$	-	Taylor-Culick velocity
McEntee & Mysels (1969)	2D	$\ll 1$ & $\gg 1$	-	Experiments
Keller (1983)	-	$\ll 1$	-	Theory
Brenner & Gueyffier (1999)	2D	$\ll 1$ & $\gg 1$	∞	Numerical
Sünderhauf et al. (2002)	2D	$\ll 1$ & $\gg 1$	∞	Numerical
Notz & Basaran (2004)	Axi	$\ll 1$ & $\gg 1$	15	Numerical
Savva & Bush (2009)	2D +	$\ll 1$ & $\gg 1$	∞	Theory / Numerical
Hoepffner & Paré (2013)	Axi	$\ll 1$	-	Experiments
Murano & Okumura (2018)	2D	$\ll 1$ & $\gg 1$	-	Experiments

Numerical simulations based on 1D models show that the Taylor-Culick velocity is always reached no matter Oh. However experiments show that Taylor-Culick velocity is only reached when $Oh \ll 1$. Goal : investigate the retraction velocity of a finite-length drop by using a combination of numerical results and scaling arguments. $Oh \in [0.1, 1, 10]$, $\mathcal{L}_0/\mathcal{R}_0 \in [5, 10, 20, 40]$

Section 2

Problem & Methods

Basilisk code (Popinet 2003, 2009)

The variable-density Navier-Stokes equations with surface tension are written in the form :

$$\boldsymbol{\nabla} \cdot \mathbf{u} = \mathbf{0} \quad ,$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \mathbf{u} \right) = -\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot (2\mu \mathbf{D}) + \mathbf{f}_\gamma \quad ,$$

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}) = 0.$$

Spatial discretization

- Finite volume with graded quadtree partitioning (Popinet 2003)
- Collocated variables at the cell center

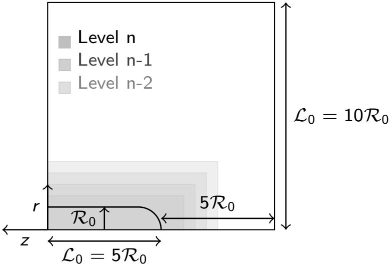
Time-stepping strategy

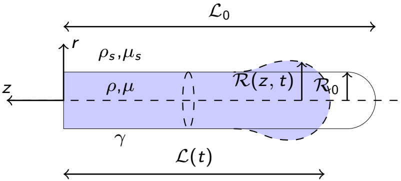
- Viscous term treated implicitly and Bell-Collela-Glaz advection scheme
- Projection technique to enforce incompressibility

Others

- Multigrid solver to solve both Helmholtz and Poisson equations (tolerance 10^{-5})
- Piecewise-linear geometrical Volume of Fluid method (Popinet 2009)+ direction split scheme

Simulation's parameters





- Cylindrical domain with locally refined mesh
- Number of points per radius : 20-80 depending on Oh
- Domain's radius : $L = \mathcal{L}_0 + 5\mathcal{R}_0$
- $\rho/\rho_s = 100$, $\mu/\mu_s = 1000$: effect of the surrounding fluid supposed to be negligible

Eggers & Dupont (1994) long-wave model

Assuming that the drop is slender $\mathcal{R} \ll \mathcal{L}$, and that the surrounding fluid has a negligible effect the equation of motion can be reduced to :

$$\begin{aligned}\frac{\partial \mathcal{R}^2}{\partial t} + \frac{\partial u_z \mathcal{R}^2}{\partial z} &= 0, \\ \frac{\partial u_z}{\partial t} + u_z \frac{\partial u_z}{\partial z} &= -\frac{2\gamma}{\rho}\frac{\partial \kappa}{\partial z} + 3\frac{\mu}{\rho \mathcal{R}^2}\frac{\partial}{\partial z}\left(\mathcal{R}^2 \frac{\partial u_z}{\partial z}\right),\end{aligned}$$

where κ is the mean curvatures which reads :

$$\kappa = \frac{1}{2}\left(\frac{1/\mathcal{R}}{\left(1+(\partial \mathcal{R}/\partial z)^2\right)^{1/2}} - \frac{\partial^2 \mathcal{R}/\partial z^2}{\left(1+(\partial \mathcal{R}/\partial z)^2\right)^{3/2}}\right).$$

The mean curvature is equal to $1/(2\mathcal{R})$ to leading order when $\mathcal{R} \ll \mathcal{L}$.

Section 3

Low Ohnersorge number : $Oh = 0.1$

Momentum analysis (Savva & Bush 2009, Pierson & Magnaudet 2018)

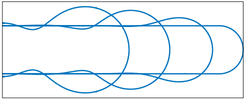
Multiplying mass conservation equation by u_z, momentum equation by $\mathcal{R}^2$, summing the resulting equation and integrating the resulting equation from $z = -\mathcal{L}(t)$ to $z = 0$:

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t} + \pi\rho\left[\mathcal{R}^2 u_z^2\right]_{z=0} = \gamma\pi\left[\frac{\mathcal{R}}{\left(1+(\partial\mathcal{R}/\partial z)^2\right)^{1/2}} + \frac{\mathcal{R}^2\partial^2\mathcal{R}/\partial z^2}{\left(1+(\partial\mathcal{R}/\partial z)^2\right)^{3/2}}\right]_{z=0} + 3\pi\mu\left[\mathcal{R}^2\frac{\partial u_z}{\partial z}\right]_{z=0}$$

where $\mathcal{P} = \pi\rho\int_{-\mathcal{L}}^{0}\mathcal{R}^2 u_z dz$ is the total momentum of the drop.

Assumptions

- viscous term negligible : $Oh \ll \mathcal{L}_0/\mathcal{R}_0$
- radius is constant near the plane of symmetry $\mathcal{R}(z=0,t) = R_0$, $\partial\mathcal{R}/\partial z(z=0,t) = 0$ and $\partial^2\mathcal{R}/\partial z^2(z=0,t) = 0$
- the blob grows spherically :



- the velocity inside the blob is constant

Drop tip's velocity

We finally get :

$$U_t = -\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}t} = \frac{U_i t}{\left(t^2 + \frac{4}{9}t_i^2\right)^{1/2}} \sim \frac{3}{2}\frac{t}{t_i} \quad \textit{for} \quad t \ll t_i,$$

Where $U_i = (\gamma/(\rho\mathcal{R}_0))^{1/2}$ is the Taylor-Culick (capillary-inertia) velocity and $t_i = \mathcal{R}_0^{3/2}\rho^{1/2}/\gamma^{1/2}$ is the capillary-inertia time scale. This velocity does not depend on the aspect ratio !

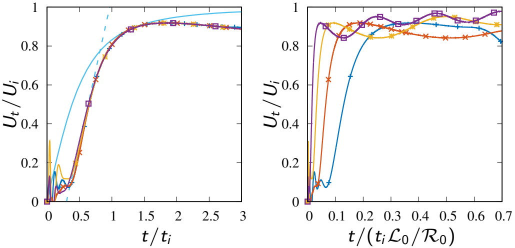


Figure : Drop tip velocity versus time for $Oh = 0.1$ and different $\mathcal{L}_0/\mathcal{R}_0$. $+$: $\mathcal{L}_0/\mathcal{R}_0 = 5$, $\times$: $\mathcal{L}_0/\mathcal{R}_0 = 10$, $*$: $\mathcal{L}_0/\mathcal{R}_0 = 20$, $\square$: $\mathcal{L}_0/\mathcal{R}_0 = 40$, —: theoretical prediction, – –: $3/2U_it/t_i$. (left) : early stages of retraction, (right) : steady state.

- initial acceleration correctly predicted by theory (after a small transient)
- the Taylor-Culick velocity is not reached : apparition of capillary waves + assumption of constant velocity not valid everywhere in the blob !

Section 4

Moderate Ohnesorge number : $Oh = 1$

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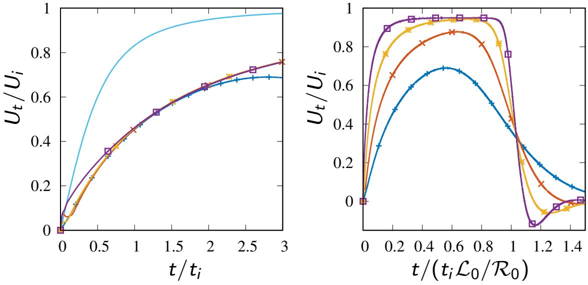


Figure : Blob velocity versus time for different $\mathcal{L}_0/\mathcal{R}_0$ and $Oh = 1$. $+$: $\mathcal{L}/\mathcal{R} = 5$, $\times$: $\mathcal{L}/\mathcal{R} = 10$, $*$: $\mathcal{L}/\mathcal{R} = 20$, $\square$: $\mathcal{L}/\mathcal{R} = 40$, —: theoretical prediction. (left) early stage of retraction, (right) steady state and return to equilibrium.

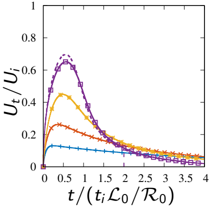
- initial acceleration not well predicted by the theory
- Taylor-Culick velocity is almost reached for high aspect ratios

Section 5

High Ohnersorge number : $Oh = 10$

Drop's tip velocity

Inertial scaling



- tip's velocity much smaller than the Taylor-Culick velocity
- large effect of the initial aspect-ratio on the velocity
- no steady state
- not the good scaling ! A balance between capillary and viscous term in the long-wave model gives $U = U_v\mathcal{L}_0/\mathcal{R}_0$ where $U_v = \gamma/\mu$ is the capillary velocity.

Viscous scaling

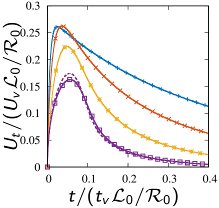
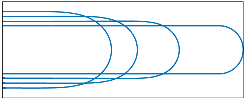


Figure : $Oh = 10$: drop tip velocity versus time for different $\mathcal{L}_0/\mathcal{R}_0$ and $Oh = 10$. $+$: $\mathcal{L}_0/\mathcal{R}_0 = 5$, $\times$: $\mathcal{L}_0/\mathcal{R}_0 = 10$, $*$: $\mathcal{L}_0/\mathcal{R}_0 = 20$, $\square$: $\mathcal{L}_0/\mathcal{R}_0 = 40$

- for $\mathcal{L}_0/\mathcal{R}_0 \leq 20$ the maximum velocity is constant $\approx 0.25 U_v\mathcal{L}_0/\mathcal{R}_0$
- for $\mathcal{L}_0/\mathcal{R}_0$ inertia effects becomes non negligible. Indeed the ratio between inertia and viscous effect is a Reynolds number based on the length of the drop.
- strong decreases of the velocity in the long time limit, why ?

Self-similar solution



In the high *Oh* limit the long wave model reads :

$$\frac{\partial \mathcal{R}^2}{\partial t} + \frac{\partial u_z \mathcal{R}^2}{\partial z} = 0,$$

$$0 = \frac{1}{3}\frac{\gamma}{\mu}\mathcal{R} + \mathcal{R}^2 \frac{\partial u_z}{\partial z}$$

By dimensional analysis we get :

$$\mathcal{R} = \frac{\gamma}{\mu}(t - t_0) R \left(\frac{\mathcal{L}_0}{\gamma/\mu(t-t_0)}, \frac{\mathcal{R}_0}{\gamma/\mu(t-t_0)}, \frac{z}{\gamma/\mu(t-t_0)} \right)$$

We look for a self similar solution of the form $\mathcal{R} = \frac{\gamma}{\mu}(t - t_0)R(\eta)$ where $\eta = \frac{z}{\gamma/\mu(t-t_0)}$ is the self-similar variable and t_0 is a shift in time to have $\mathcal{R}(t=0) = \mathcal{R}_0$. The last system of equation can be simplified as :

$$R''(1 - 6R) = 2R'^2/R,$$

which has been recently obtained by Eggers (2014) when studying the post break-up solution of thread. For $\eta \ll 1$, $R \sim 1/6$, thus :

$$\boxed{\mathcal{R} \sim \frac{1}{6}\frac{\gamma}{\mu}(t - t_0)}$$

Using mass conservation equation and assuming that the ending blob remains hemi-spherical we get :

$$\boxed{\frac{d\mathcal{L}}{dt} \sim U_v \left(\frac{1}{18} - \frac{1}{3} \frac{\mathcal{L}_0/\mathcal{R}_0 - 1/3}{(1 + t/(6t_v))^3} \right) \sim -\frac{1}{3} \frac{\mathcal{L}_0}{\mathcal{R}_0} \quad \textit{for} \quad \frac{\mathcal{L}_0}{\mathcal{R}_0} \gg 1}$$

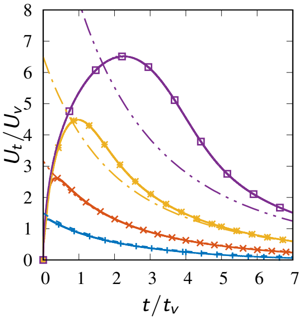


Figure : Drop tip velocity versus time for different $\mathcal{L}_0/\mathcal{R}_0$ and $Oh = 10$. $+$: $\mathcal{L}_0/\mathcal{R}_0 = 5$, $\times$: $\mathcal{L}_0/\mathcal{R}_0 = 10$, $*$: $\mathcal{L}_0/\mathcal{R}_0 = 20$, $\square$: $\mathcal{L}_0/\mathcal{R}_0 = 40$, $-$: theoretical prediction

Section 6

Conclusion

Concluding remarks and future works

Concluding remarks

- The Taylor-Culick velocity gives reasonable agreement with the numerical results for $Oh \ll \mathcal{L}_0/\mathcal{R}_0$.
- For $Oh \gg 1$ there is no blob formation and the tip's velocity differs significantly from the Taylor-Culick velocity. Capillary-viscous scaling should be prefer in this regime
- The large Ohnesorge number self-similar solution gives reasonable agreement with the numerical solution
- The disagreement between numerical results and experiments is due to the significant effect of the aspect ratio at large Oh.

Future works

- Include the effect of the surrounding fluids
- Investigate non-newtonian effects (plasticity, viscoelasticity, ...)

Acknowledgements

- Mijail Febres & Hiranya Deka for fruitful discussions
- Alexis Berny for having provided me a first example to run this kind of configuration

We introduce the viscous effect of the surrounding fluid in the right hand side of momentum equation by means of the tangential stress boundary condition. It reads (see Pierson 2015 and Lister & Stone (1998) for a derivation when the surrounding fluids is very viscous):

$$\frac{\partial u_z}{\partial t} + u_z \frac{\partial u_z}{\partial z} = -\frac{2\gamma}{\rho}\frac{\partial \kappa}{\partial z} + 3\frac{\mu}{\rho \mathcal{R}^2}\frac{\partial}{\partial z}\left(\mathcal{R}^2 \frac{\partial u_z}{\partial z}\right) + \frac{2\mu_s}{\rho \mathcal{R}}\frac{\partial u_{sz}}{\partial r}$$

where u_{sz} is the velocity of the surrounding fluid in z direction.

Scaling analysis

$u_{sz} \sim U_i$ (due to the continuity of velocity at the interface), $\partial r \sim \delta \sim \mathcal{R}_0/(Re_s)^{1/2}$, where δ is an estimation of the thickness of the boundary layer and $Re_s = \rho_s U_i \mathcal{R}_0/\mu_s$. Hence $\delta = \mathcal{R}_0\,(\mu_s/\mu \times \rho/\rho_s \times Oh)^{1/2}$.

The ratio between (capillary term) / (viscous stress due to the surrounding fluid) is :

$$Oh^{-1/2}\mathcal{R}_0/\mathcal{L}_0(\mu_s/\mu \times \rho/\rho_s)^{1/2}\mu/\mu_s$$

Scaling [1]: $u_z = U u_z^*$, $\kappa = 1/\mathcal{R}_0 \kappa^*$, $z = \mathcal{L}_0 z^*$, $\mathcal{R} = \mathcal{R}_0 \mathcal{R}^*$ and $t = Tt^* = \mathcal{L}_0/Ut^*$.

$$\frac{\rho \mathcal{R}_0 U^2}{\gamma}\left(\frac{\partial u_z^*}{\partial t^*} + u_z^* \frac{\partial u_z^*}{\partial z^*}\right) = -2\frac{\partial \kappa^*}{\partial z^*} + \frac{\mu U \mathcal{R}_0}{\gamma \mathcal{L}_0}\frac{3}{\mathcal{R}^{*2}}\frac{\partial}{\partial z^*}\left(\mathcal{R}^{*2}\frac{\partial u_z^*}{\partial z^*}\right).$$

Two distinct regimes can appear whether inertia or viscous effects are negligible.

Capillary - Inertia balance

$U = U_i = (\gamma/(\rho \mathcal{R}_0))^{1/2}$ and $T = \mathcal{L}_0/\mathcal{R}_0 t_i$ where $t_i = \mathcal{R}_0^{3/2}\rho^{1/2}/\gamma^{1/2}$ is the capillary-inertia time scale.

$$\frac{\partial u_z^*}{\partial t^*} + u_z^* \frac{\partial u_z^*}{\partial z^*} = -2\frac{\partial \kappa^*}{\partial z^*} + \frac{Oh \mathcal{R}_0}{\mathcal{L}_0}\frac{3}{\mathcal{R}^{*2}}\frac{\partial}{\partial z^*}\left(\mathcal{R}^{*2}\frac{\partial u_z^*}{\partial z^*}\right)$$

Viscous term becomes effectively negligible if $Oh \ll \mathcal{L}_0/\mathcal{R}_0$. For moderate Ohnesorge number ($Oh = \mathcal{O}(1)$) inertia effect can still be predominant if the length of the drop is sufficiently high !

Capillary - Viscous balance

$U = \gamma/\mu \mathcal{L}_0/\mathcal{R}_0 = U_v \mathcal{L}_0/\mathcal{R}_0$ which is based on the capillary velocity $U_v = \gamma/\mu$ and on a viscous time scale $T = \mathcal{L}_0/U_v = t_v \mathcal{L}_0/\mathcal{R}_0$ where $t_v = \mathcal{R}_0 \mu/\gamma$.

$$\frac{1}{Oh^2}\frac{\mathcal{L}_0^2}{\mathcal{R}_0^2}\left(\frac{\partial u_z^*}{\partial t^*} + u_z^* \frac{\partial u_z^*}{\partial z^*}\right) = -2\frac{\partial \kappa^*}{\partial z^*} + \frac{3}{\mathcal{R}^{*2}}\frac{\partial}{\partial z^*}\left(\mathcal{R}^{*2}\frac{\partial u_z^*}{\partial z^*}\right)$$

Inertia becomes effectively negligible if $Oh \gg \mathcal{L}_0/\mathcal{R}_0$. Hence even for large Oh, if the drop is sufficiently long, inertia cannot be neglected.

[1] The characteristic scale for the velocity is *a priori* unknown and must be determined *a posteriori*