Basilisk compressibility

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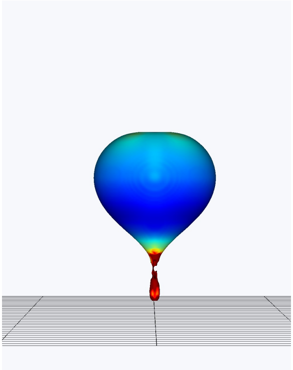
Objective:

Test a compressible solver accounting for surface tension effects



NUMERICAL METHOD

Compressible flow formulation: Homogeneous one fluid model equations

[Fuster & Popinet, JCP, 2018]

Continuity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

Momentum:

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \boldsymbol{u}) = -\nabla p + \nabla \cdot \tau + \sigma \kappa \nabla c$$

Total Energy:

$$\frac{\partial \rho e + 1/2 \rho \boldsymbol{u}^2}{\partial t} + \nabla \cdot (\rho e \boldsymbol{u} + 1/2 \rho \boldsymbol{u}^2) = -\nabla \cdot (\boldsymbol{u} p) + \nabla \cdot (\tau \boldsymbol{u}) + \boldsymbol{u} \cdot \sigma \kappa \nabla c$$

Color Function:

$$\frac{\partial c}{\partial t} + \boldsymbol{u} \cdot \nabla \cdot c = 0$$

State equation:

$$f(p, \rho, T) = 0$$

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We solve for: $$\frac{\partial \boldsymbol{Y}}{\partial t} + \nabla \cdot \boldsymbol{F} = \boldsymbol{S}$$

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We solve for: $\frac{\partial \boldsymbol{Y}}{\partial t} + \nabla \cdot \boldsymbol{F} = \boldsymbol{S}$ where $\boldsymbol{F}$ obtained from a semi-implicit method where p:

$$\frac{1}{\rho c_{\text{eff}}^2} \frac{Dp}{Dt} - \frac{\beta_T \Phi_v}{\rho c_p} = -\nabla \cdot \boldsymbol{u}$$

1) Advection step

Continuity: $$\frac{\rho^{n+1} - \rho^n}{\Delta t} + \nabla \cdot (\rho u) = 0$$

Momentum: $$\frac{(\rho \boldsymbol{u})^* - \rho \boldsymbol{u}}{\Delta t} + \nabla \cdot (\rho \boldsymbol{u} \boldsymbol{u}) = 0$$

Total Energy: $$\frac{(\rho e_T)^* - (\rho e_T)^n}{\Delta t} + \nabla \cdot (\rho e_T \boldsymbol{u}) = 0$$

Color Function: $$\frac{c^{n+1} - c^n}{\Delta t} + \nabla \cdot (\boldsymbol{u} c) = c^* \nabla \cdot \boldsymbol{u}$$

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2) Surface tension and viscous terms

$$\rho^{n+1} \frac{\boldsymbol{u}^{**} - \boldsymbol{u}^*}{\Delta t} = \nabla \cdot \tau + \sigma \kappa^{n+1} \nabla c^{n+1}$$

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3) Projection step: To compute $(p^{n+1})_{est}$ implicitly

$$\frac{p^{n+1}}{\rho c^2\Delta t}-\nabla\cdot\left(\frac{\Delta t}{\rho^{n+1}}\nabla p^{n+1}\right)=\frac{p^{adv}}{\rho c^2\Delta t}-\nabla\cdot u^{*}$$

1) Advection step

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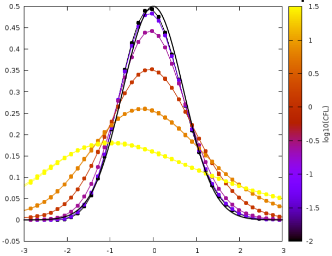
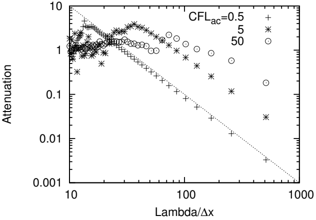
4) Update $$(\rho \boldsymbol{u})^{n+1} = (\rho \boldsymbol{u})^{**} - \Delta t \nabla p^{n+1}$$

$$(\rho e_T)^{n+1} = (\rho e_T)^* - \Delta t \nabla \cdot (\boldsymbol{u} p)^{n+1} + \boldsymbol{u} \cdot \sigma \kappa \nabla c \rightarrow p^{n+1} = EOS(\rho, \rho e)^{n+1}$$

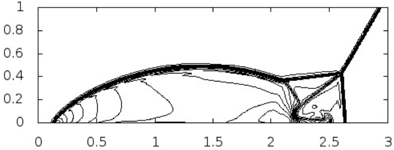
Test cases for single phase flows

- Linear problems (non-linear effects are negligible): Propagation of a linear disturbance in pure liquid





- Non-linear problems: Classical shock tests



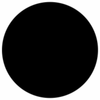
Tests for two–phase flows in presence of surface tension:

1. Equilibrium in a static configuration
2. Oscillation of a deformed droplet
3. Peak pressures generated by the impact of a liquid jet into a wall

$\gamma_l = 7.14 \; \Pi_l = 300$

$p_0 = 1$

Laplace number $\mathrm{La} = \dfrac{\rho \mathrm{D}_0 \sigma}{\mu^2}$



$\gamma_g = 1.4$

$\sigma = 1$

$p_{g,0} = p_0 + \dfrac{\sigma}{R_0}$

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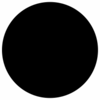

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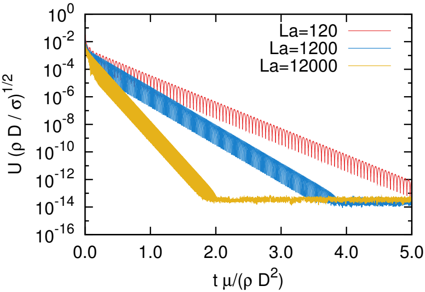
Current



-Spurious currents are damped by viscosity within the viscous time–scale

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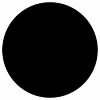

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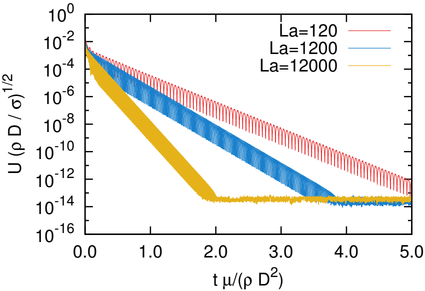
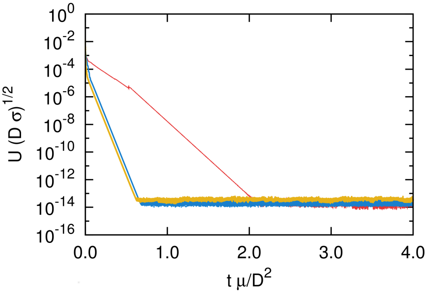


Incompressible



-Spurious currents are damped by viscosity within the viscous time–scale

-Similar results than the incompressible formulation except for large La numbers

-At large La the viscous boundary layer is not well-resolved

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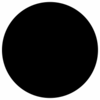

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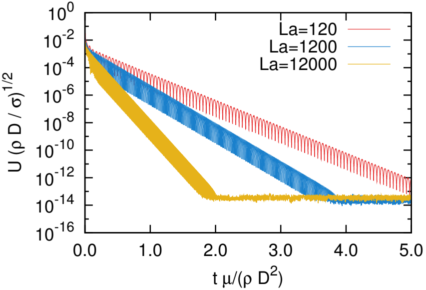

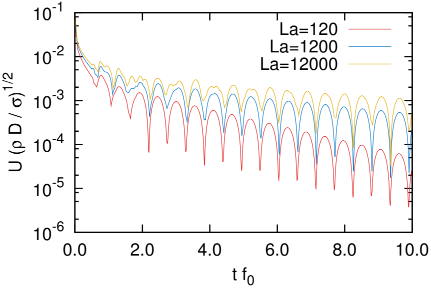

-Bubble naturally oscillates at its resonance frequency

$$\omega_0 = 2\pi f_0 = \frac{1}{R_0}\sqrt{\frac{3\gamma_g p_0}{\rho_l}}$$

Test II) Oscillation of a deformed droplet

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Slightly compressible liquid

$\Gamma_l = 7.14$, $\Pi_l = 300$

Ambient: ideal gas

$\gamma_g = 1.4$

$\rho_{g,0}/\rho_{l,0} = 10^{-3}$

$R(x, y) = 0.1\ (1 + 0.05 \cos(2\ \mathrm{arctan}(\mathrm{y}, \mathrm{x})))$

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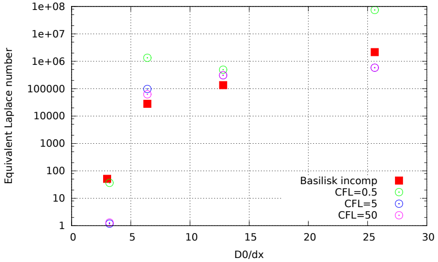
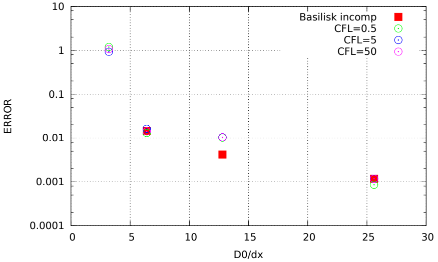
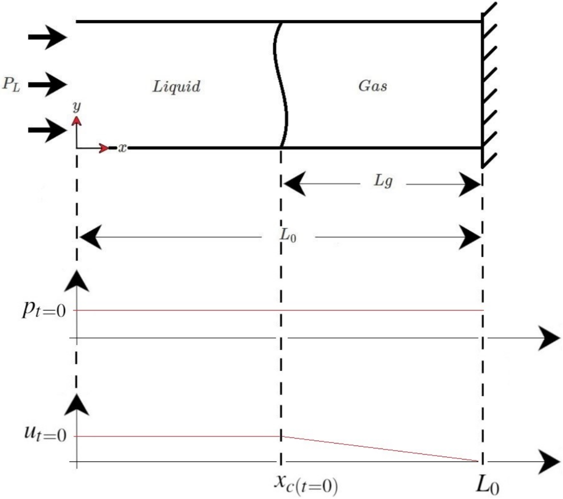




Test III) PEAK PRESSURES GENERATED BY A LIQUID JET

Influence of gas on the impact of a liquid jet into a wall:



Low Mach impacts $U_0 \approx 1 - 10$ m/s $\rightarrow$ Ma$_l \approx 0.01$

Bagnold problem (low Ma)

The liquid jet is finite (pressure is constant at a given distance)

$$\frac{\partial u}{\partial x} = 0, \tag{1}$$

$$\frac{Du_l}{Dt} = -\frac{1}{\rho_l}\frac{\partial p_l}{\partial x}. \tag{2}$$

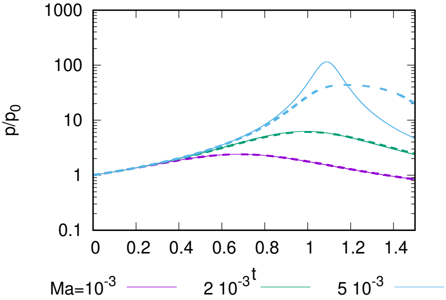
Integrating....

$$I\ddot{\chi}(\chi - L_R) \quad = 1 - \chi^{-\gamma} \tag{3}$$

where $LR = \dfrac{L_0}{Lg_0}$

The solution depends on the gas layer thickness

Line: theory, dash line: simulation



Simulations require a finite Ma_l (the theory is only formally valid for $Ma_l \to \infty$

Impact of an infinite weakly compressible liquid column on a wall

$$\frac{\partial u}{\partial x} = -\frac{1}{\rho_l c_l^2}\frac{Dp}{Dt}, \qquad (4)$$

$$\frac{Du_l}{Dt} = -\frac{1}{\rho_l}\frac{\partial p_l}{\partial x}. \qquad (5)$$

Because the flow is potential these equations reduce to the transient Bernouilli equation

$$\Phi_{,t} = \frac{u^2}{2} + h \qquad (6)$$

where Φ is the liquid's potential and h is the liquid's enthalpy

$$h = \int_{p_0}^{p}\frac{dp}{\rho_l} = \frac{p - p_0}{\rho_l} \qquad (7)$$

In the quasi–accoustic approximation, both Φ and $\Phi_{,t}$ propagate at the liquid's sound speed and therefore

$$\frac{D}{Dt}\left(h + \frac{u^2}{2}\right) + c_l\frac{\partial}{\partial x}\left(h + \frac{u^2}{2}\right) = 0. \qquad (8)$$

$$\ddot{L}_g = \frac{1}{\rho_l c_l}\dot{p}_g. \tag{9}$$

Assuming uniform pressure within the gas and adiabatic gas compression:

$$p_g L_g^\gamma = C_0 \tag{10}$$

Using the initial velocity U_0, $L_{g,0}$ and ρ_l to render variables dimensionless

$$\ddot{\chi} = -\frac{\gamma}{K_l Ma_l}\frac{\dot{\chi}}{\chi^{\gamma+1}} \tag{11}$$

No influence of the gas layer!

The peak pressure is the well-known water-hammer pressure
Balancing the initial kinetic energy and the elastic energy:

$$\frac{1}{2}\frac{(p_{max}-p_0)^2}{\rho_l c_l^2} = \frac{1}{2}\rho_l U_0^2 \qquad (12)$$

$$p_{max}-p_0 = \rho_l c_l U_0 \qquad (13)$$

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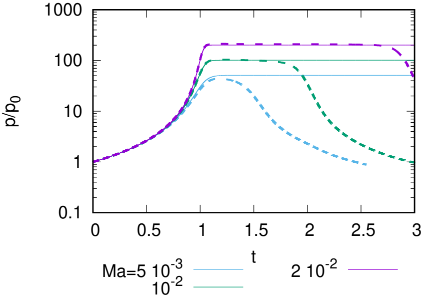

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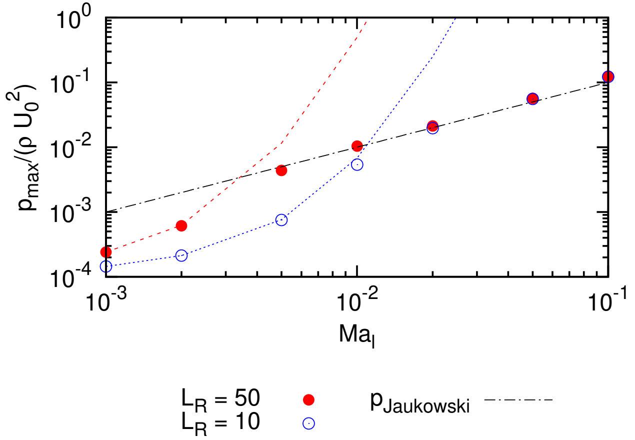
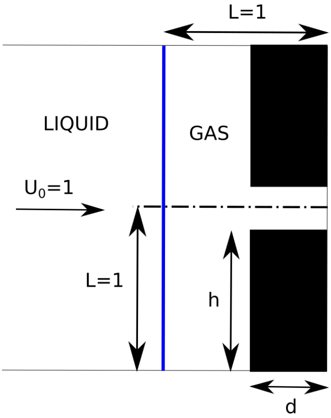


In 1D, the presence of the gas acts as a damping mechanism

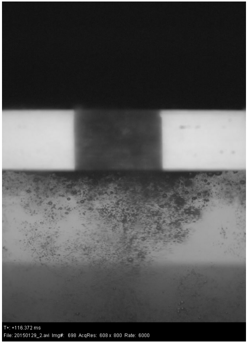
2D problem

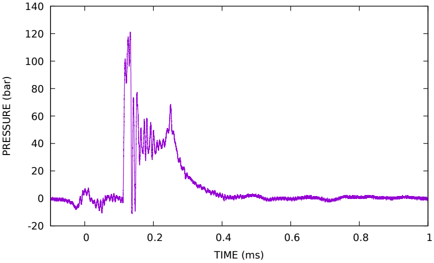


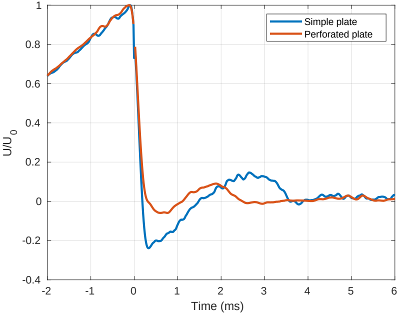
Roughness $r = 1 + d$, Hole aspect ratio $a_R = d/(1 - h)$

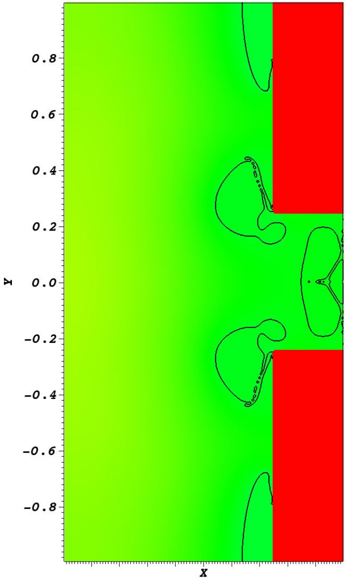
Problem examples

Plate impact on a surface

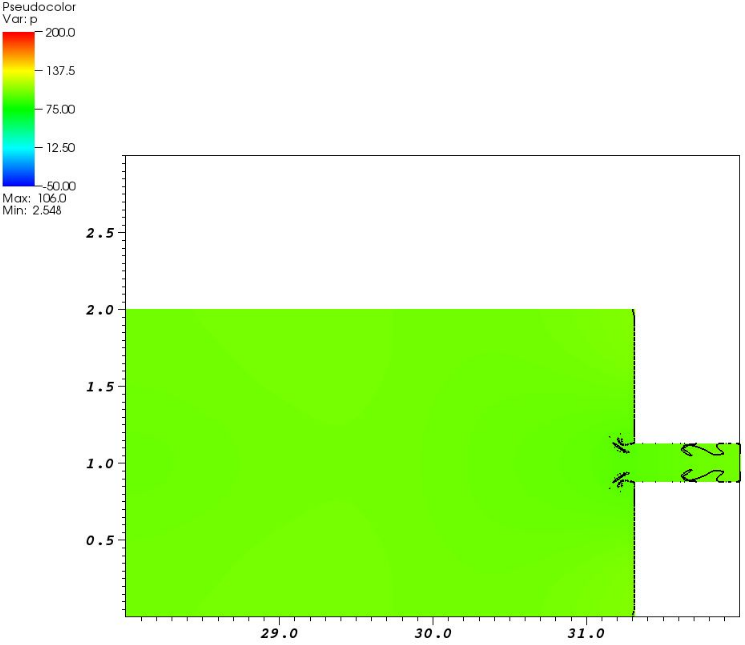








0.8
0.6
0.4
0.2
Y
0.0
-0.2
-0.4
-0.6
-0.8
X



Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 106.0
Min: 2.548
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

Ma=0.01 $\rightarrow p_{max} - p_0 = 100$ $\frac{h}{L_0} = 0.9$ $\frac{\rho_{g,0}}{\rho_l} = 10^{-2}$

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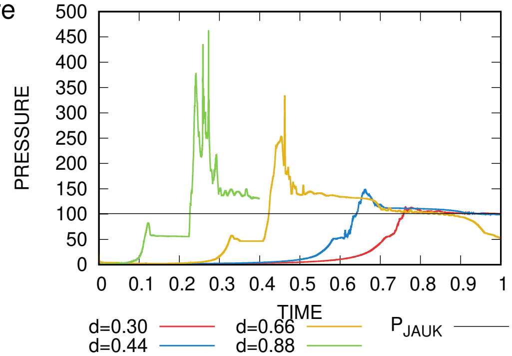
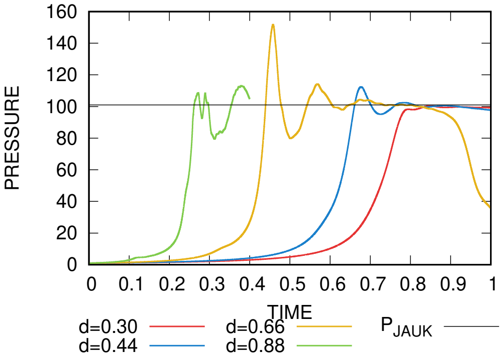
Peak pressure



We can obtain pressures larger than the Jaukowski pressure!!

Ma=0.01 $\rightarrow p_{max} - p_0 = 100$ $\qquad \frac{h}{L_0} = 0.9 \qquad \frac{\rho_{g,0}}{\rho_l} = 10^{-2}$

Gas Avg pressure



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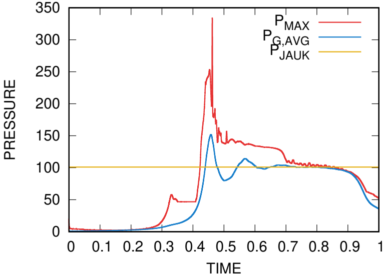
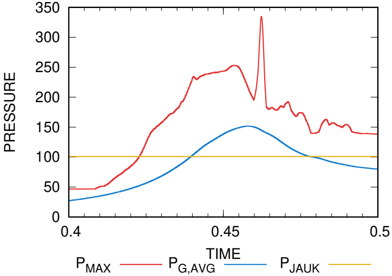
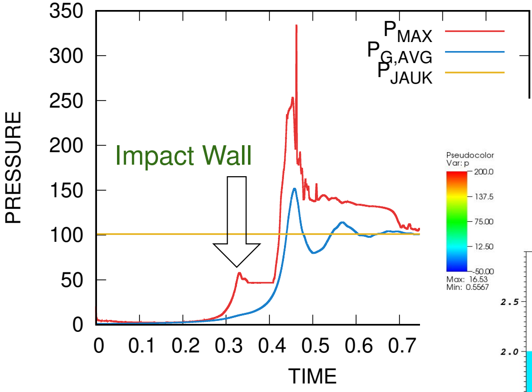


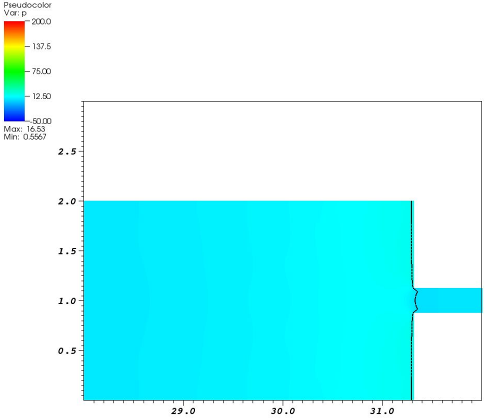
Re=100

h×d = 0.9×0.666

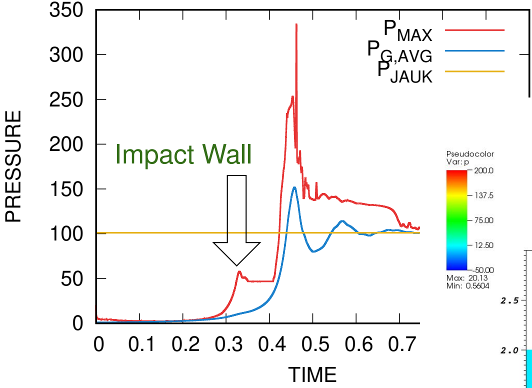



PRESSURE
TIME
0
50
100
150
200
250
300
350
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
P MAX
P G,AVG
P JAUK
Impact Wall

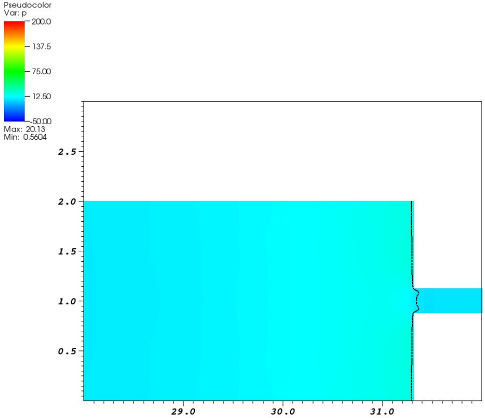
t=0.3


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 16.53
Min: 0.5567
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

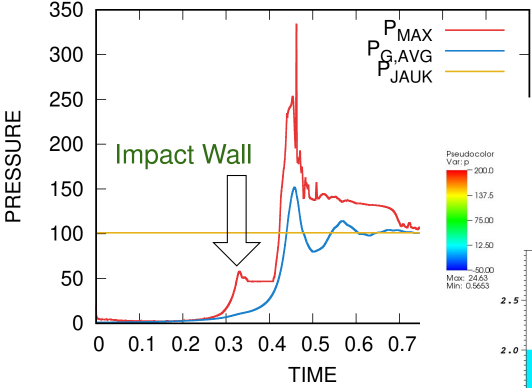
t=0.305



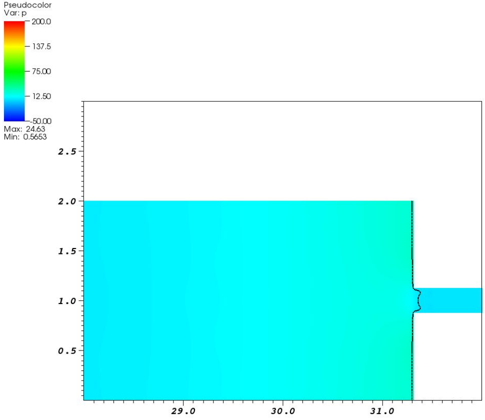
PRESSURE
TIME
Impact Wall
P_{MAX}
$P_{G,AVG}$
P_{JAUK}
350
300
250
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

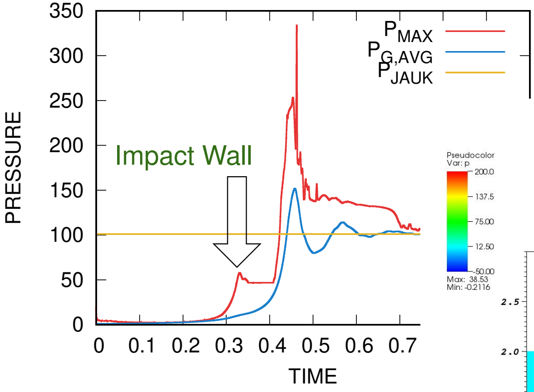


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 20.13
Min: 0.5604
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

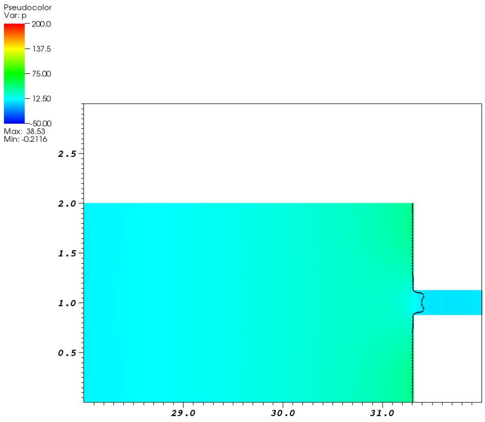

PRESSURE
TIME
350
300
250
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
P_{MAX}
$P_{G,AVG}$
P_{JAUK}
Impact Wall

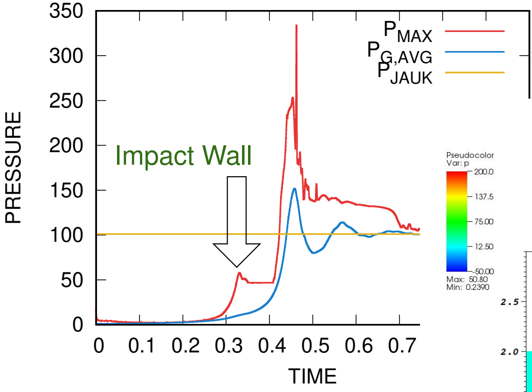
t=0.31


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 24.63
Min: 0.5653
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

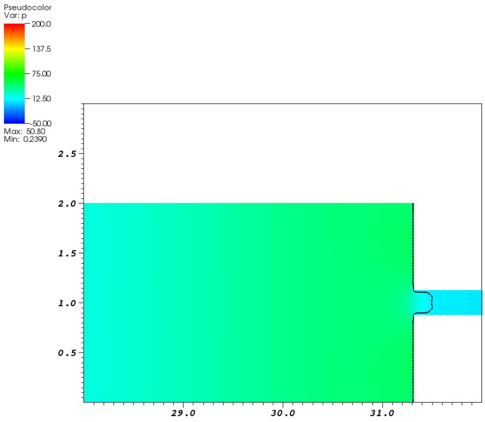

PRESSURE
TIME
350
300
250
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
P_{MAX}
$P_{G,AVG}$
P_{JAUK}
Impact Wall

t=0.32

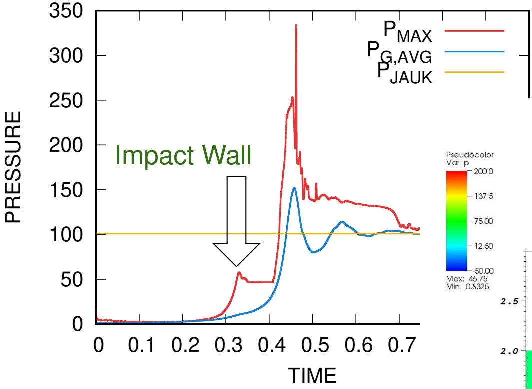

Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 38.53
Min: -0.2116
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0


350
300
250
200
150
100
50
0
PRESSURE
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
TIME
P_{MAX}
$P_{G,AVG}$
P_{JAUK}
Impact Wall

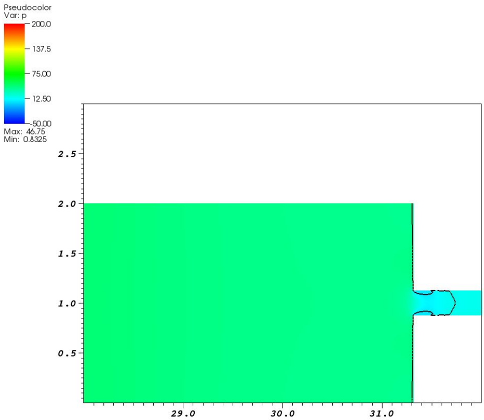
t=0.34


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 50.80
Min: 0.2390
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

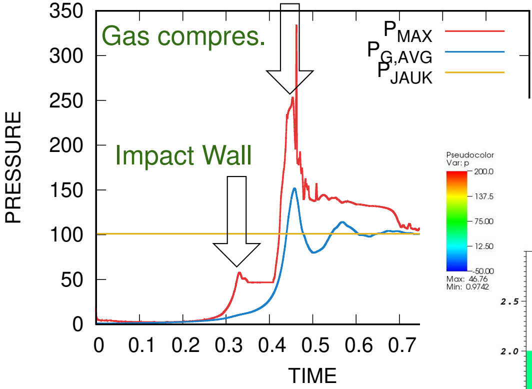
t=0.38



PRESSURE
TIME
Impact Wall
P_{MAX}
$P_{G,AVG}$
P_{JAUK}
350
300
250
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

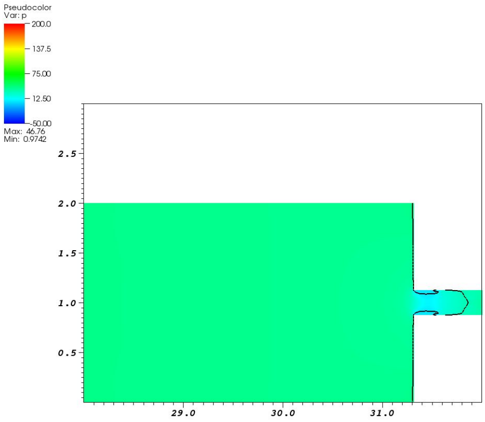


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 46.75
Min: 0.8325
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

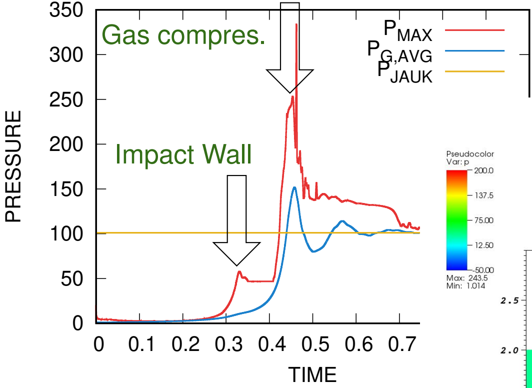


Gas compres.
Impact Wall
PRESSURE
TIME
P_MAX
P_G,AVG
P_JAUK

t=0.40

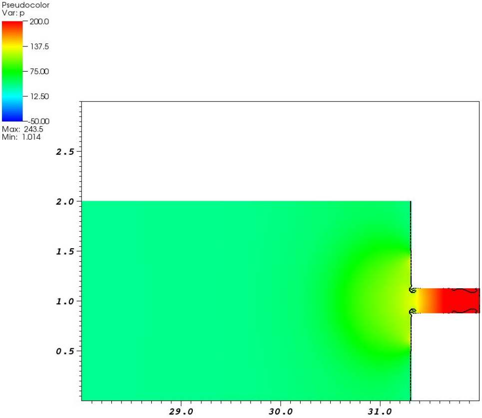


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 46.76
Min: 0.9742

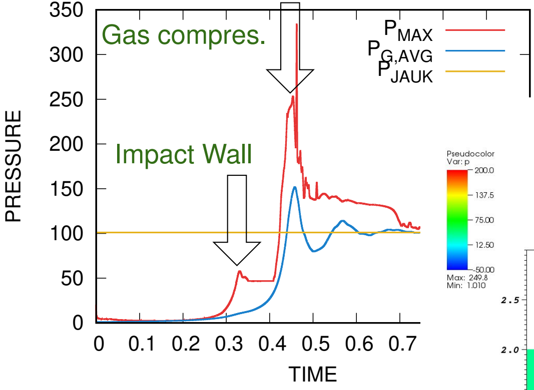


Gas compres.
Impact Wall
PRESSURE
TIME
P_{MAX}
$P_{G,AVG}$
P_{JAUK}
0
50
100
150
200
250
300
350
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

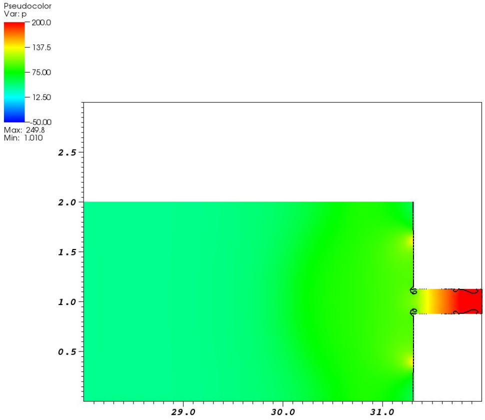
t=0.45

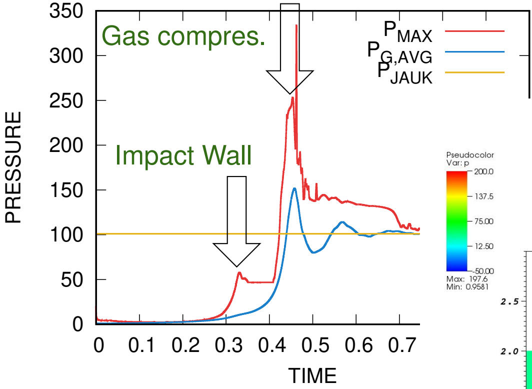


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 243.5
Min: 1.014
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0


Gas compres.
Impact Wall
PMAX
PG,AVG
PJAUK
PRESSURE
TIME
350
300
250
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

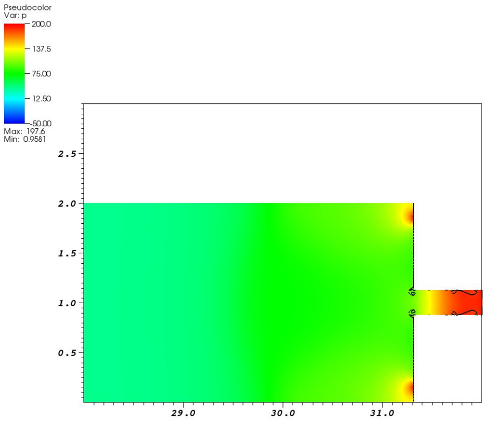
t=0.455


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 249.8
Min: 1.010
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

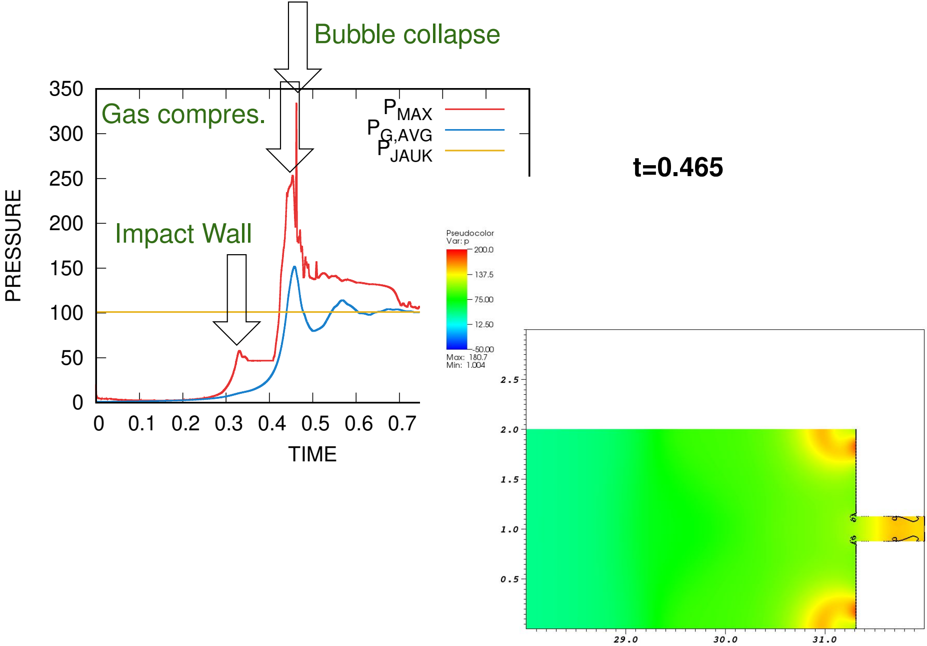


350
300
250
200
150
100
50
0
PRESSURE
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
TIME
Gas compres.
Impact Wall
P_{MAX}
$P_{G,AVG}$
P_{JAUK}

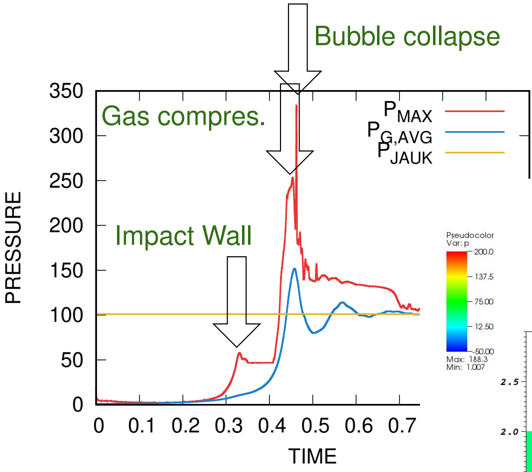
t=0.46



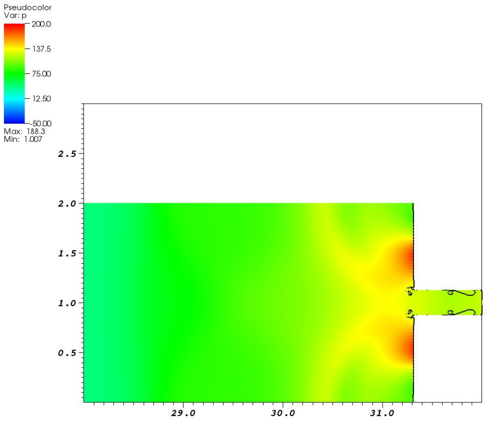
Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 197.6
Min: 0.9581
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

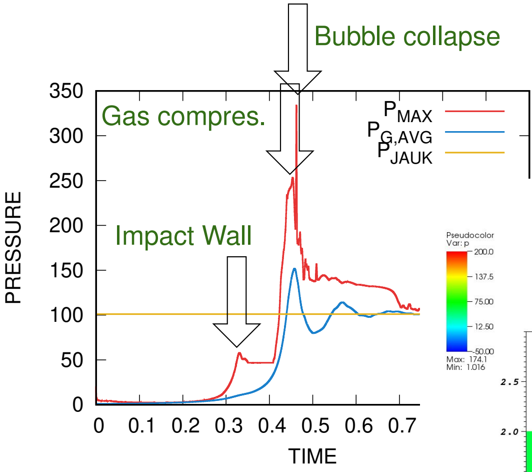


Bubble collapse
Gas compres.
Impact Wall
PRESSURE
TIME
350
300
250
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
P_{MAX}
$P_{G,AVG}$
P_{JAUK}
t=0.465
Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 180.7
Min: 1.004
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0


Bubble collapse
Gas compres.
Impact Wall
PRESSURE
TIME
P MAX
P G,AVG
P JAUK
350
300
250
200
150
100
50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

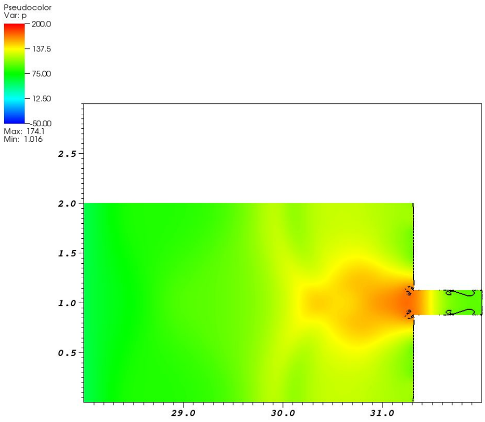
t=0.47


Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 188.3
Min: 1.007
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0



Bubble collapse
Gas compres.
Impact Wall
PMAX
PG,AVG
PJAUK
PRESSURE
TIME
350
300
250
200
150
100
50
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

t=0.475



Pseudocolor
Var: p
200.0
137.5
75.00
12.50
-50.00
Max: 174.1
Min: 1.016
2.5
2.0
1.5
1.0
0.5
29.0
30.0
31.0

BUBBLE COLLAPSE PROBLEM

Rayleigh–Plesset (RP) model vs. compressibility effects

Rayleigh–Plesset (RP) model vs. compressibility effects

All RP versions accounting for compressibility are approximations

For intermediate collapses they should be fine

Dimensionless parameters

$$\frac{p_{l,0}}{p_{b,0}} = 20$$

$$\mathrm{We} = \frac{\Delta \mathrm{p}_0 \mathrm{R}_0}{\sigma} = 1900$$

$$\mathrm{Ma} = \sqrt{\frac{\Delta \mathrm{p}_0}{\rho_\mathrm{l} \mathrm{c}_{\mathrm{l},0}^2}} = 5 \times 10^{-2}$$

Rayleigh–Plesset (RP) model vs. compressibility effects

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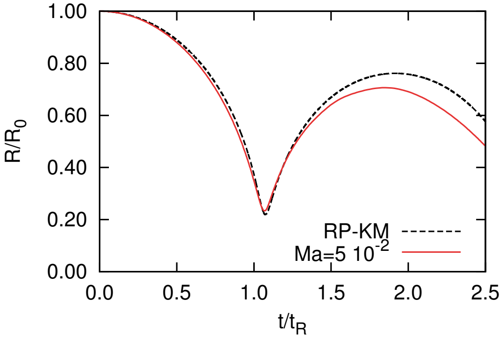
Dimensionless parameters

$$\frac{p_{l,0}}{p_{b,0}} = 20$$

$$\mathrm{We} = \frac{\Delta \mathrm{p}_0 \mathrm{R}_0}{\sigma} = 1900$$

$$\mathrm{Ma} = \sqrt{\frac{\Delta \mathrm{p}_0}{\rho_\mathrm{l} \mathrm{c}_{\mathrm{l},0}^2}} = 5 \times 10^{-2}$$

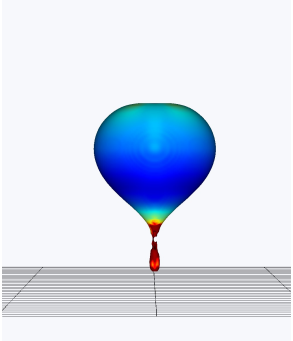


Bubble collapse close to a wall

$$p_l(r) = p_{l,0} + \left(p_{l,0}^I - p_{l,0}\right)\frac{R_0}{r}$$

Slightly compressible liquid

$$\Gamma_l = 7.14,\ \Pi_l$$



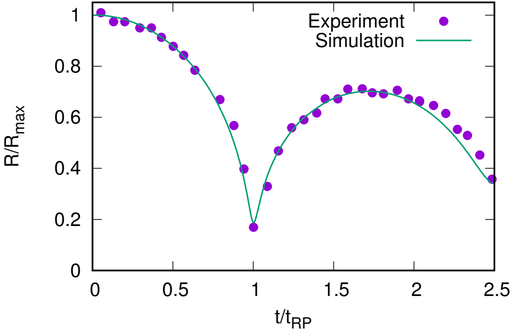
Dimensionless parameters

$$\frac{p_{atm}}{p_{vap}} = 40$$

$$\mathrm{We} = \frac{\Delta \mathrm{p_0 R_0}}{\sigma} \sim 10^3$$

$$\mathrm{Ma} = \sqrt{\frac{\Delta \mathrm{p_0}}{\rho_\mathrm{l} \mathrm{c_{l,0}^2}}} = 10^{-2}$$

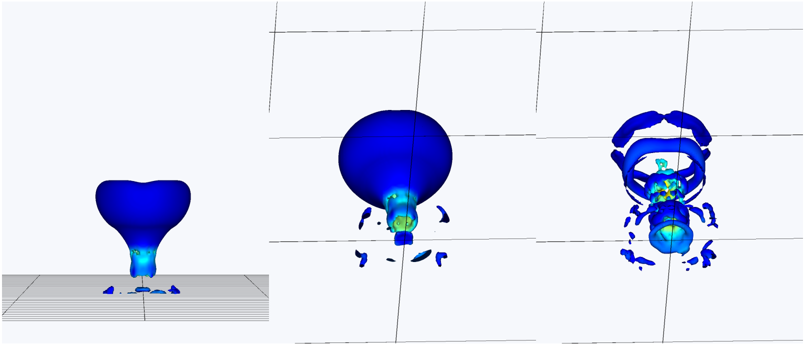
$$\mathrm{Re} = \frac{\sqrt{\rho_\mathrm{l} \Delta \mathrm{p_0}}\mathrm{R_0}}{\mu_\mathrm{l}} = \infty$$



[Yang et al; Ultr. Sonoch., 2013]

We can investigate the turbulence generated during the collapse

$\lambda_2 < 0$ criterion



Conclusions:

- A well balanced compressible solver with surface tension is proposed
- The code is validated for classical test accounting for surface tension
- We have shown the capabilities of the solver for two problems:
    1. The influence of a gas layer on the peak pressures generated by a liquid jet
    2. Problems related to the collapse of a bubble