Retraction of a viscoplastic sheet

By

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Introduction

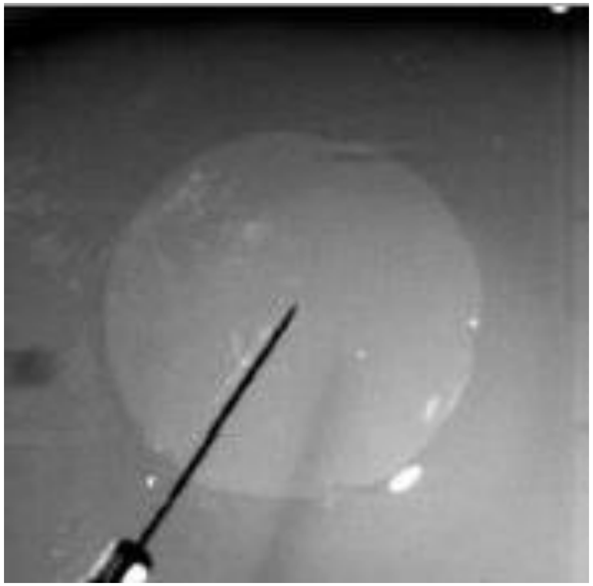

Fig. Photograph of a bursting soap film (Savva and Bush (2009))

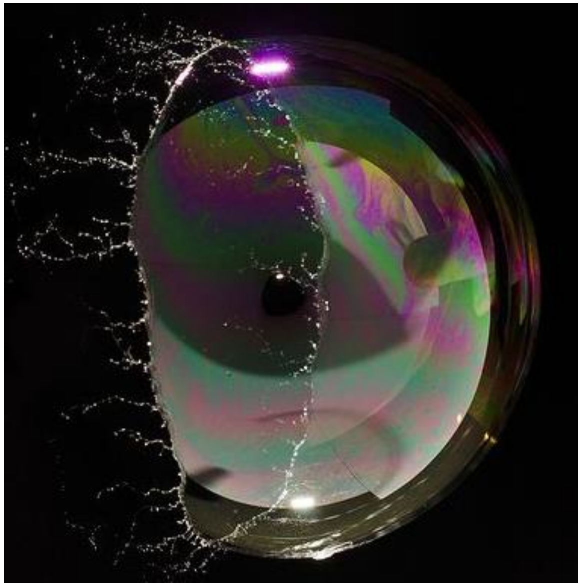

Fig. Photograph of a bursting bubble (www.flickr.com)

- Sheet fragmentation and atomization.
- Fragmentation is desirable in spray formation and undesirable in curtain coating.

Dupré (1867)	$\sqrt{2\sigma / \rho h}$
Ranz (1959)	The retraction velocity is less than that predicted by Dupré's formula.
Taylor (1959), Culick (1959)	$\sqrt{\sigma / \rho h}$
McEntree & Mysels (1969)	Experimentally verified the Taylor-Culick velocity.
Debrégeas et al. (1995)	Rupture of polymeric films; No rim formation and exponential hole growth with time.
Brenner and Gueyffier (1999), Savva and Bush (2009)	Absence of rim can be a result of pure viscous effect.

Bingham model (Bingham (1922)) :

$$\mu_1 = \begin{cases} \infty, & \text{if} \quad \tau \le \tau_y, \\ \mu_p + \tau_y / \dot{\gamma}, & \text{if} \quad \tau \ge \tau_y. \end{cases}$$

Deformation rate,

$$\dot{\gamma}_{i,j} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \quad \text{and} \quad \dot{\gamma} = \sqrt{\sum \dot{\gamma}_{i,j}^2 / 2}$$

Regularized version of Bingham model (Allouche et al. (2000), Frigaard et al (2005)) :

$$\mu_1 = \mu_p + \frac{\tau_y}{\dot{\gamma} + \epsilon},$$

$$\epsilon = \tau_y / \mu_{max}$$

Characteristic shear rate $$\dot{\gamma}_c = \sqrt{\sigma / \rho h^3}$$

Characteristic viscosity $$\eta_c = \mu_p + \tau_y / \dot{\gamma}_c$$

Plastic number $$Pl = \tau_y / (\tau_y + \mu_p \dot{\gamma}_c)$$

Ohnesorge number $$Oh = \eta_c / \sqrt{\rho_1 \sigma h}$$

Viscosity $$\mu_1 = \mu_p \left(1 + \frac{Pl}{(1 - Pl)\bar{\dot{\gamma}}} \right)$$

Governing equations:

$$\bar{\rho}\left[\frac{\partial \bar{u}_i}{\partial \bar{t}} + \bar{u}_i \frac{\partial \bar{u}_j}{\partial \bar{x}_j} \right] = \frac{\partial \bar{P}}{\partial \bar{x}_i} + Oh \frac{\partial}{\partial \bar{x}_j} \left[\bar{\mu} \left(\frac{\partial \bar{u}_i}{\partial \bar{x}_j} + \frac{\partial \bar{u}_j}{\partial \bar{x}_i} \right) \right] + \bar{\kappa} \mathbf{n} \delta_s$$

$$\frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = 0;$$ c is volume fraction and is 1 for fluid 1 and 0 for fluid 2

$$\rho = \rho_1 c + \rho_2 (1 - c), \qquad \mu = \mu_1 c + \mu_2 (1 - c).$$

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Collapse of a Bingham flow

Physcical problem

application to mud flows: slump, application to wet concrete flow: collapse of columns (Abrahams cone test).

equations

We propose an implementation of the Bingham rheology. For those flows, when stress is larger than a yield value, the media flows. For non Newtonian fluids, the "viscosity" is at least a function of the second principal invariant of the shear strain rate tensor that we define here as (other definitions proportional to this one are possible):

$$D_2 = \sqrt{\sum_{i,j} D_{ij} D_{ij}}$$

In littérature two norms are used, the Euclidian:

$$||D|| = \sqrt{\frac{1}{2} \sum_{i,j} D_{ij} D_{ij}}$$

and the Frobenius one:

$$|D| = \sqrt{\sum_{i,j} D_{ij} D_{ij}}$$

Obviously

$$||D|| = \sqrt{\frac{1}{2}}|D| = \frac{D_2}{\sqrt{2}}$$

The strain rate tensor is $D_{ij} = (\partial_i u_j + \partial_j u_i)/2$, it has unit of s^{-1}, the components in 2D:

$D_{11} = \frac{\partial u}{\partial x}$, $D_{12} = \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$, $D_{21} = D_{12} = \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$, $D_{22} = \frac{\partial v}{\partial y}$

And where the second invariant is $D_2 = \sqrt{D_{ij} D_{ij}}$

$$D_2^2 = D_{ij} D_{ij} = D_{11} D_{11} + D_{12} D_{21} + D_{21} D_{12} + D_{22} D_{22}$$

hence, as $D_{12} D_{21} = D_{21} D_{12}$:

$$D_2^2 = D_{ij} D_{ij} = (\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 + \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2$$

We have by definition of the Bingham rheology

$$\tau_{ij} = 2\mu_1 D_{ij} + \tau_y \frac{D_{ij}}{||D||}$$

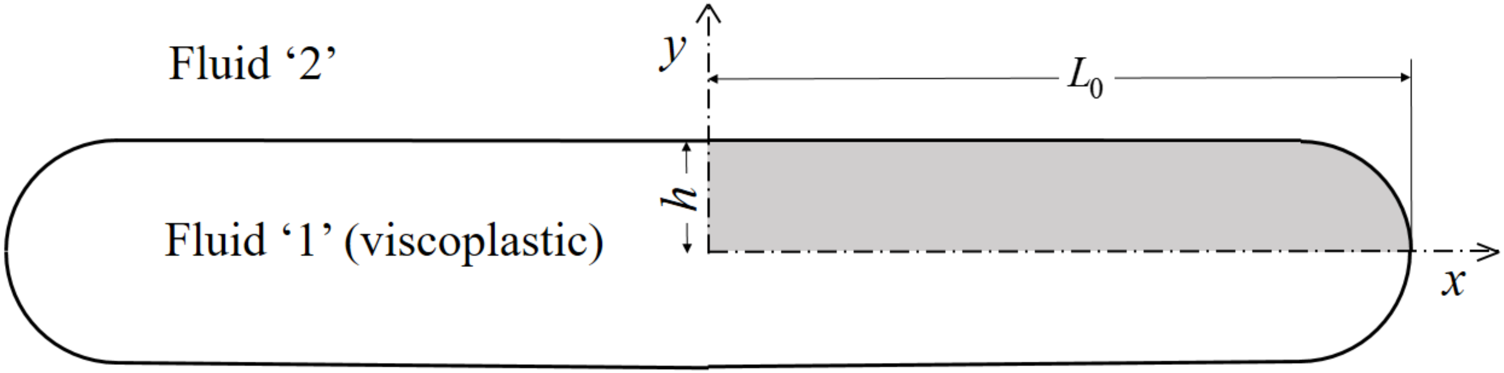
(hence if $||\tau_{ij}|| > \tau_y$ there is a flow). In order to identify this as an effective viscosity, `$(ij) = 2 (eff) D_{ij} $`,

$$\tau_{ij} = 2\mu D_{ij} + \tau_y \frac{D}{||D||} = 2(\mu + \frac{\tau_y}{2||D||}) D_{ij}$$

then the equivalent, or effective, of apparent viscosity is with $D_2 = \sqrt{2}||D||$

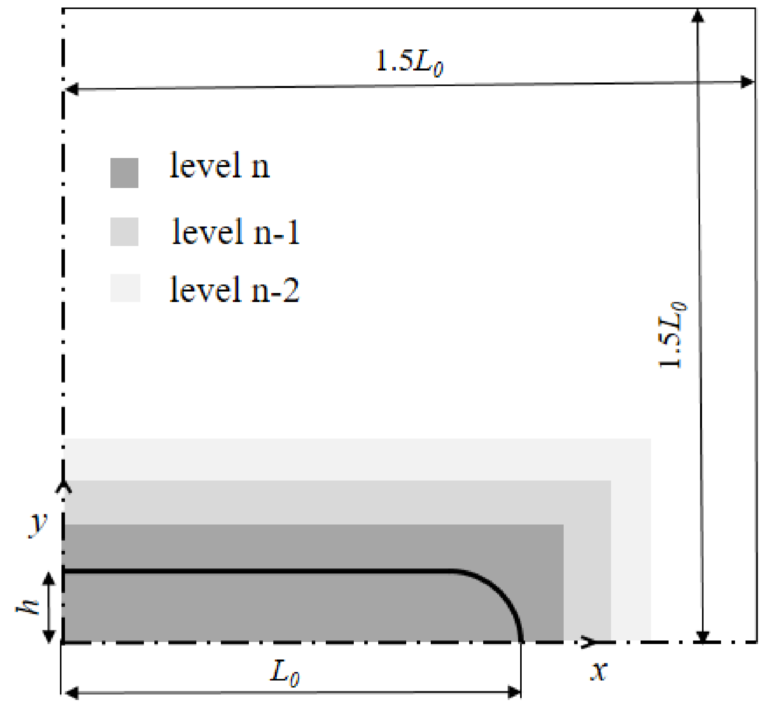
$$\mu_{eq} = \mu_1 + \frac{\tau_y}{\sqrt{2} D_2}$$

We can use a general Herschel-Bulkley formulation, with here $N = 1$,



Non-dimensional Parameter	Range
Oh	0.1-10
Pl	0-0.9
L_0 / h	10

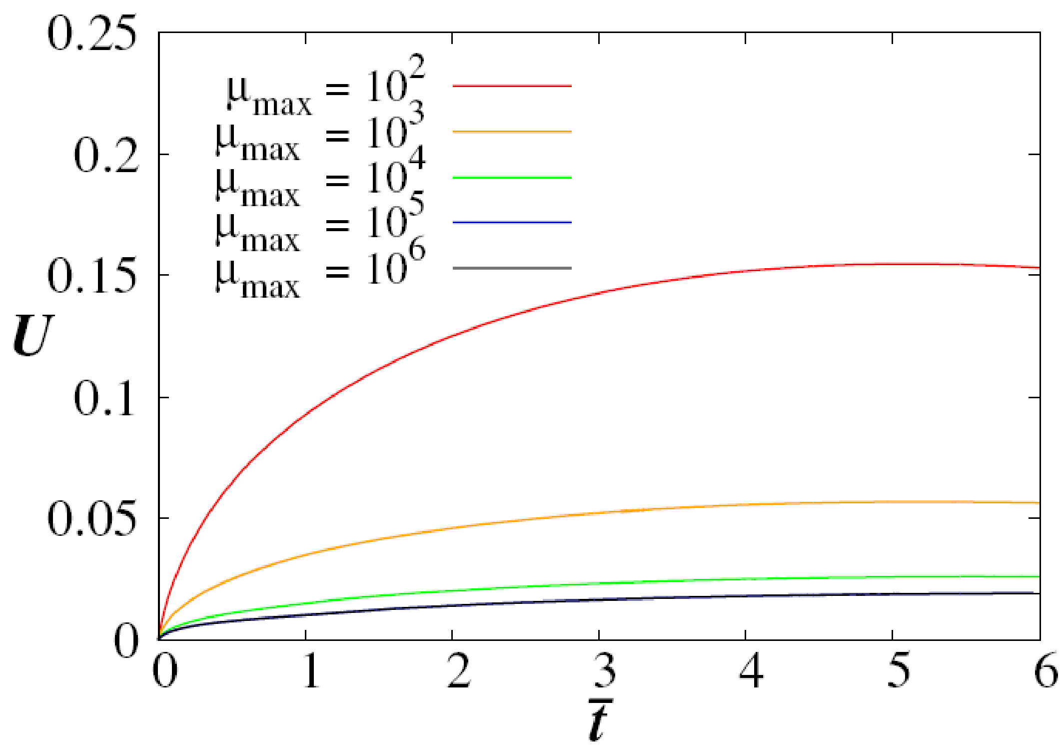
Grid cell size = $10^{-3} \times h$



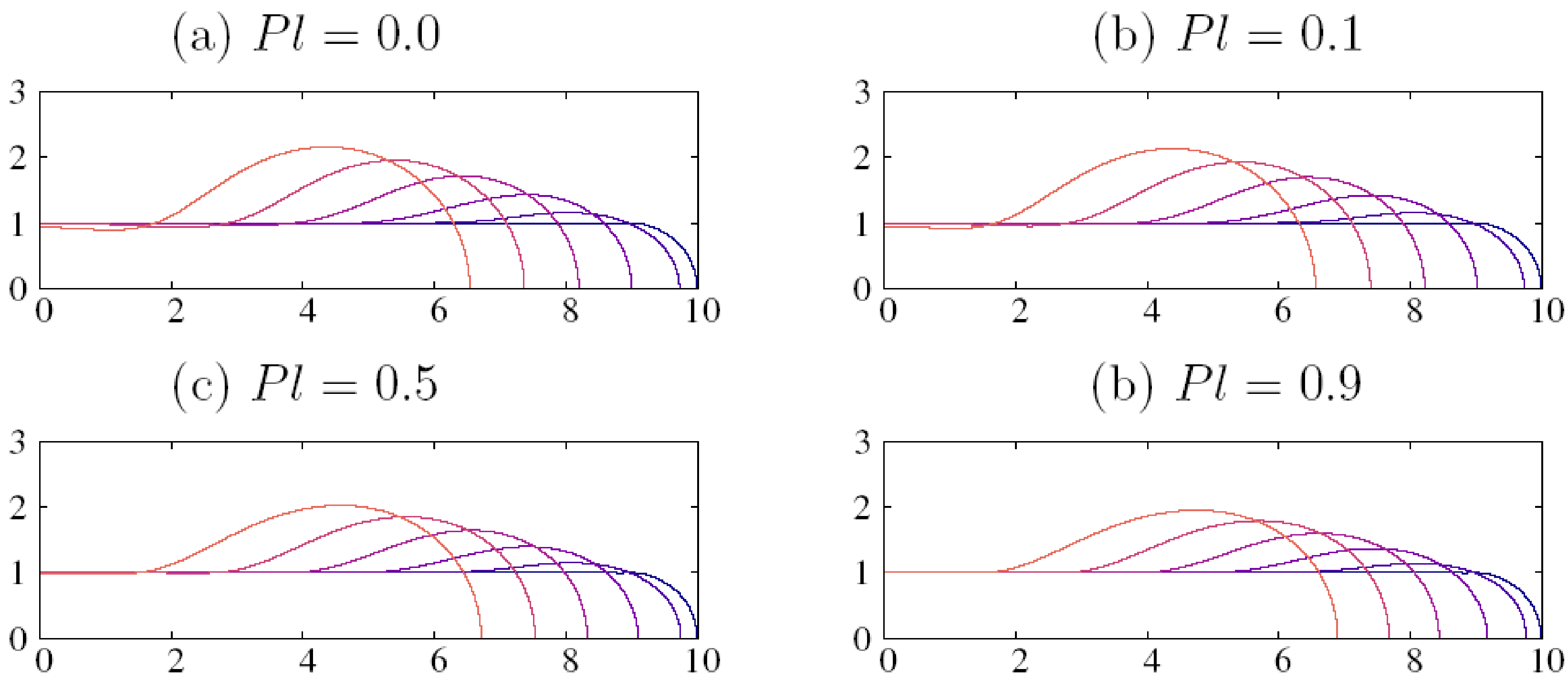
Optimization of infinite viscosity :

$Oh = 1.0, \quad Pl = 0.5$

$\mu_{max} = 10^5 \times \eta_c$



$Oh = 0.1$



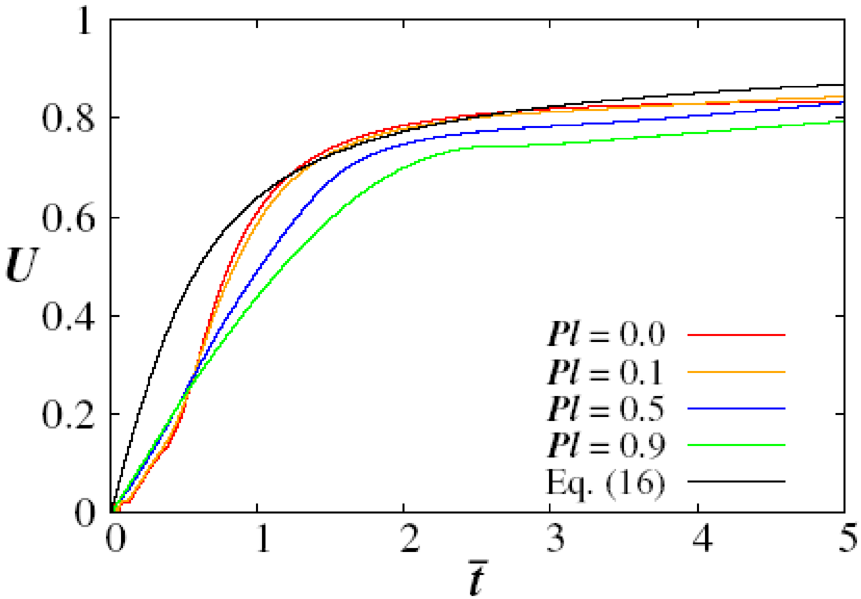
Momentum balance

$$-\frac{d}{dt}\left(2\rho h(L_0 - x_m(t))\frac{d}{dt}(x_m(t))\right) = 2\sigma$$

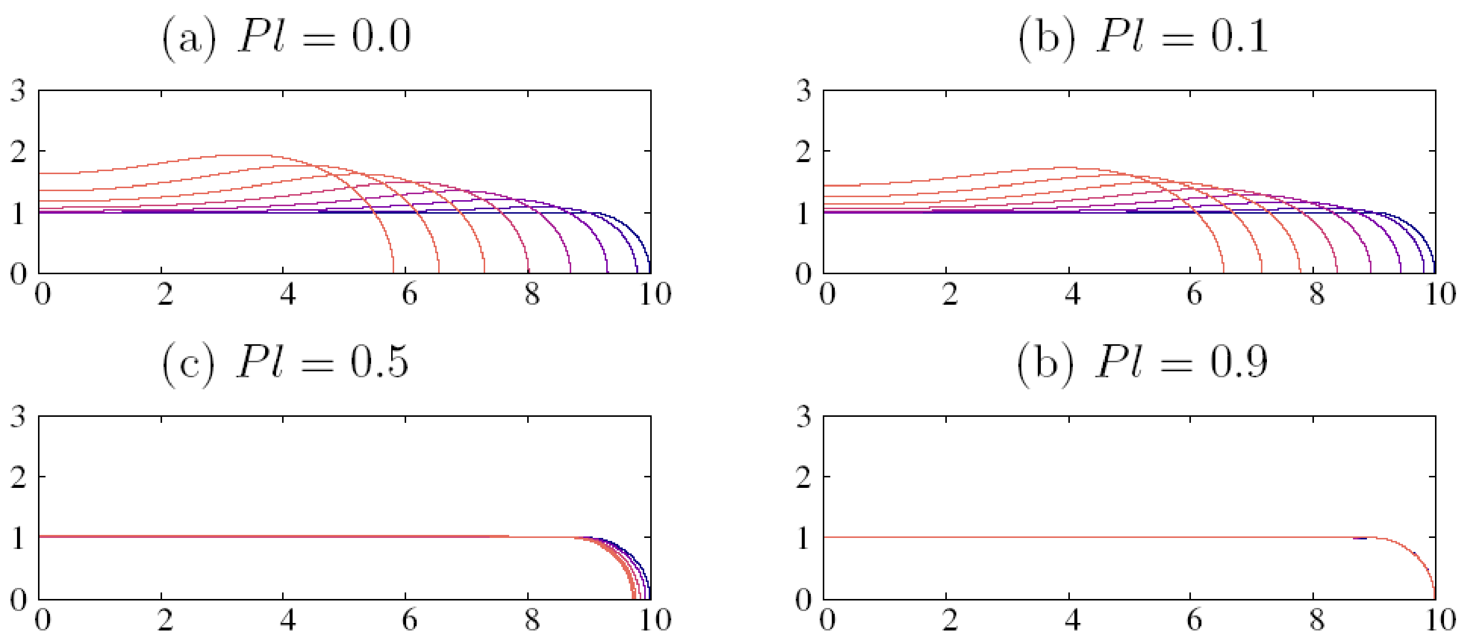
Assumption:
Mass of the blob = $2\rho h(L_0 - x_m(t))$

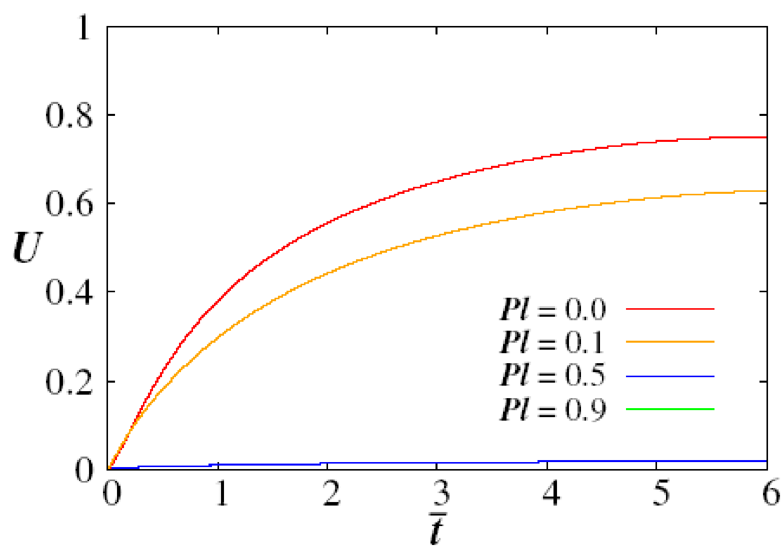
$$u_{tip}(t) = -\frac{dx_m(t)}{dt} - \frac{dR(t)}{dt}$$

$$\bar{u}_{tip}(\bar{t}) = \left(1 - \frac{1}{2\sqrt{\pi}}\left((\bar{L}_0 - \bar{x}_m(0))^2 + \bar{t}^{\,2}\right)^{-1/4}\right)\frac{\bar{t}}{\sqrt{(\bar{L}_0 - \bar{x}_m(0))^2 + \bar{t}^{\,2}}} \quad .....(16)$$



$Oh = 1$


(a) Pl = 0.0
(b) Pl = 0.1
(c) Pl = 0.5
(b) Pl = 0.9
0
1
2
3
0
2
4
6
8
10


U
t̄
0
0.2
0.4
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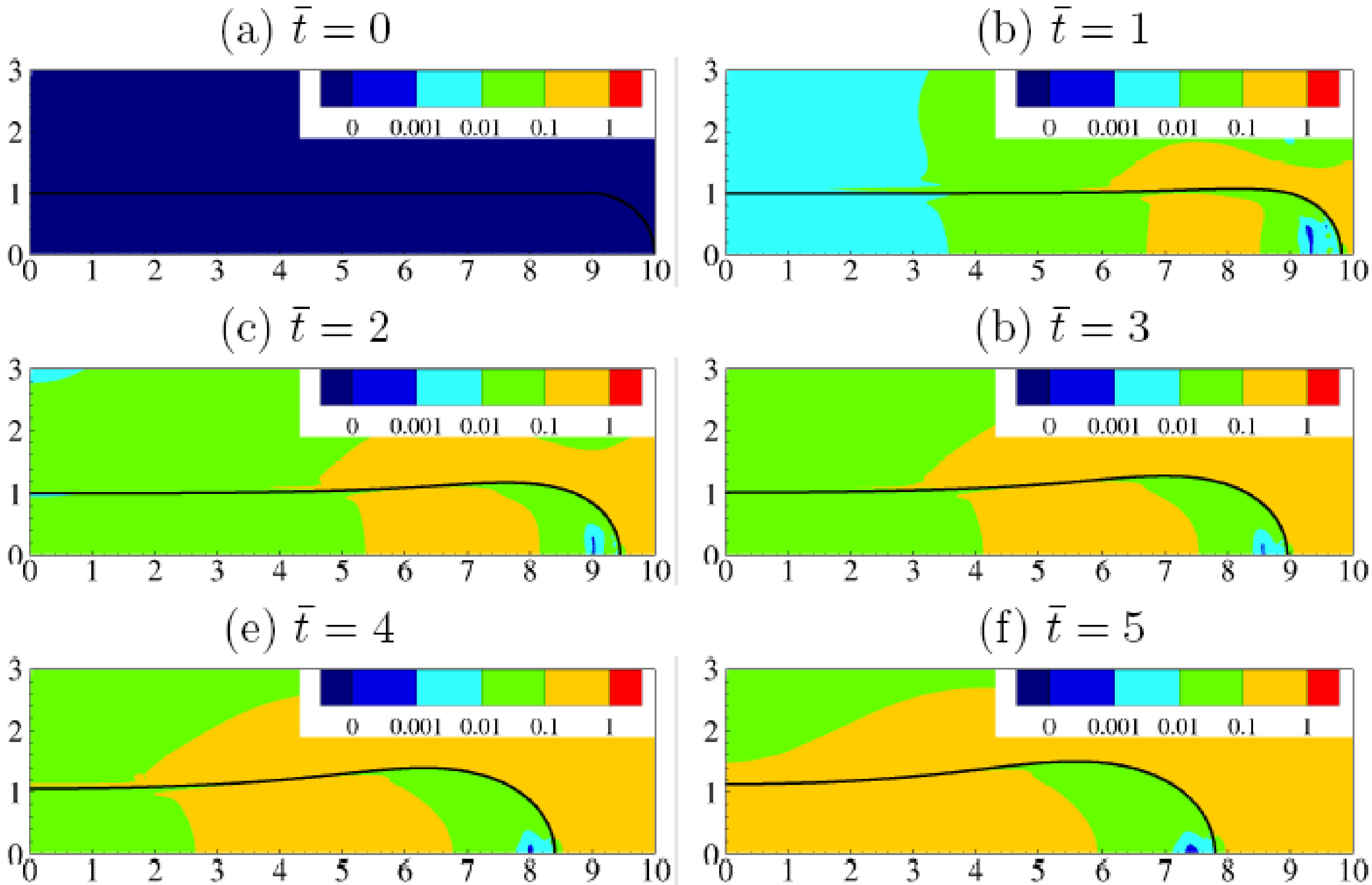
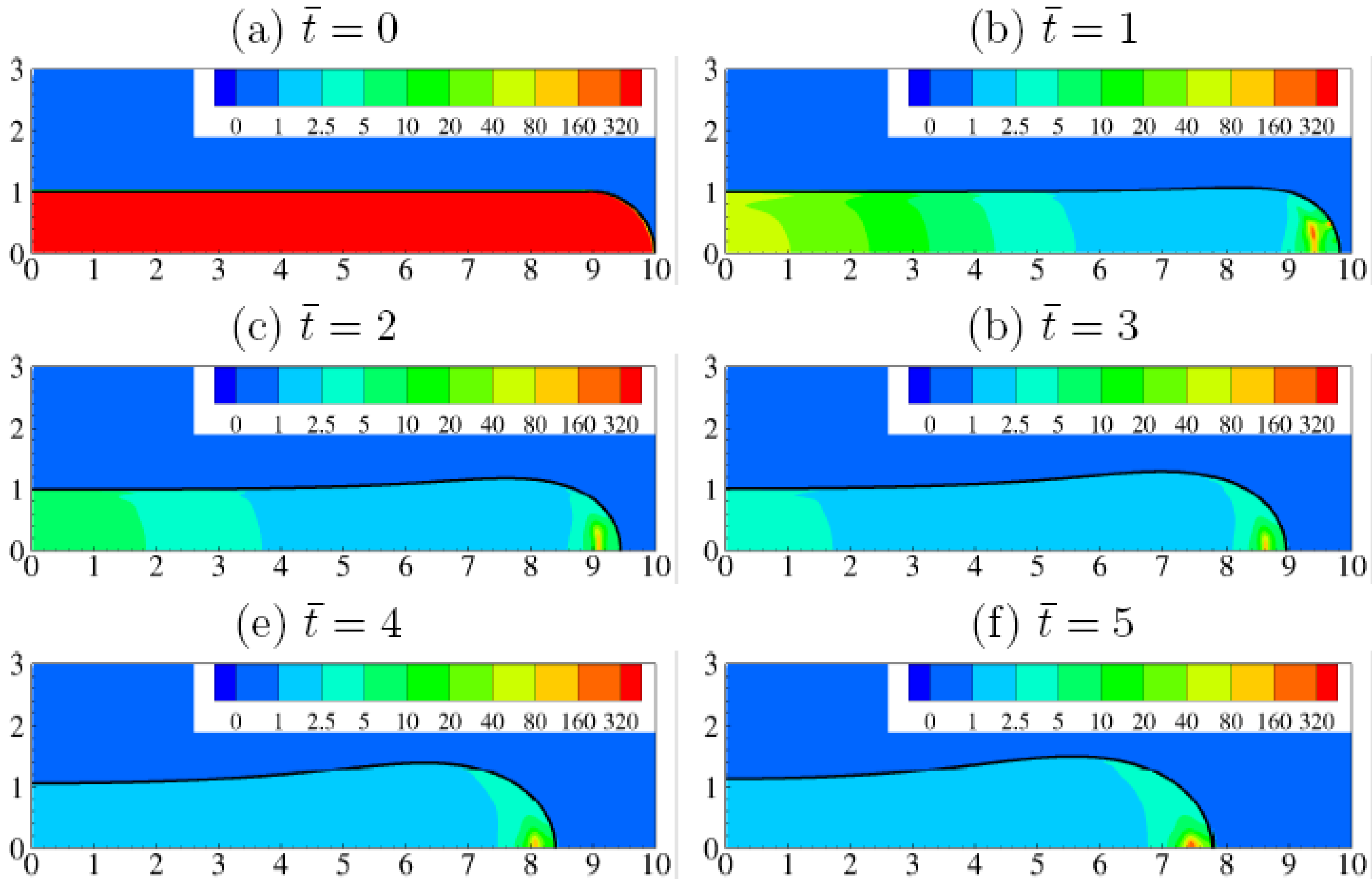
6
Pl = 0.0
Pl = 0.1
Pl = 0.5
Pl = 0.9


(a) $\bar{t} = 0$
(b) $\bar{t} = 1$
(c) $\bar{t} = 2$
(b) $\bar{t} = 3$
(e) $\bar{t} = 4$
(f) $\bar{t} = 5$
0 0.001 0.01 0.1 1
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3

$$Oh = 1 \, , \, Pl = 0.1$$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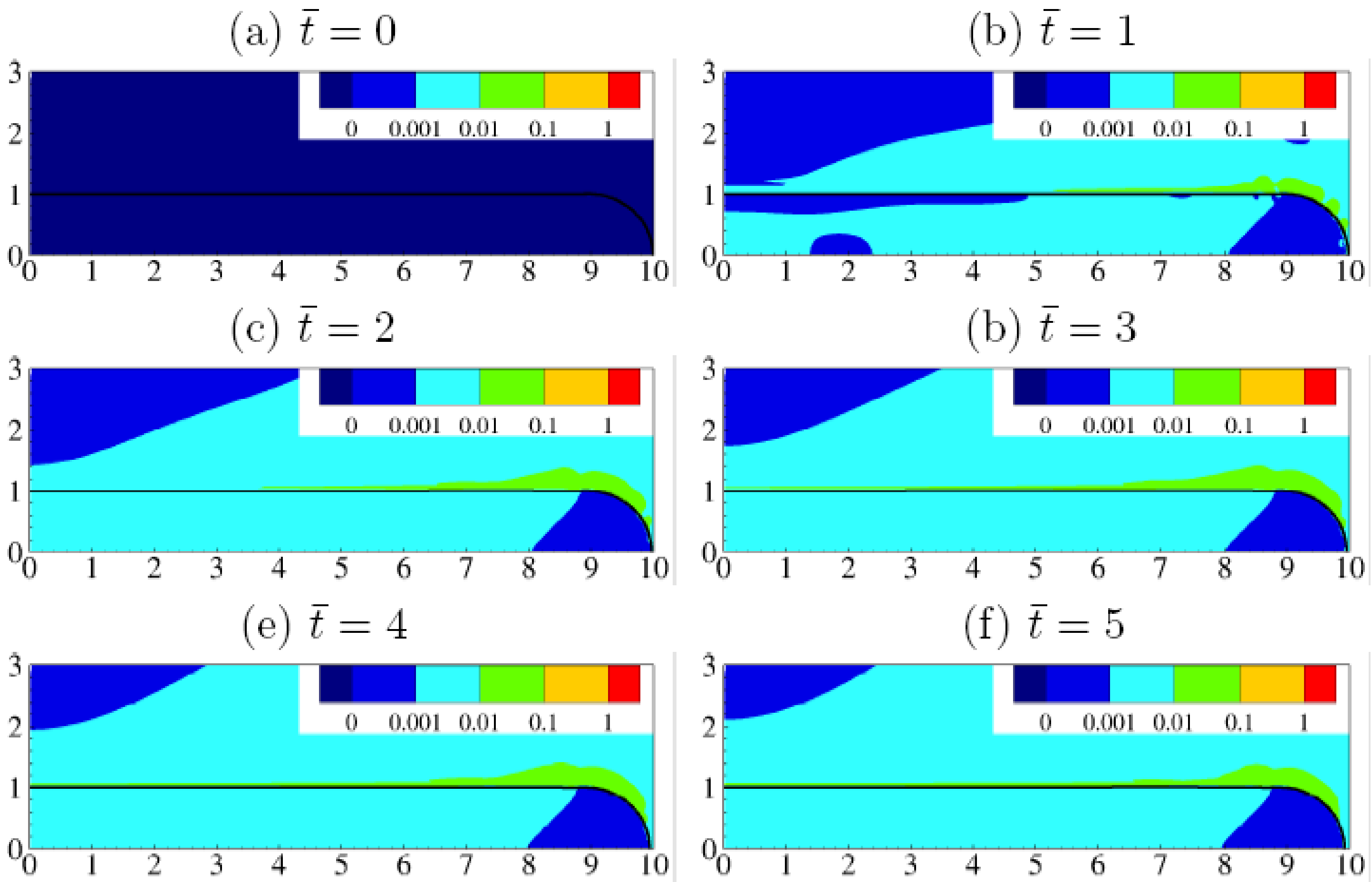
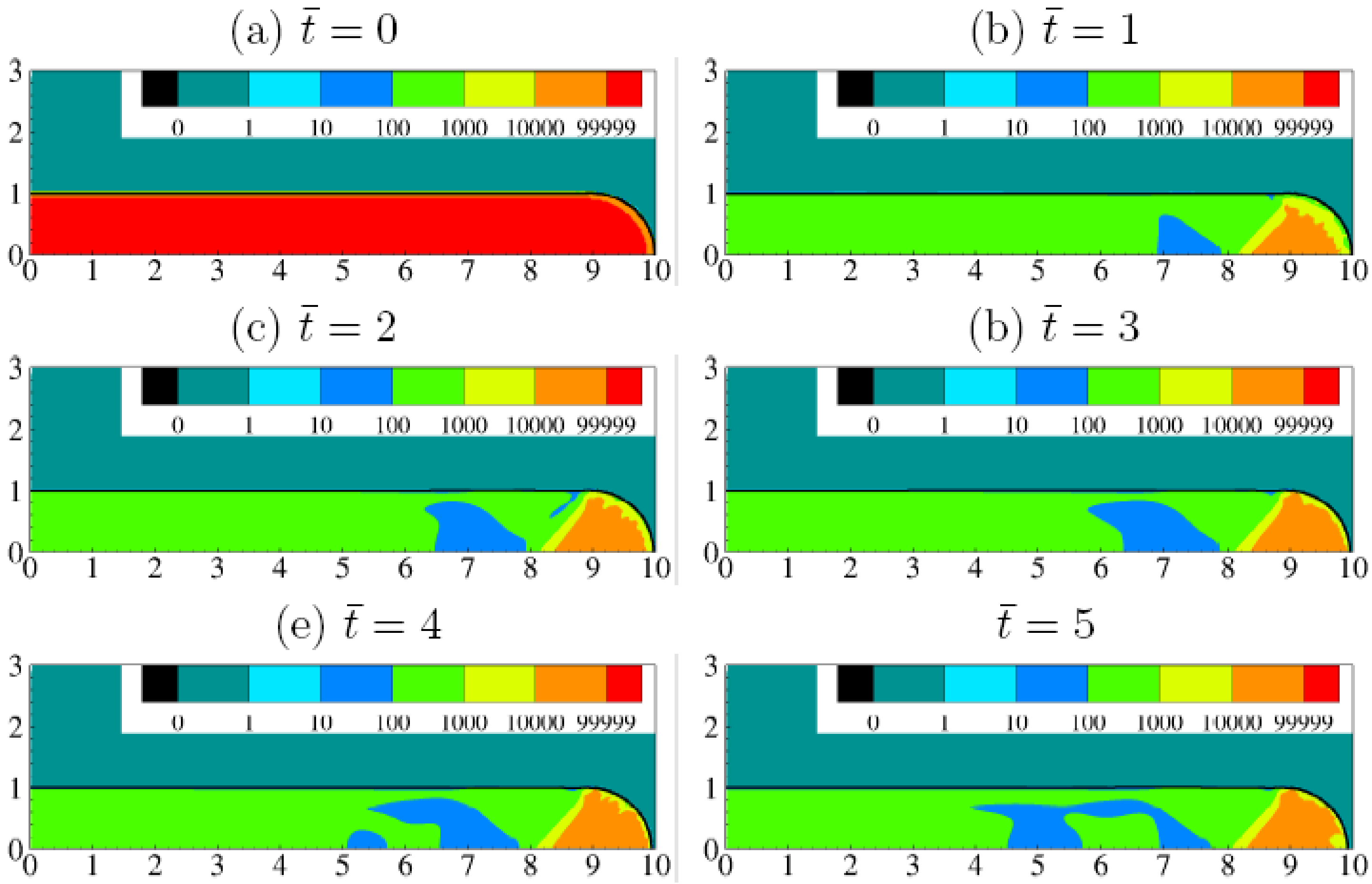
(b) $\bar{t} = 1$
(c) $\bar{t} = 2$
(b) $\bar{t} = 3$
(e) $\bar{t} = 4$
(f) $\bar{t} = 5$
0 1 2.5 5 10 20 40 80 160 320
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3

$$Oh = 1\,,\ Pl = 0.1$$



$$Oh = 1\,,\ Pl = 0.5$$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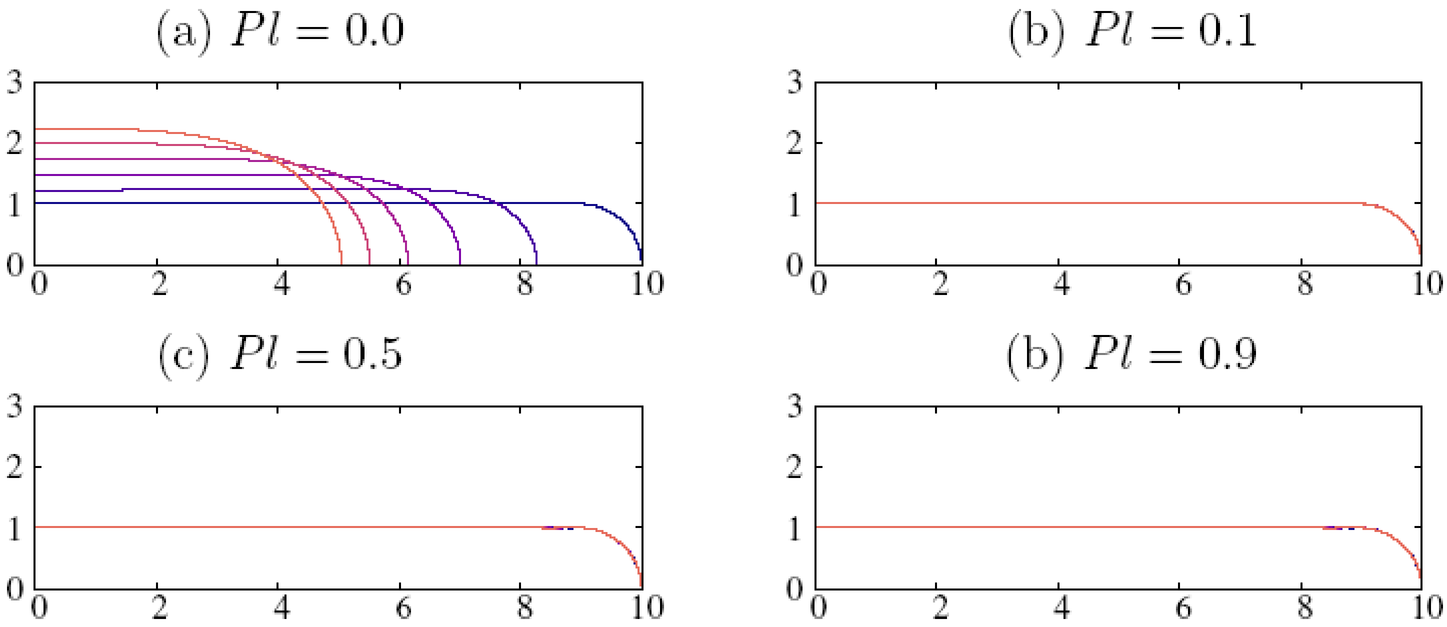
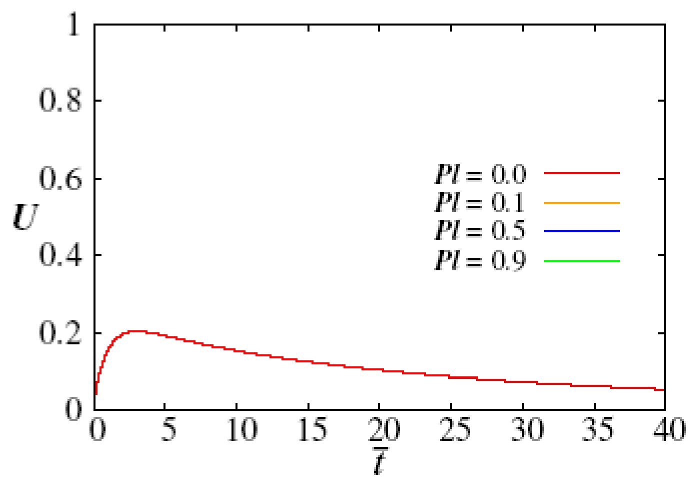
(c) $\bar{t} = 2$
(b) $\bar{t} = 3$
(e) $\bar{t} = 4$
$\bar{t} = 5$
0 1 10 100 1000 10000 99999

$$Oh = 1 \, , \, Pl = 0.5$$


(a) $Pl = 0.0$
(b) $Pl = 0.1$
(c) $Pl = 0.5$
(b) $Pl = 0.9$
0
1
2
3
0
2
4
6
8
10

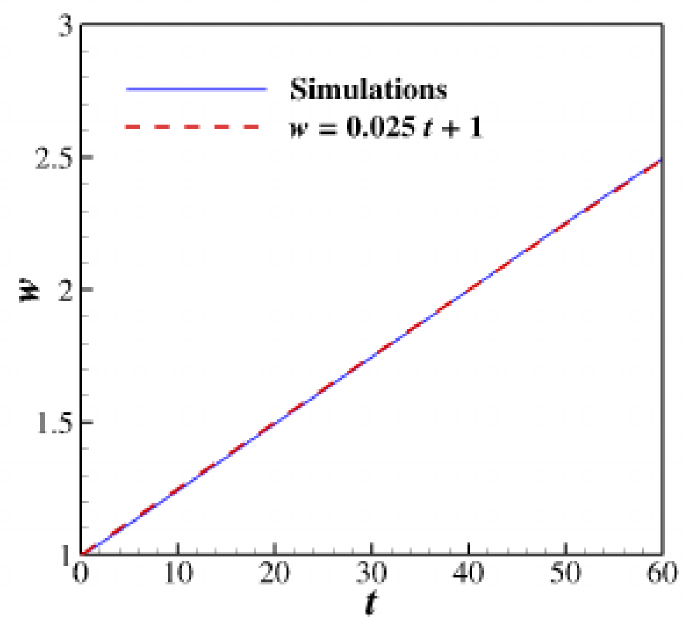
$Oh = 10$

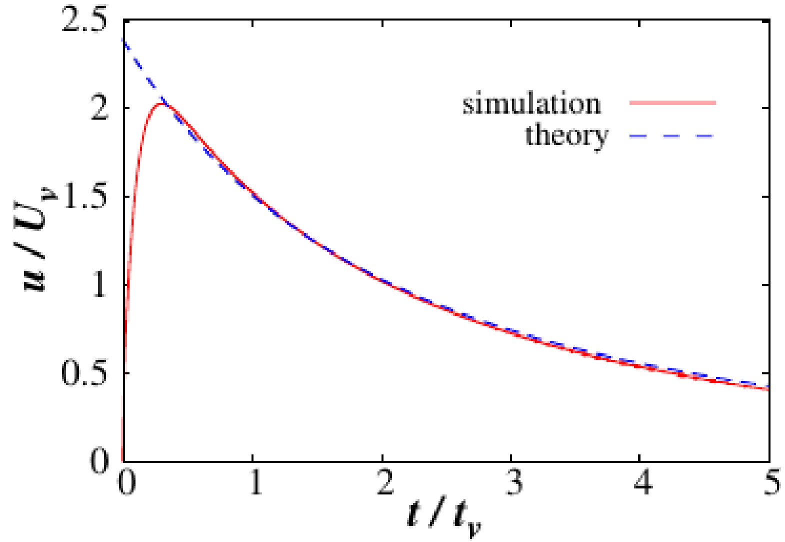

U
$\bar{t}$
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
35
40
$Pl = 0.0$
$Pl = 0.1$
$Pl = 0.5$
$Pl = 0.9$

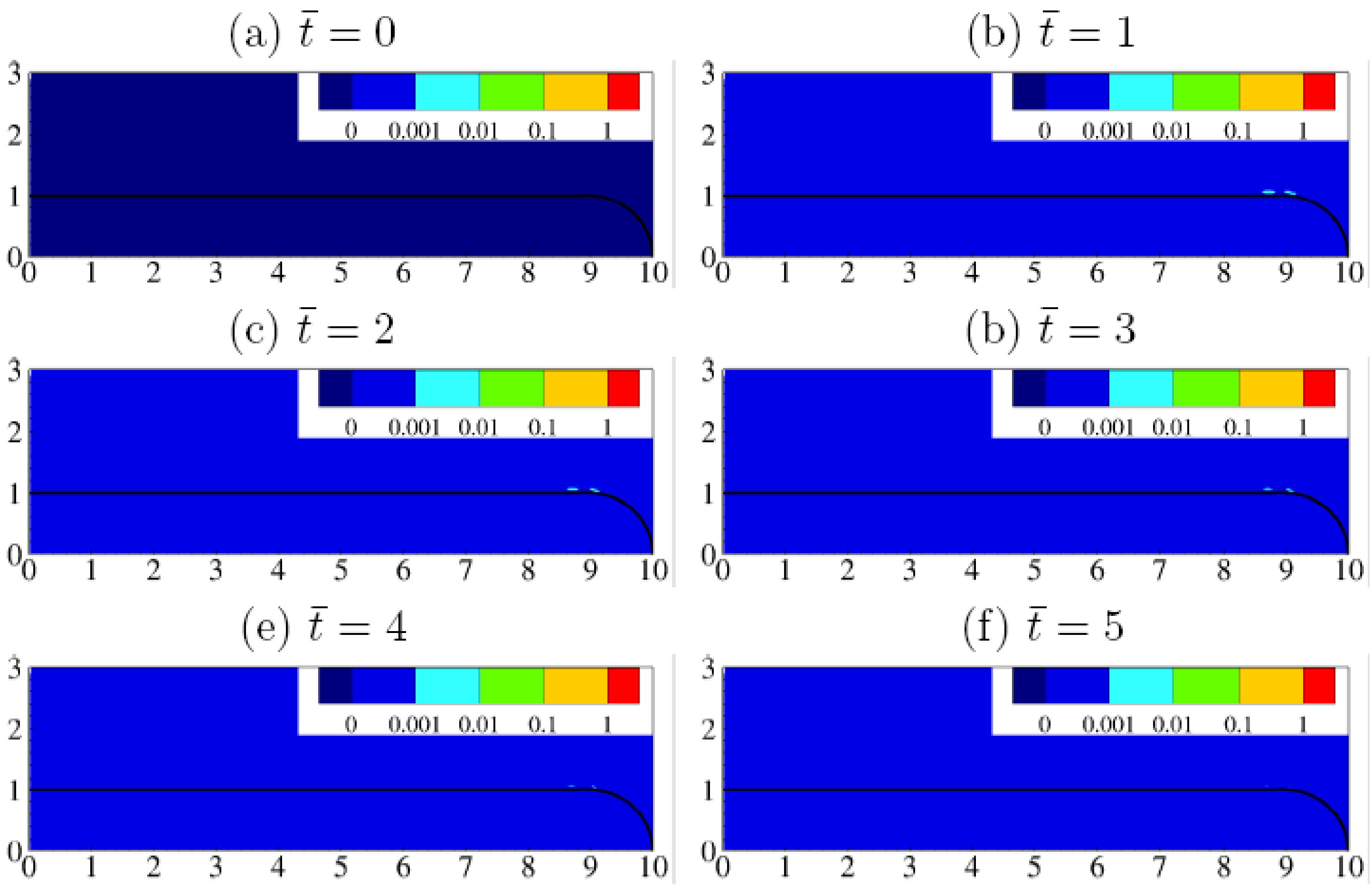
$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial t} = \frac{4\nu}{h}\frac{\partial}{\partial x}\left(h\frac{\partial u}{\partial x}\right) - \frac{\sigma}{\rho}\frac{\partial \kappa}{\partial x}$$

$$-\frac{\partial L}{U_v \partial t} = \frac{4(L_0/W_0 - 1) + \pi}{(t/t_v - 4)^2} - \frac{1}{4}\left(1 - \frac{\pi}{4}\right)$$



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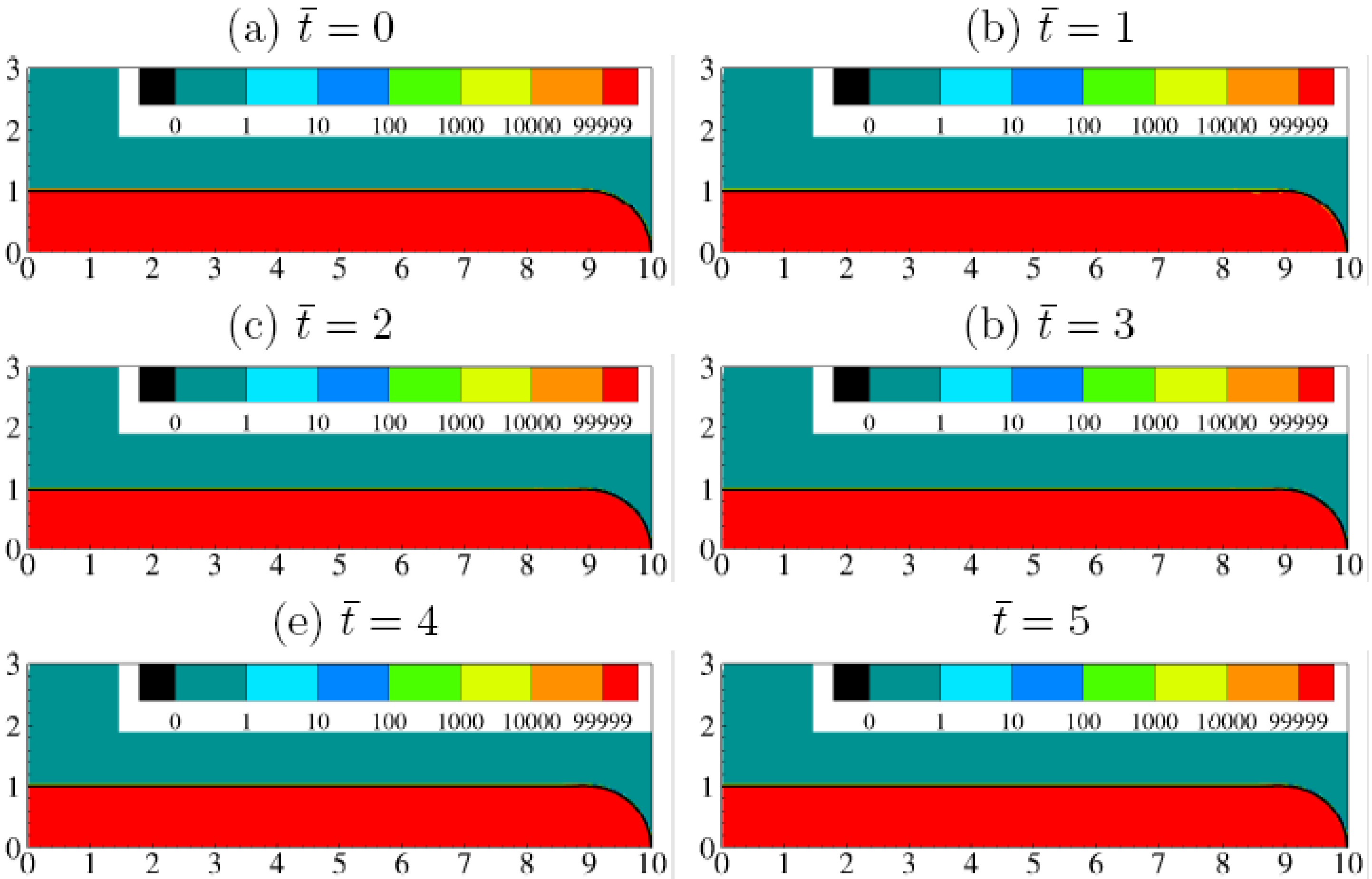
(f) $\bar{t} = 5$
0 0.001 0.01 0.1 1
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3

$$Oh = 10 \;,\; Pl = 0.5$$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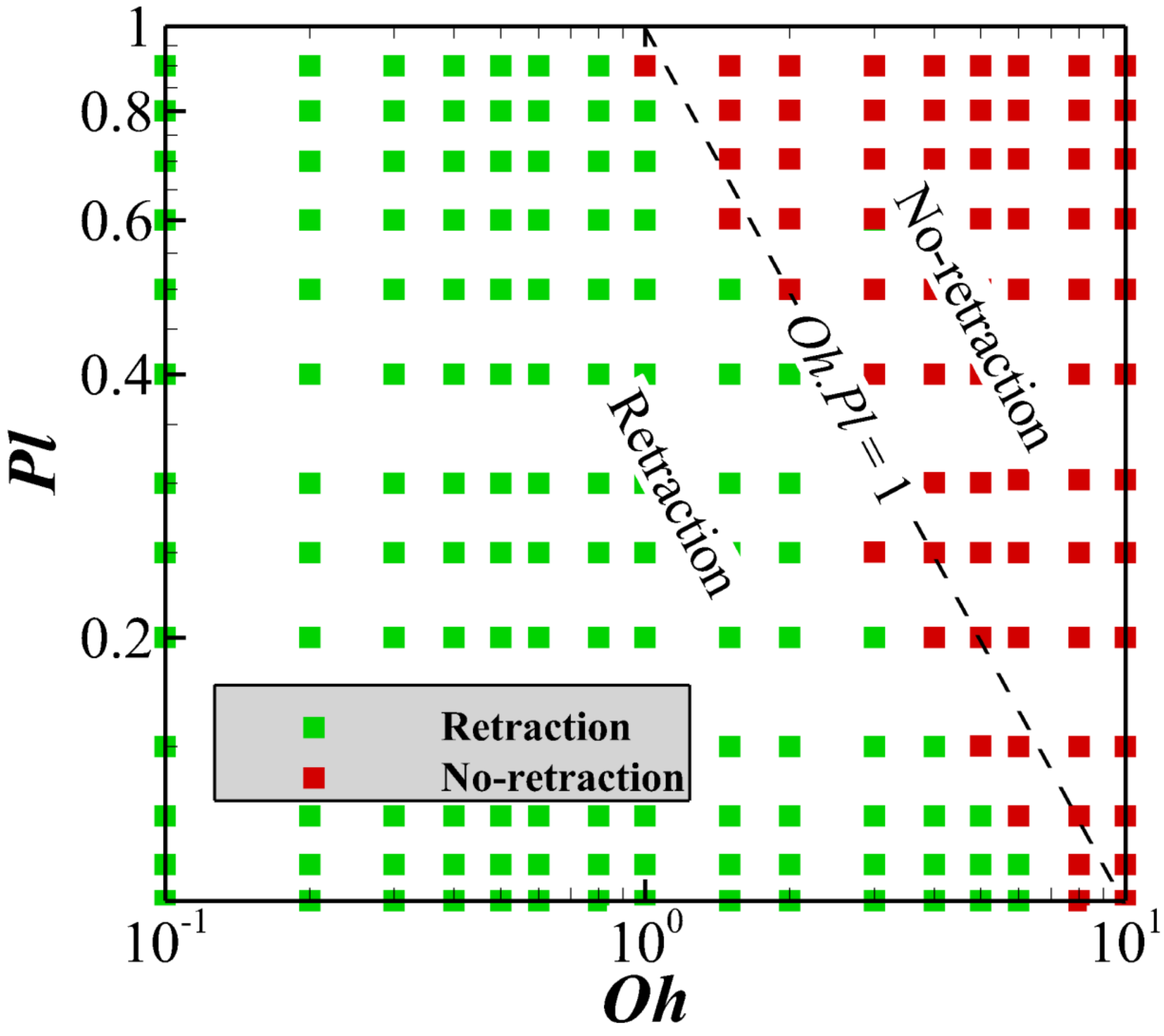
(c) $\bar{t} = 2$
(b) $\bar{t} = 3$
(e) $\bar{t} = 4$
$\bar{t} = 5$
0 1 10 100 1000 10000 99999

$$Oh = 10 \ , \ Pl = 0.5$$

$$\tau_y h / \sigma < 1$$

$$\tau_y h / \sigma = Oh \times Pl < 1$$



Acknowledgement

✓ I acknowledge the financial support of IFPEN.

THANK YOU