

PRINCETON
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Some similarity problems involving bubbles and filaments

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Two mini-stories involving similarity solutions

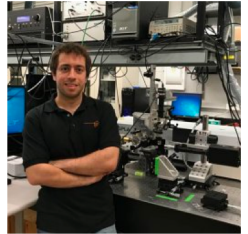
Visco-elastic filament thinning

E. Turkoz, J.M. Lopez-Herrera, J. Eggers, C.B. Arnold and L. Deike, Journal of Fluid Mechanics Rapids, 851, R2, 2018.

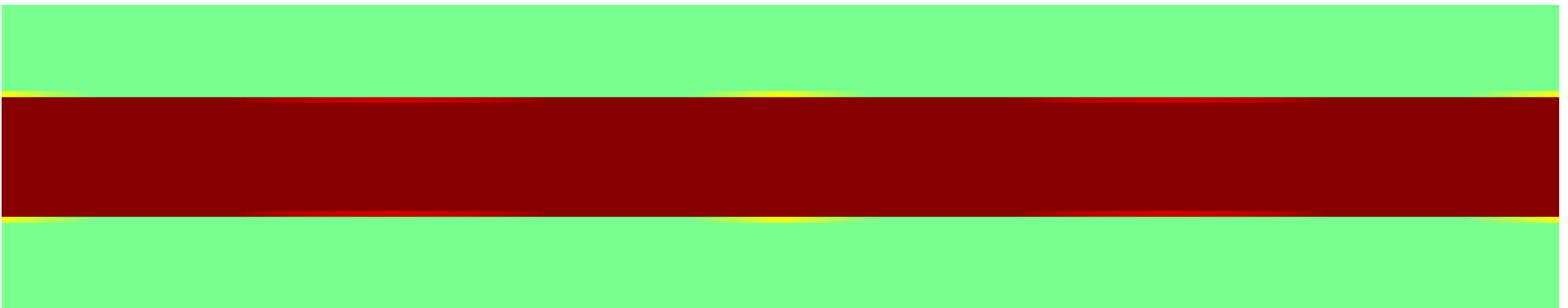
Inertio-capillary cavity collapse and jet ejection during bursting bubble

C-Y. Lai, J. Eggers and L. Deike, Physical Review Letters, 121, 2018.

Visco-elastic filament thinning



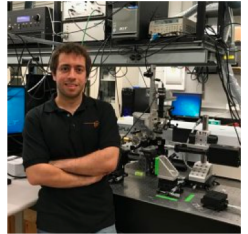
Exponential thinning of a viscoelastic filament ($De = 60$) in ambient air



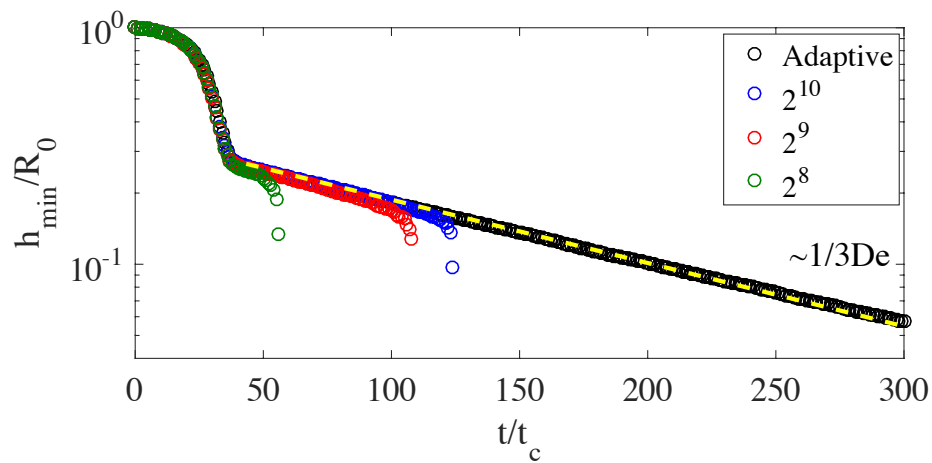
E. Turkoz, J. M. Lopez-Herrera, J. Eggers, C. B. Arnold and L. Deike, "Axisymmetric simulation of viscoelastic filament thinning with the Oldroyd-B model," **J. Fluid. Mech.**, 851, R2 (2018)

Uses the implementation from **López-Herrera, J.M.**, Popinet, S. and Castrejón-Pita, A.A., 2019. *An adaptive solver for viscoelastic incompressible two-phase problems applied to the study of the splashing of weakly viscoelastic droplets*. Journal of Non-Newtonian Fluid Mechanics, 264, pp.144-158.

Visco-elastic filament thinning: similarity solution

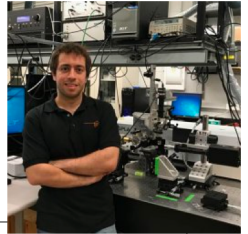


Filament thinning: $h_{min}(t) = h_0 \exp[-t/(3\lambda)],$

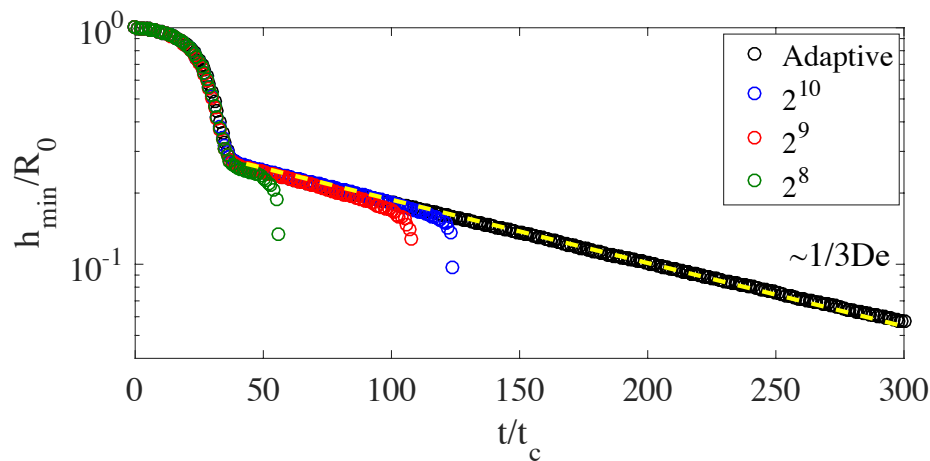


In the end of the simulation ($t/t_c = 300$):
 $h_{min}/R_0 \approx 0.06$, which corresponds to ~ 40
grid points in radial direction.

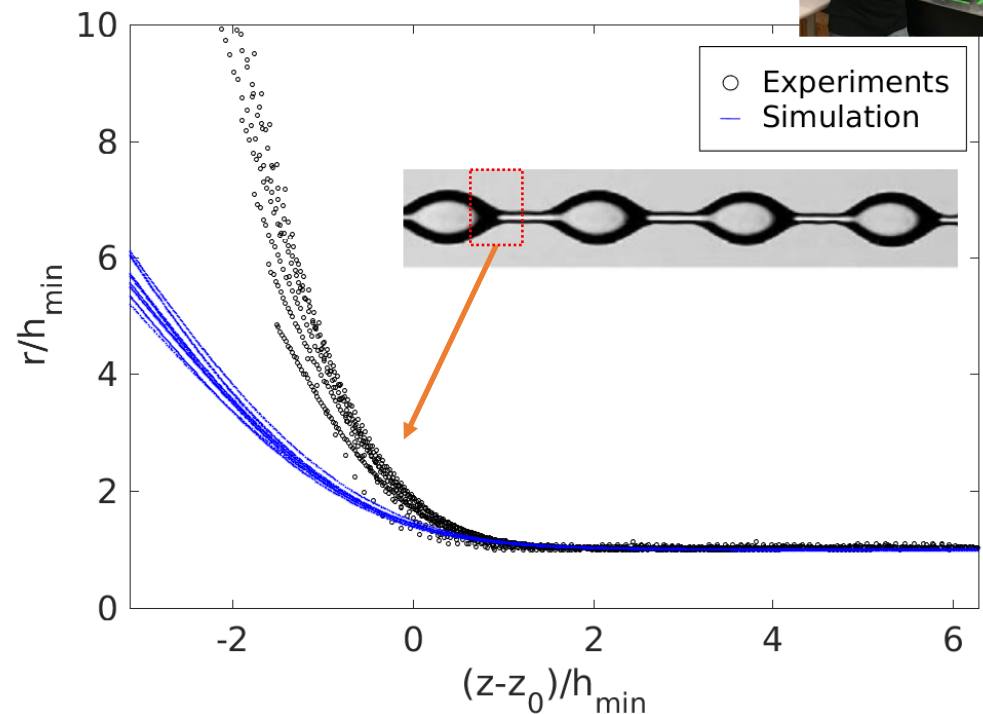
Visco-elastic filament thinning: similarity solution



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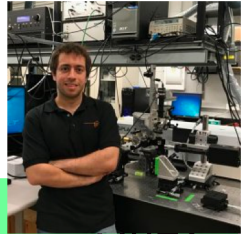


In the end of the simulation ($t/t_c = 300$):
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Both experimental and simulation profiles exhibit self-similarity. However, experimental profiles exhibit a larger curvature at the drop-thread connection

Moving forward: visco-elastic jetting



$Oh=2$
 $De=0.75$

$Oh=2$
 $De=5$

$Oh=2$
 $De=20$

Two mini-stories involving similarity solutions

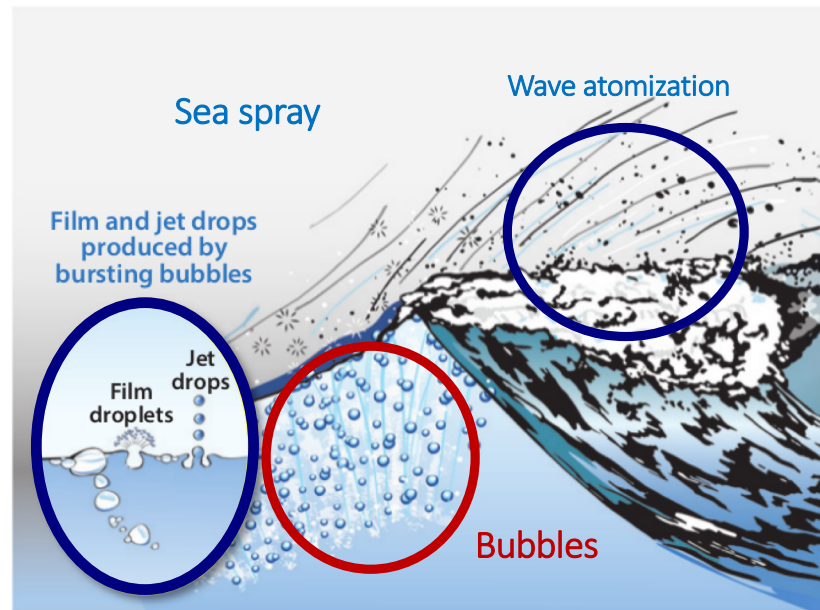
Visco-elastic filament thinning

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Inertio-capillary cavity collapse and jet ejection during bursting bubble

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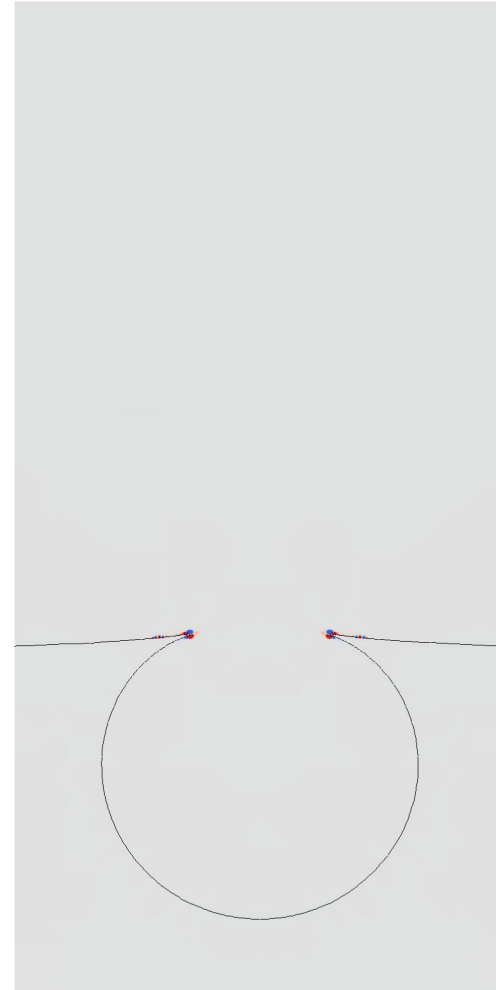
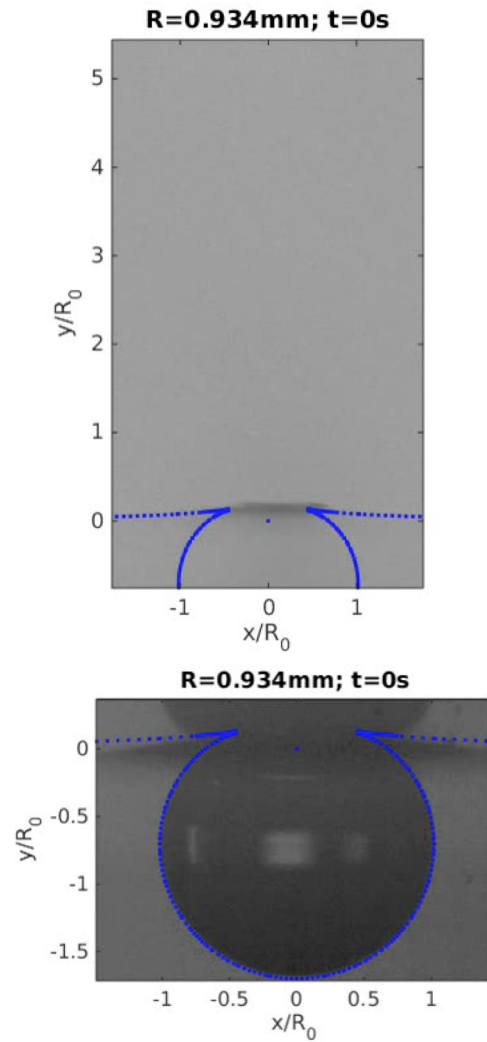
Context: Mass transfers at the ocean-atmosphere interface



From water to air: Transfer of momentum, heat, moisture
Production of aerosols (sea salt, biological particles)
→ climate impact (cloud nucleation & radiative balance)

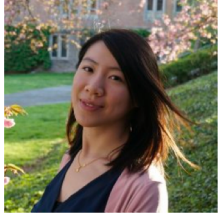
From air to water: Air entrainment & gas transfer
→ climate impact (carbon uptake)

Bubble bursting: jet dynamics and jet drops



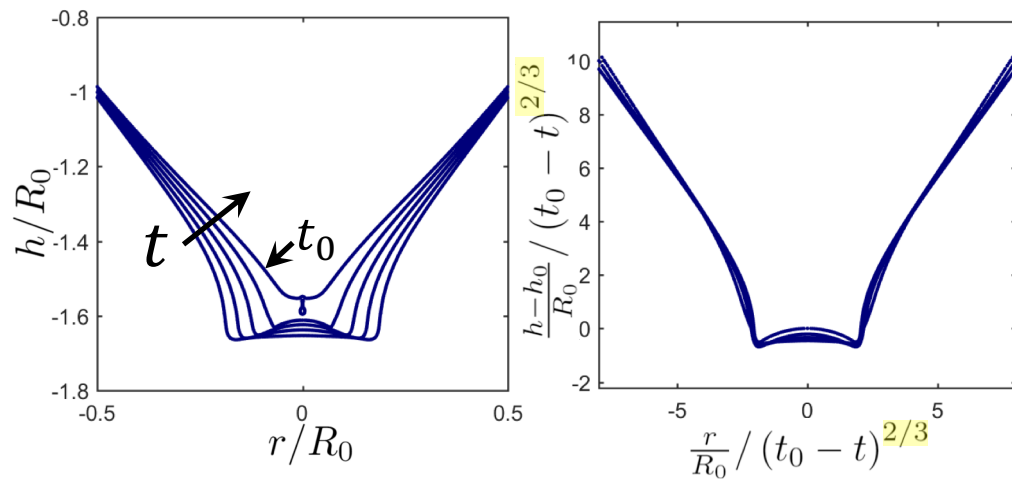
Deike et al 2018
Lai, Eggers and Deike, 2018

Bubble bursting: time evolution and similarity solutions



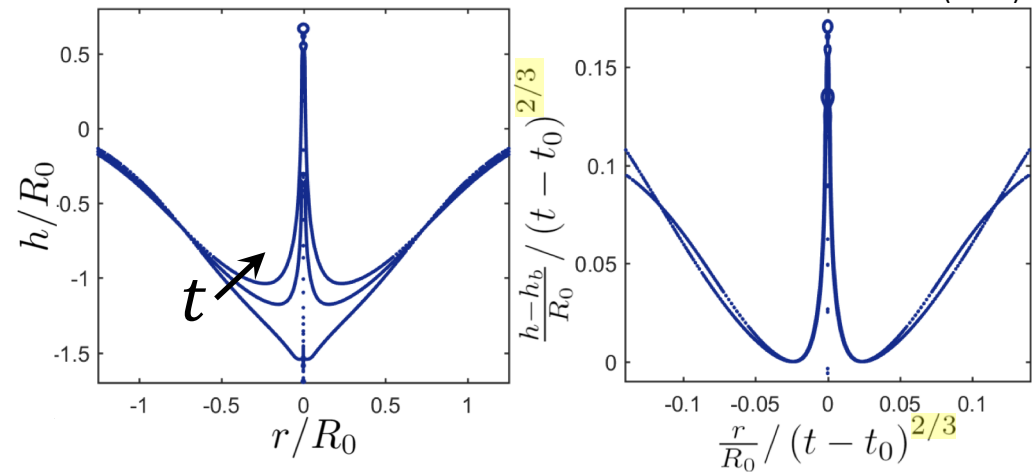
I. Cavity Collapse ($t < t_0$)

Duchemin (2002)
Ghabache et al. (2014)



II. Jet Ejection ($t > t_0$)

Brasz et al. (2018)



- Irrotational & Incompressible

- **Inertia** vs **surface tension** Zeff et. al. (2000)

Universal profiles:

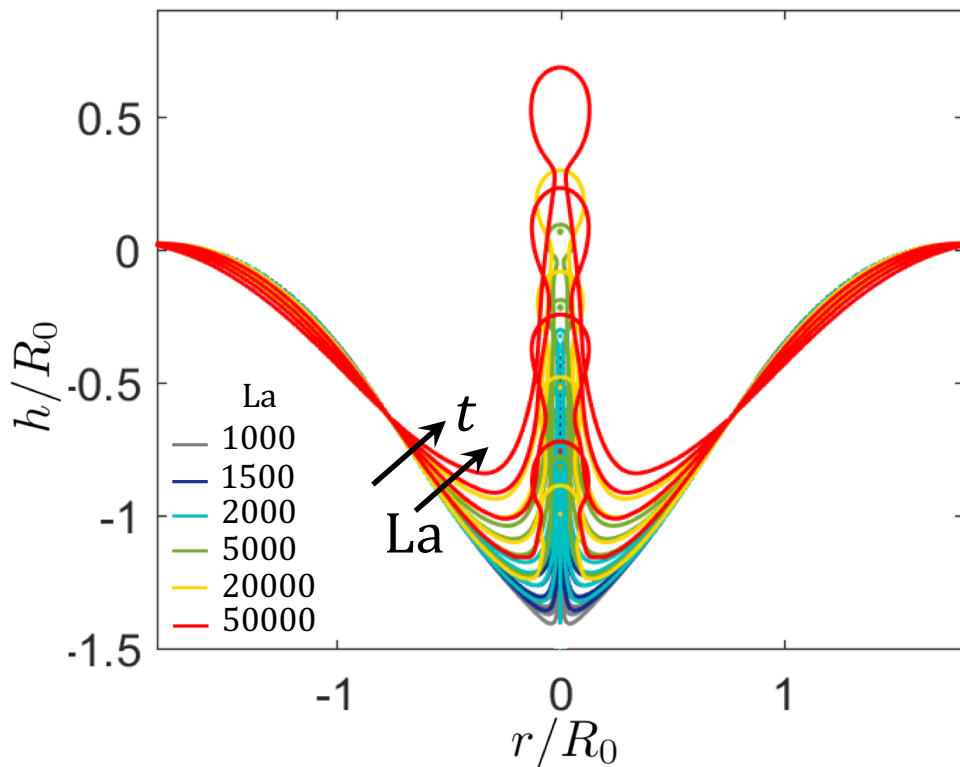
$$\frac{h(r, t)}{|t - t_0|^{2/3}} = f\left(\frac{r}{|t - t_0|^{2/3}}\right) \xrightarrow{r^*, h^*, t^*}$$

$$\frac{h(r, t)/h^*}{|(t - t_0)/t^*|^{2/3}} = f\left(\frac{r/r^*}{|(t - t_0)/t^*|^{2/3}}\right)$$

Universal jet profiles

Lai, Eggers and Deike, PRL, 2018

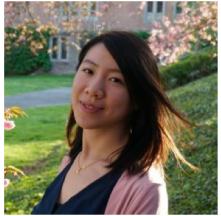
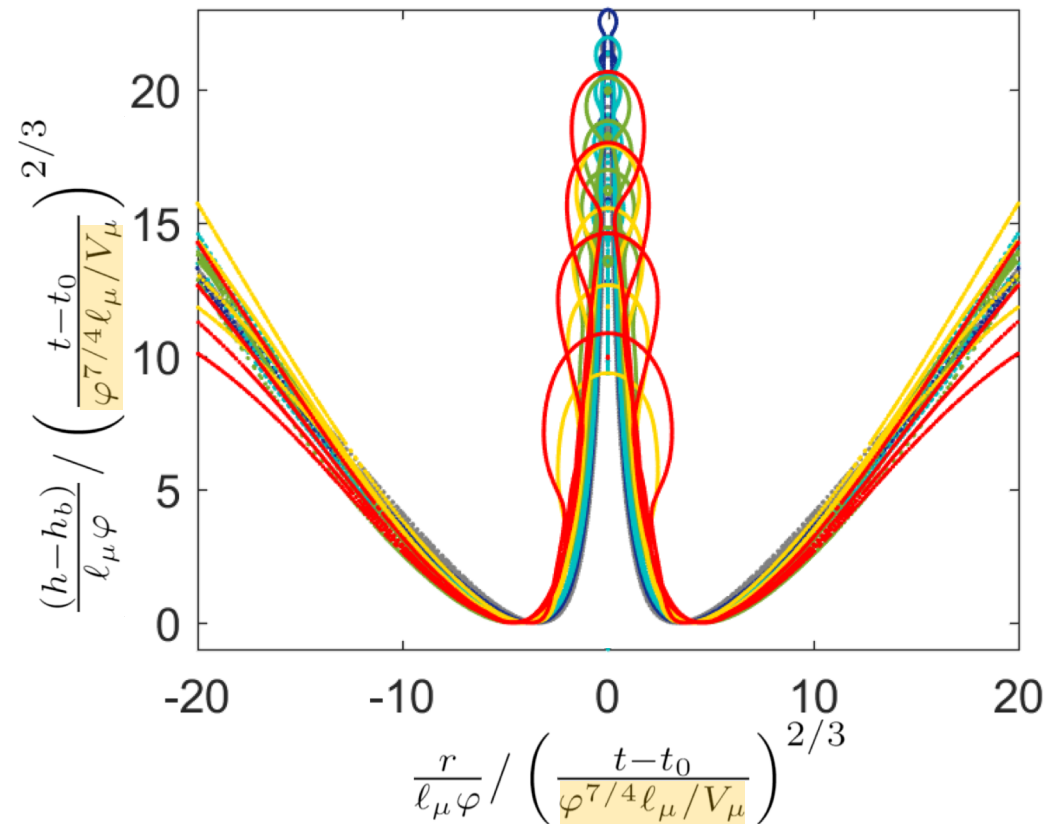
$$La = \rho\gamma R_0/\mu^2 \quad Oh = La^{-2}$$



$$\frac{(h - h_b)/h^*}{|(t - t_0)/t^*|^{2/3}} = f\left(\frac{r/r^*}{|(t - t_0)/t^*|^{2/3}}\right)$$

$$t^* = t_j \equiv \varphi^{7/4} l_\mu / V_\mu$$

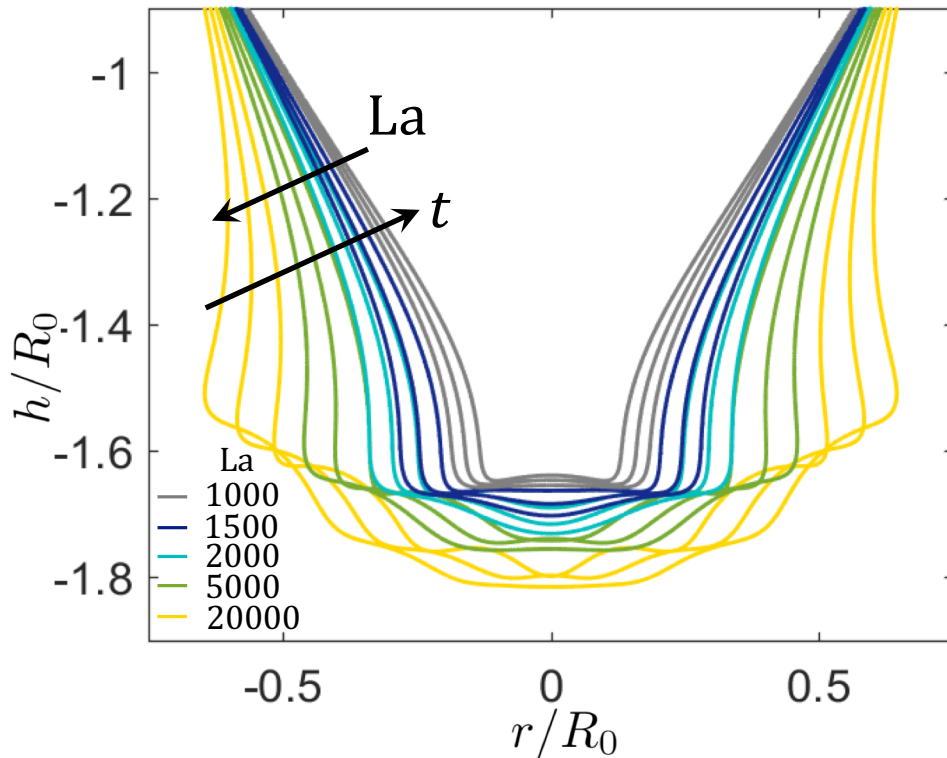
$$h^* = r^* \equiv l_\mu \varphi$$



Universal cavity profiles

Lai, Eggers and Deike, PRL, 2018

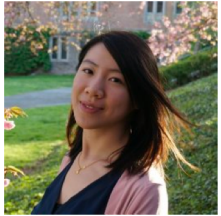
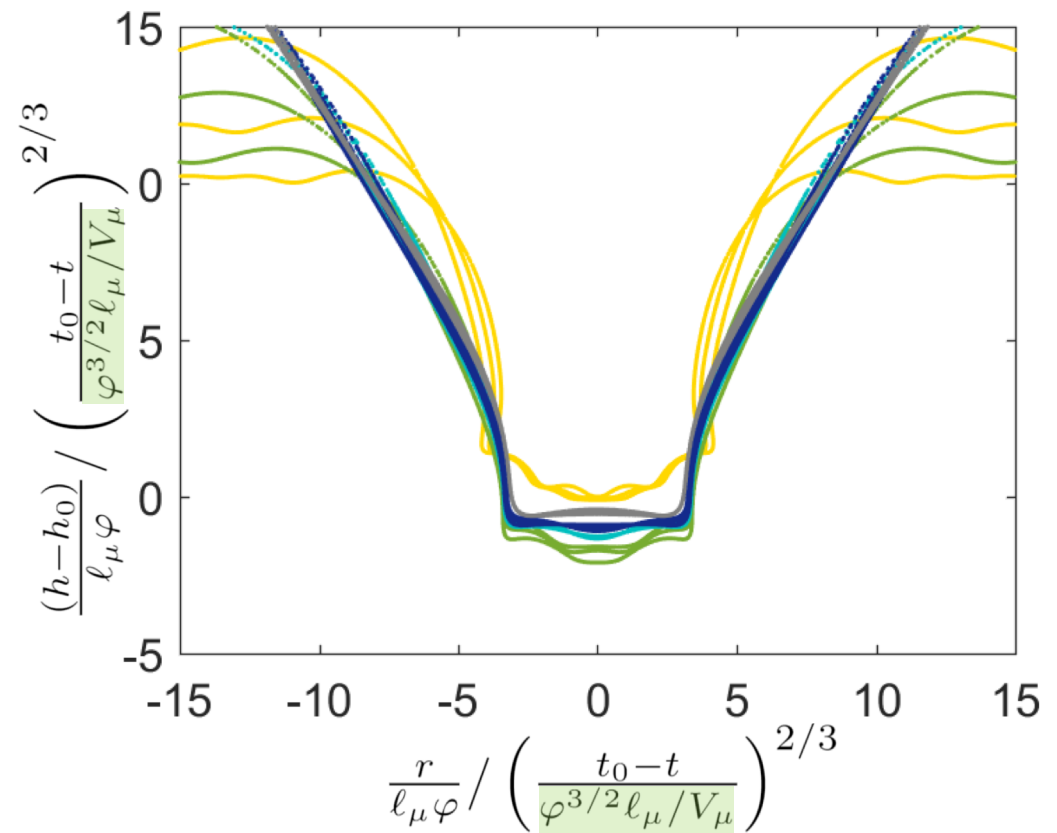
$$La = \rho\gamma R_0/\mu^2 \quad Oh = La^{-2}$$



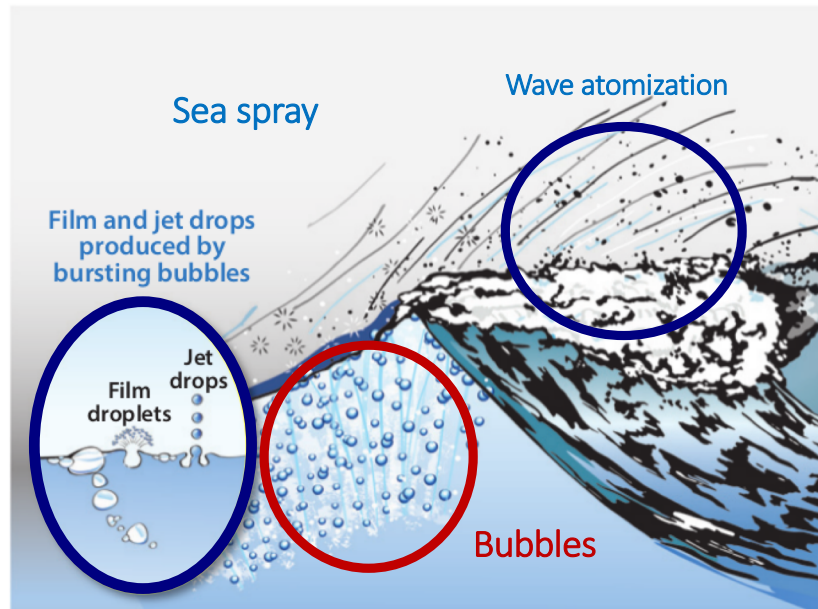
$$\frac{(h - h_0)/h^*}{|(t - t_0)/t^*|^{2/3}} = f\left(\frac{r/r^*}{|(t - t_0)/t^*|^{2/3}}\right)$$

$$t^* = t_c \equiv \varphi^{3/2} l_\mu / V_\mu$$

$$h^* = r^* \equiv l_\mu \varphi$$



And now bubble break-up in turbulence by Alienor Riviere



From water to air: Transfer of momentum, heat, moisture
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From air to water: Air entrainment & gas transfer
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