

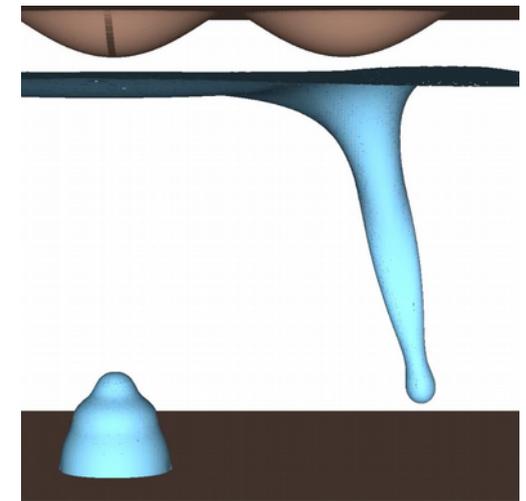
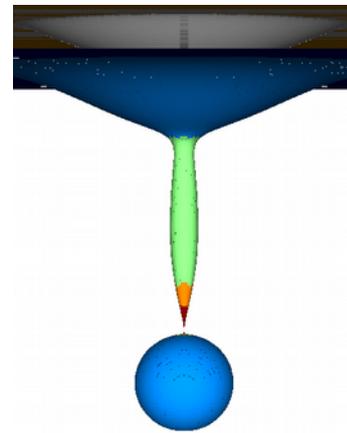
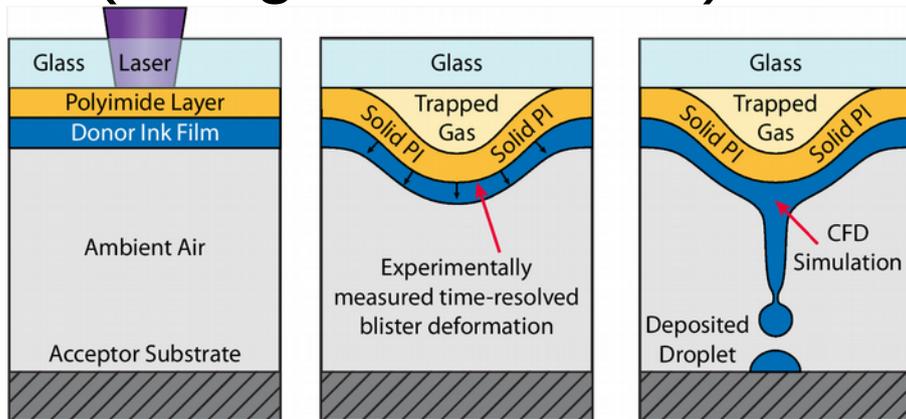
# Jets and droplets from bursting bubbles

**Frederik Brasz, Casey Bartlett, Peter Walls,  
Elena Flynn, Estella Yu, James Bird**  
*Boston University*

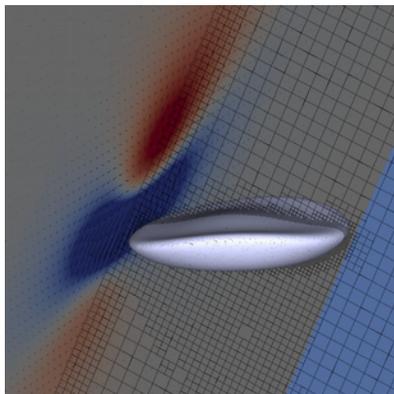
Basilisk/Gerris User's Meeting  
Princeton, NJ  
November 15, 2017

# Introduction

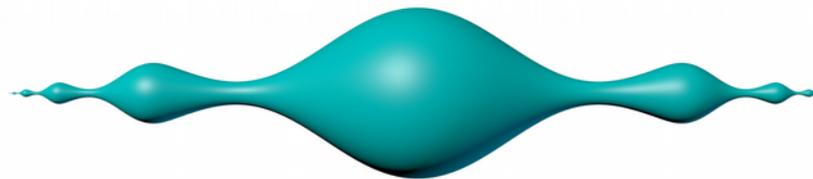
- Started using Gerris for my PhD work here in Princeton (Craig Arnold's lab) in 2012



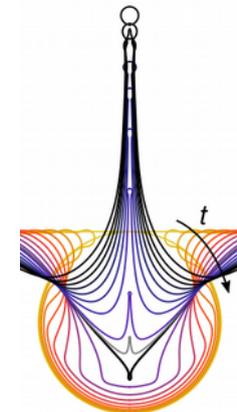
- Started postdoc at BU (Jacy Bird's lab) August 2016



Rising bubbles beneath inclined walls

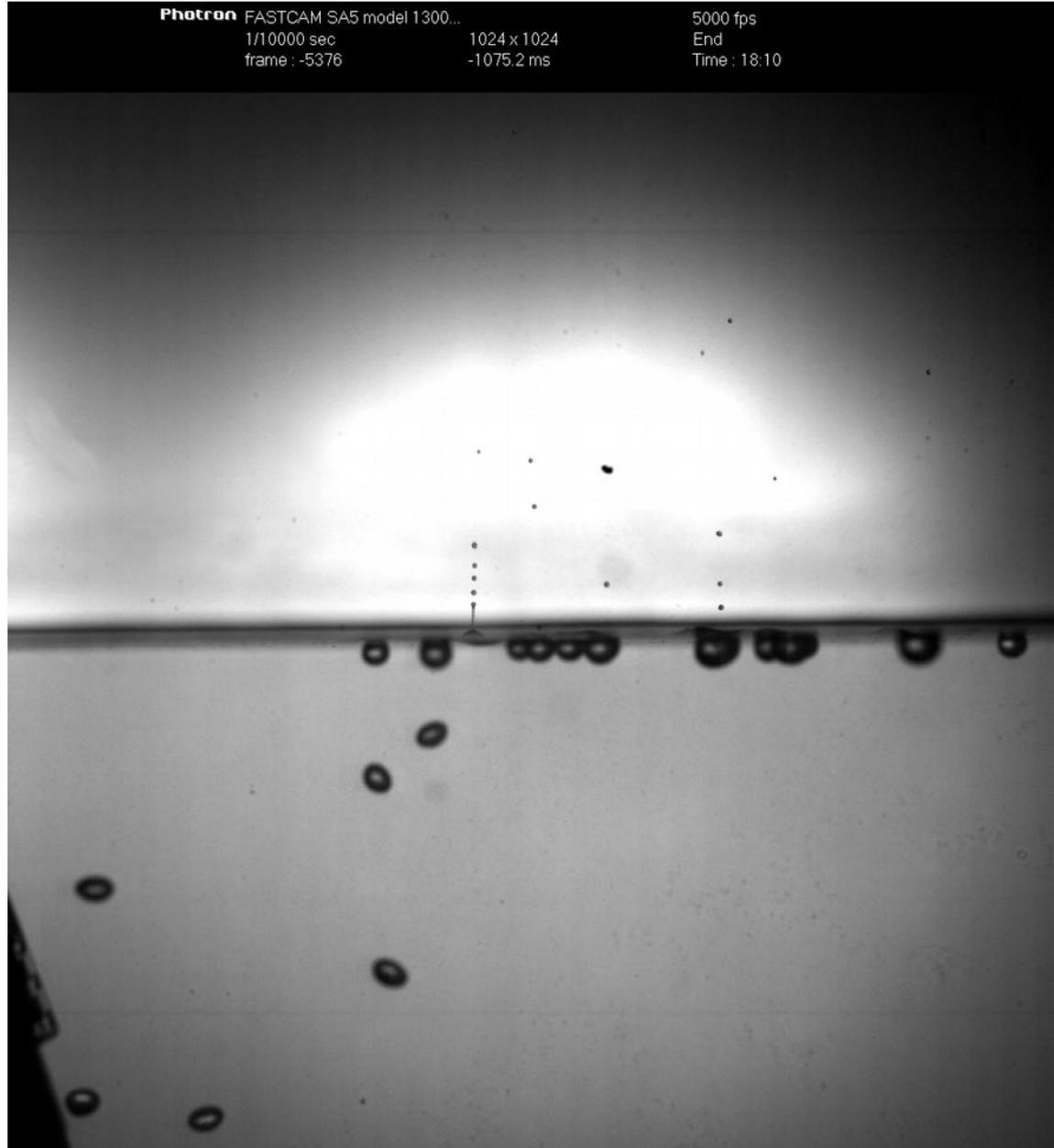
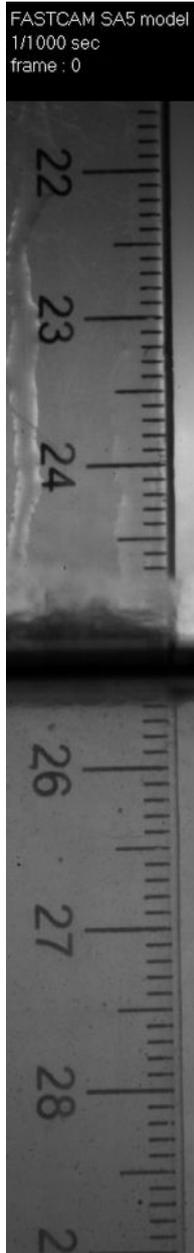


Self-similar breakup of liquid cones

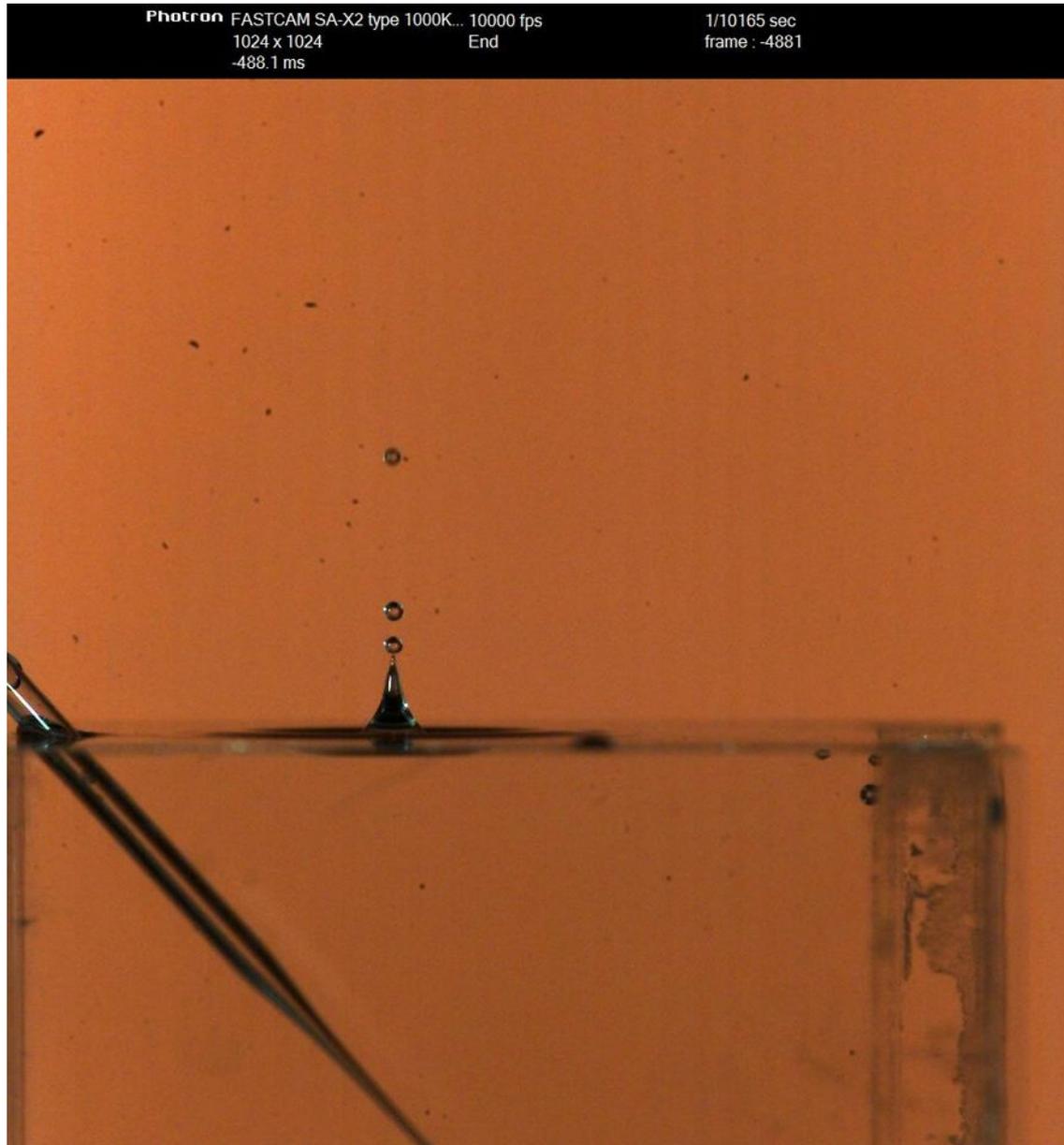


Jet drops from bursting bubbles

# Jet drops from bursting bubbles



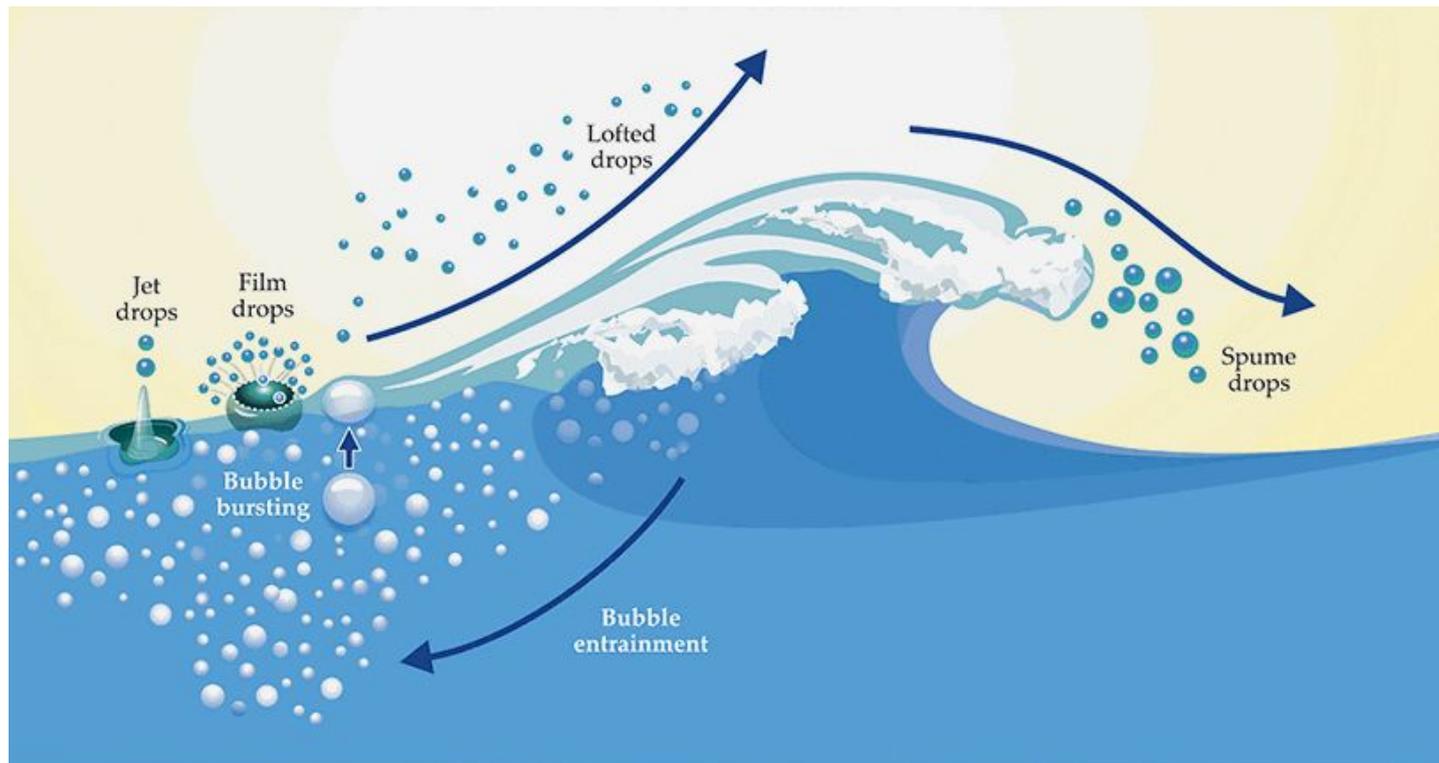
# Jet drops from bursting bubbles



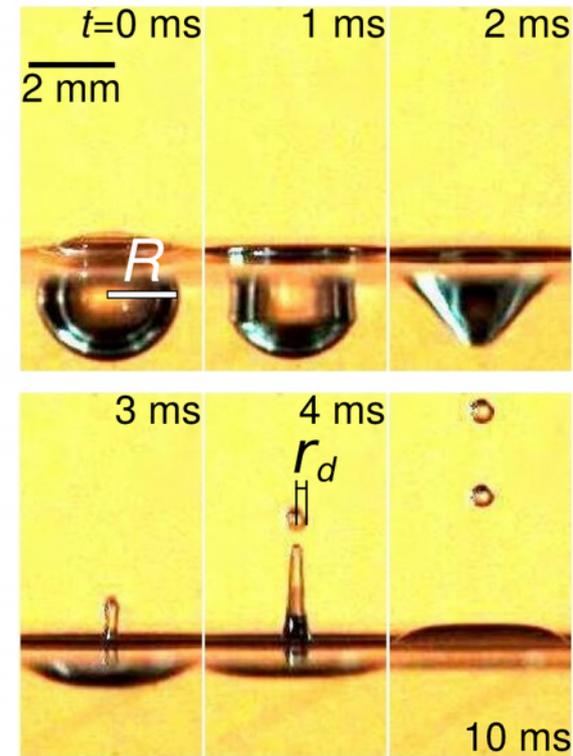
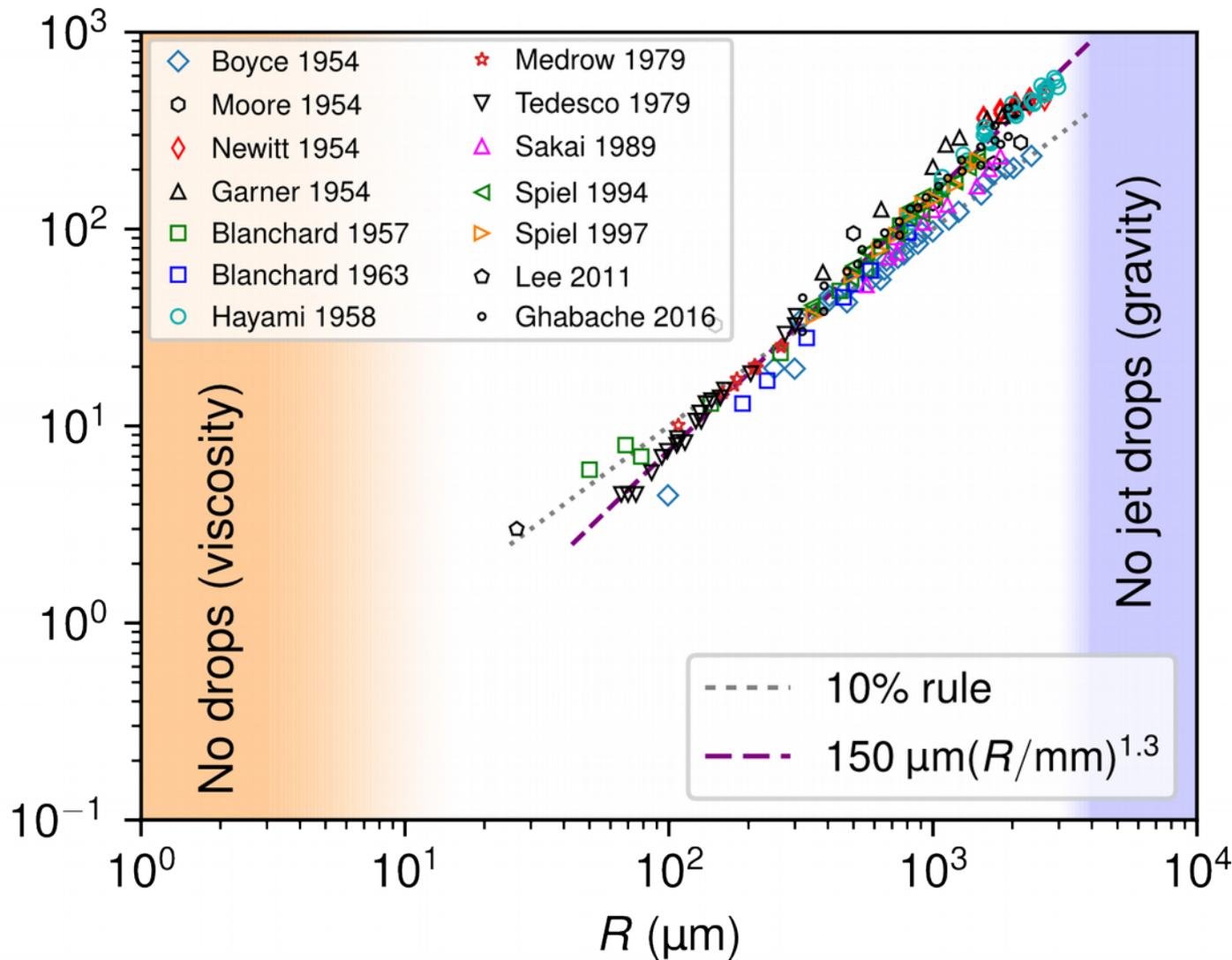
©Alain Cornu/Collection CIVC

# Motivation

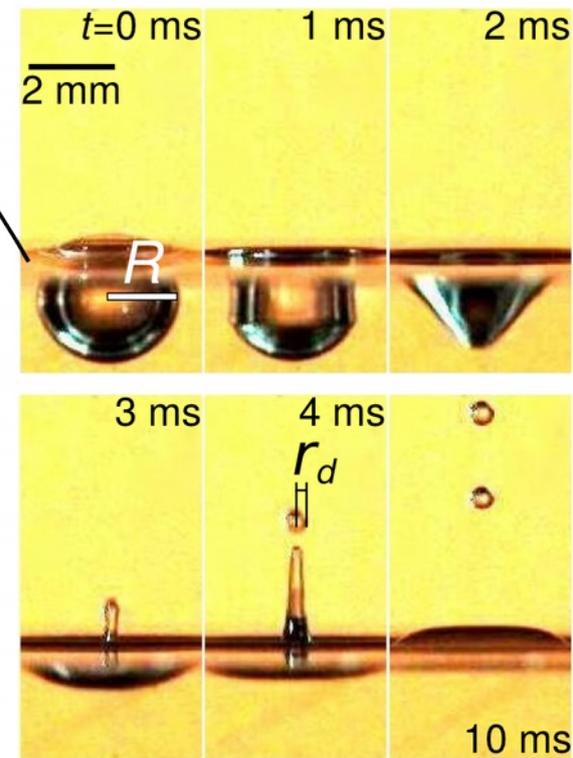
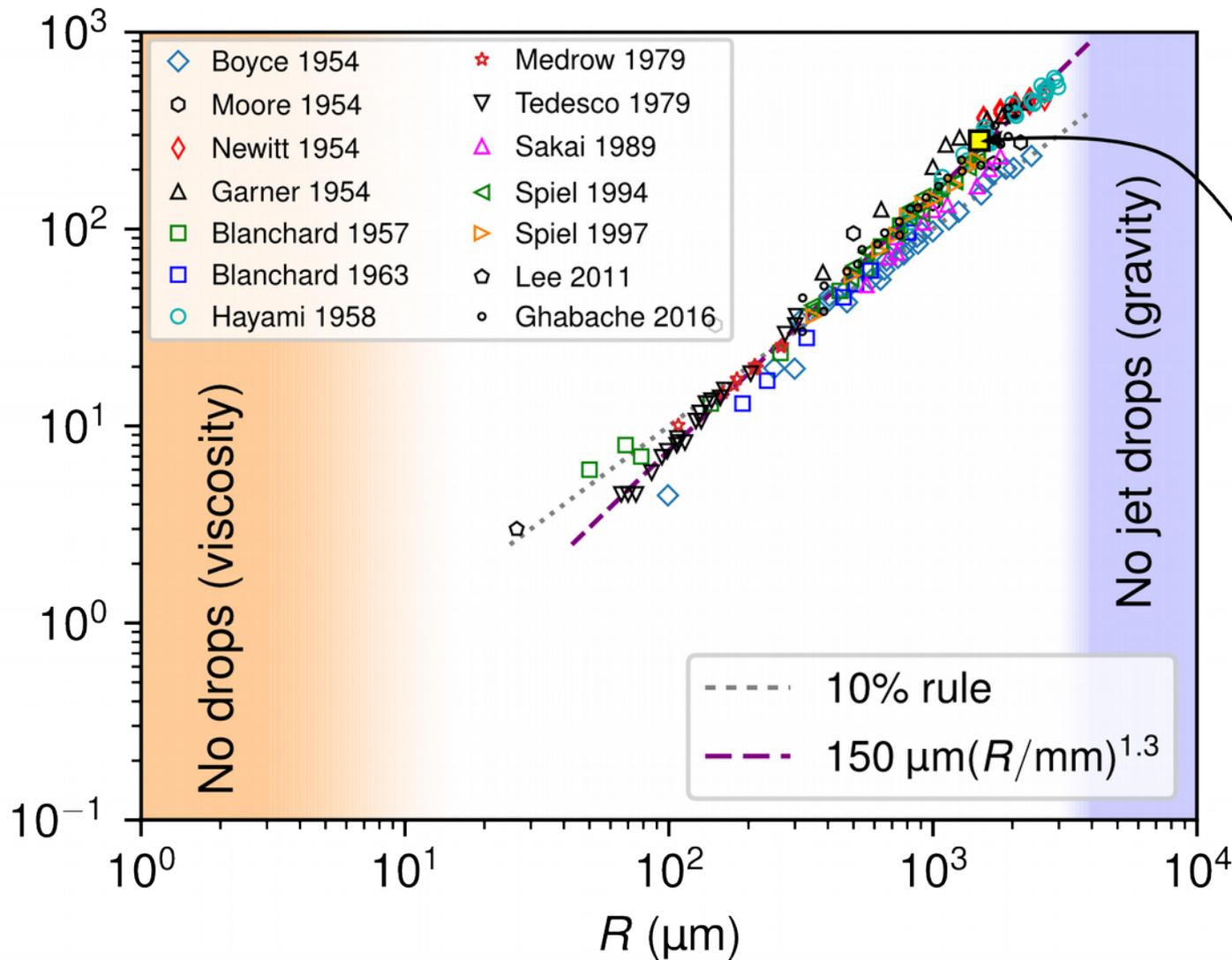
- Atmospheric science
  - Sea spray aerosol particles act as cloud condensation nuclei, scatter radiation
    - Still significant uncertainties in climate forcing by aerosols



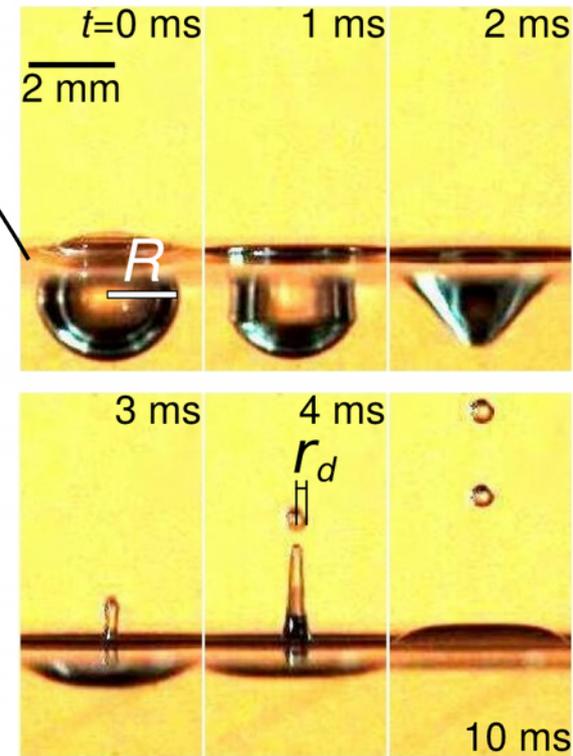
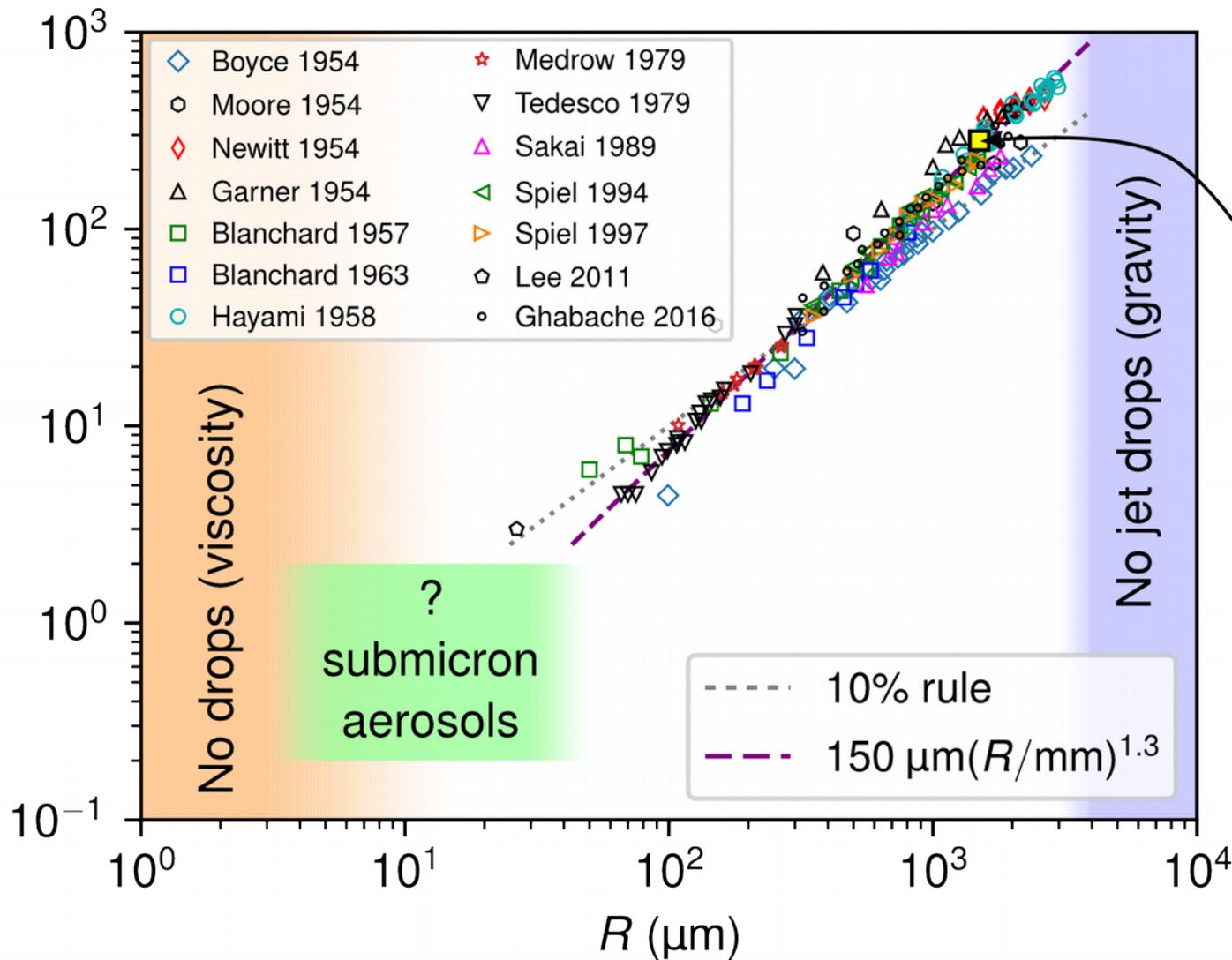
# Jet drop radius vs. bubble radius



# Jet drop radius vs. bubble radius



# Jet drop radius vs. bubble radius



# Dynamic similitude

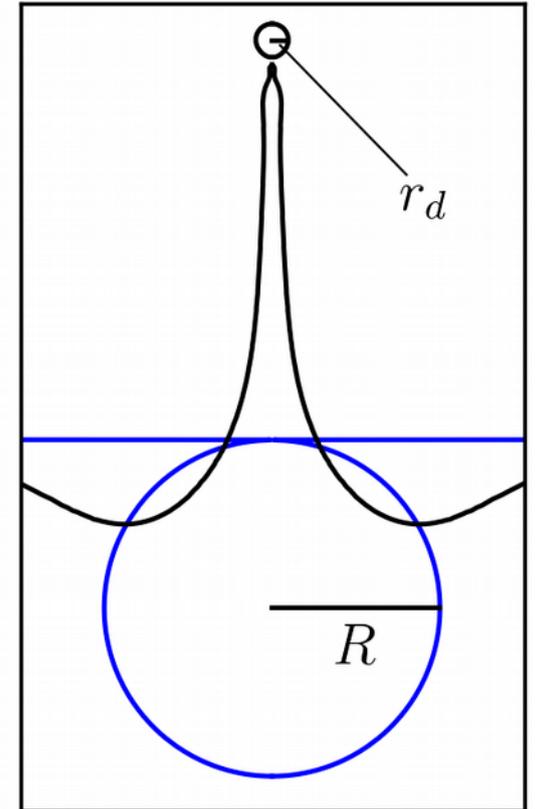
- Nondimensionalization:

- Length:  $R$

- Time:  $\tau \equiv \sqrt{\rho R^3 / \gamma}$

- Neglect gravity (valid for  $Bo \lesssim 0.01$ )  
( $Bo \equiv \rho g R^2 / \gamma \rightarrow R \lesssim 0.3 \text{ mm}$ )

- Only dimensionless parameter:  
Laplace number  $La \equiv \rho \gamma R / \mu^2$ 
    - Note:  $La = 1 / Oh^2$



- **Increasing  $\mu$  equivalent to decreasing  $R$**

Density, surface tension, viscosity:

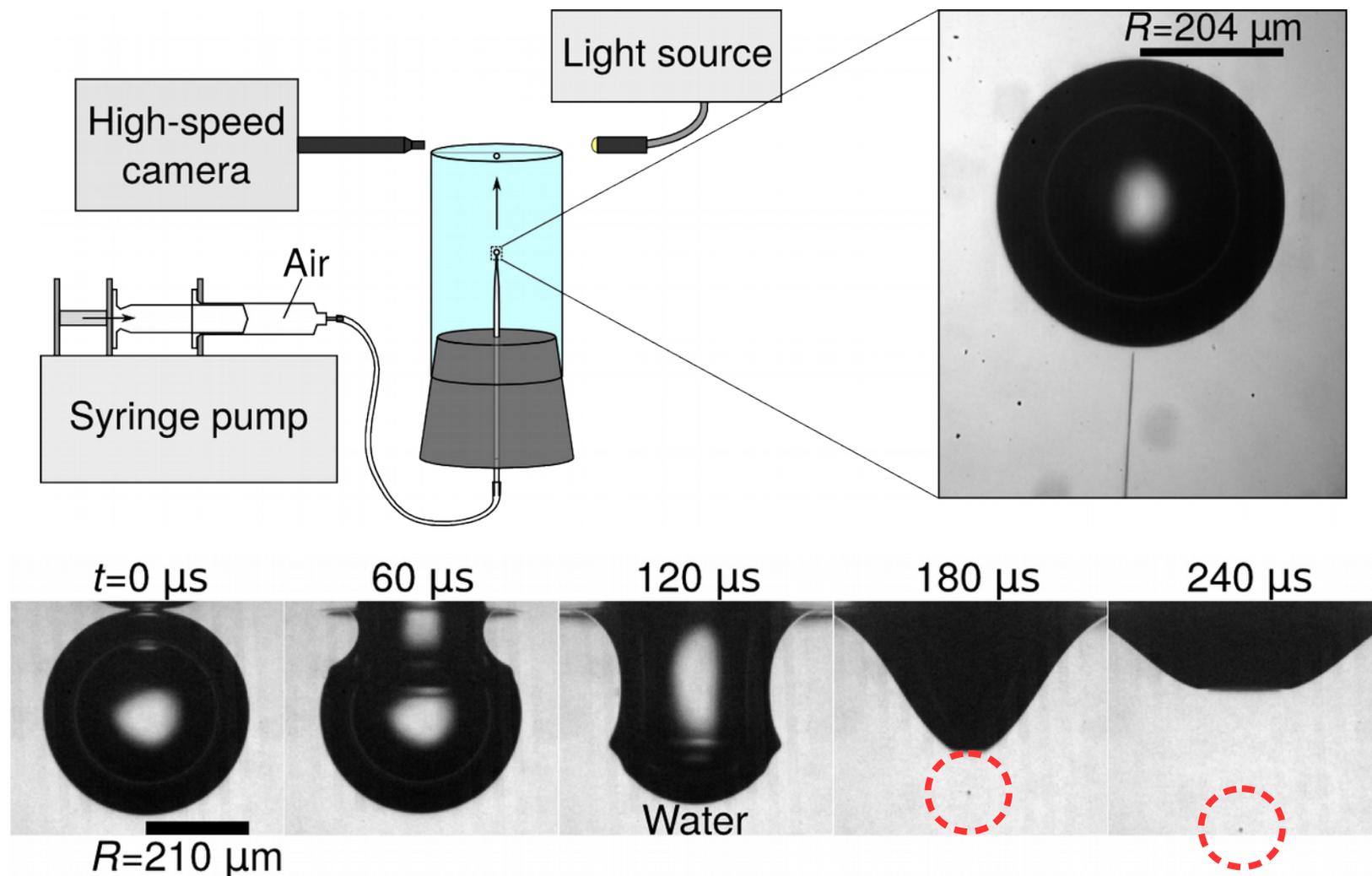
$\rho$

$\gamma$

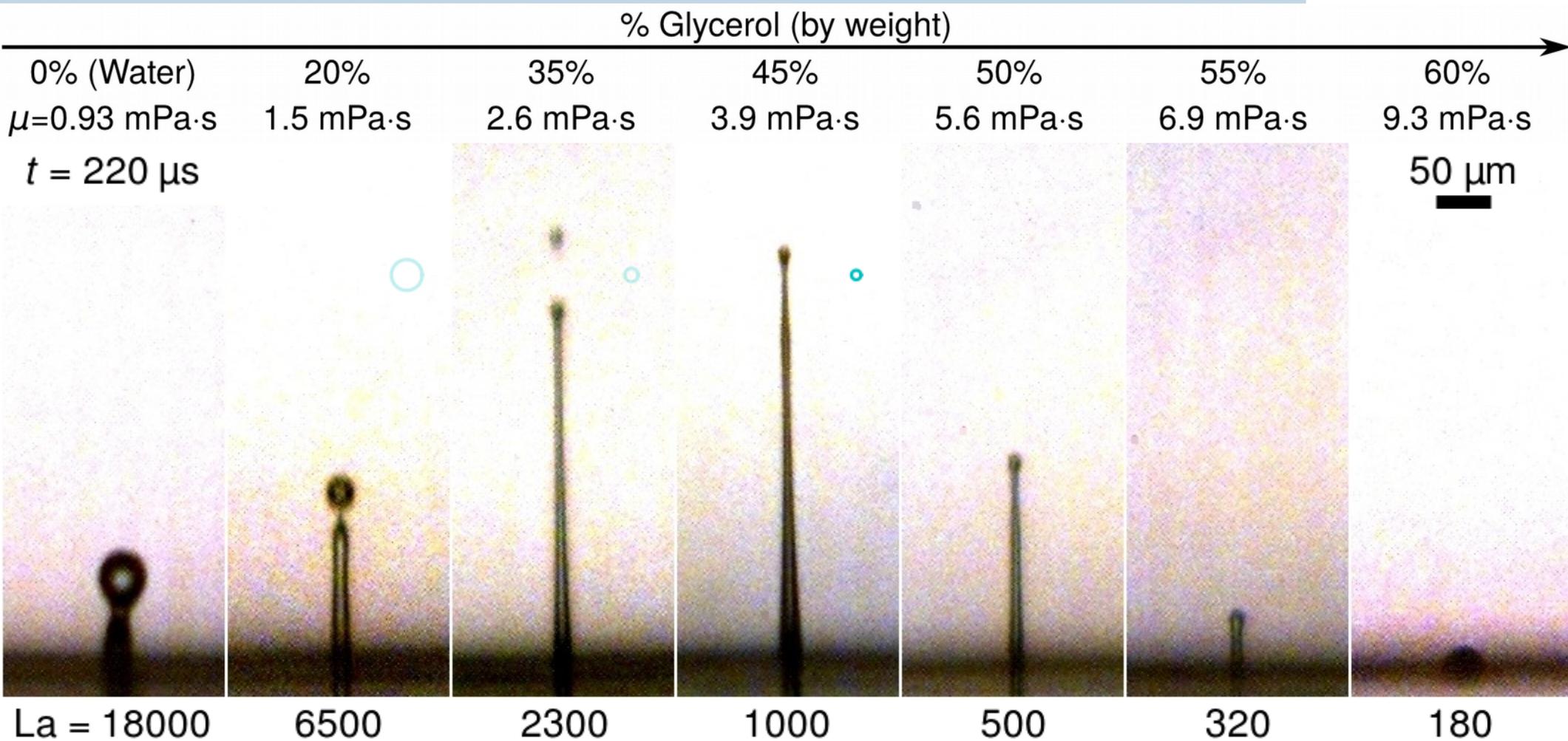
$\mu$

# Bubble bursting experiments

- Use glycerol-water solutions of varying concentrations to change viscosity, keeping  $R \approx 200 \mu\text{m}$

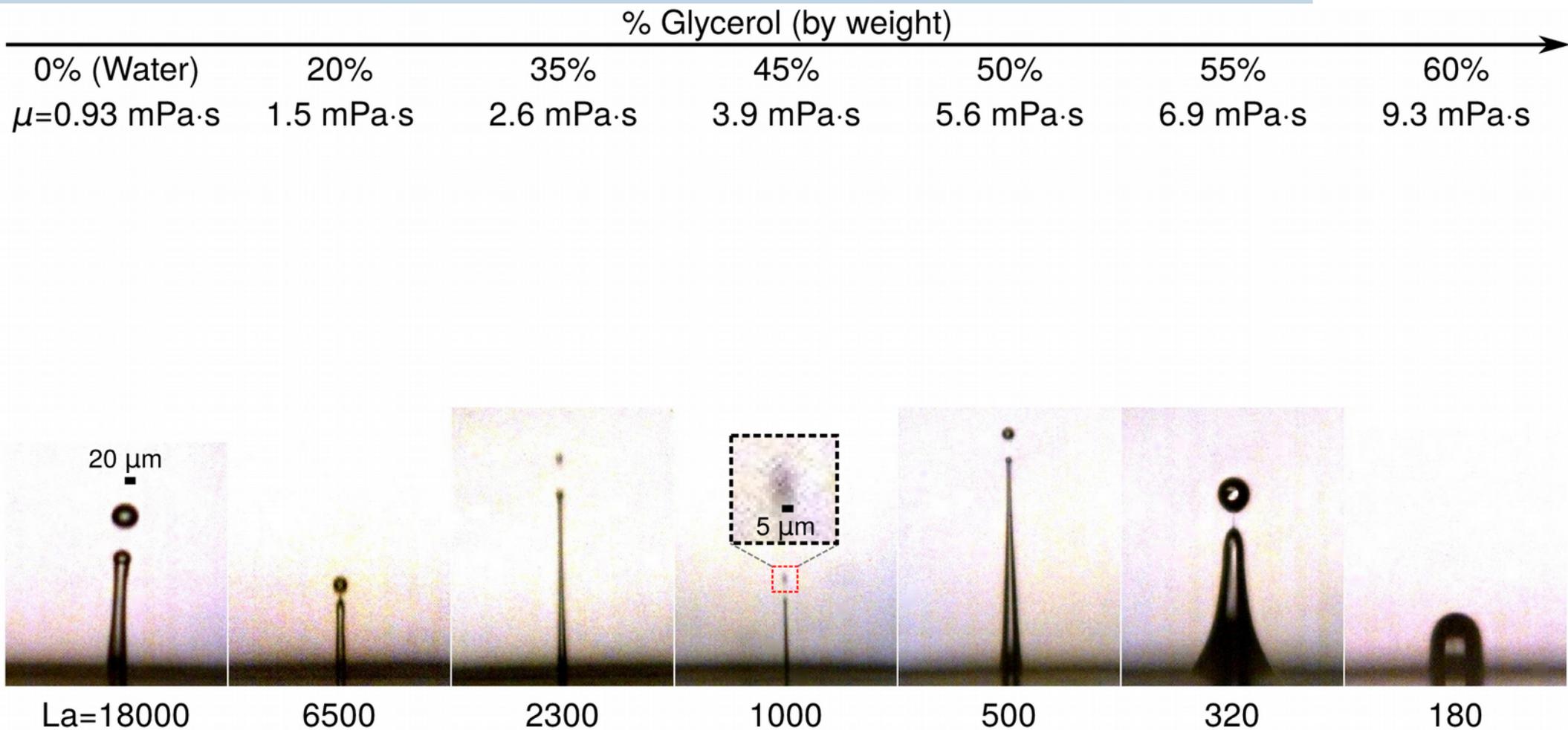


# Bubble bursting experiments



- Non-monotonic relationship between size of the top jet drop and Laplace number

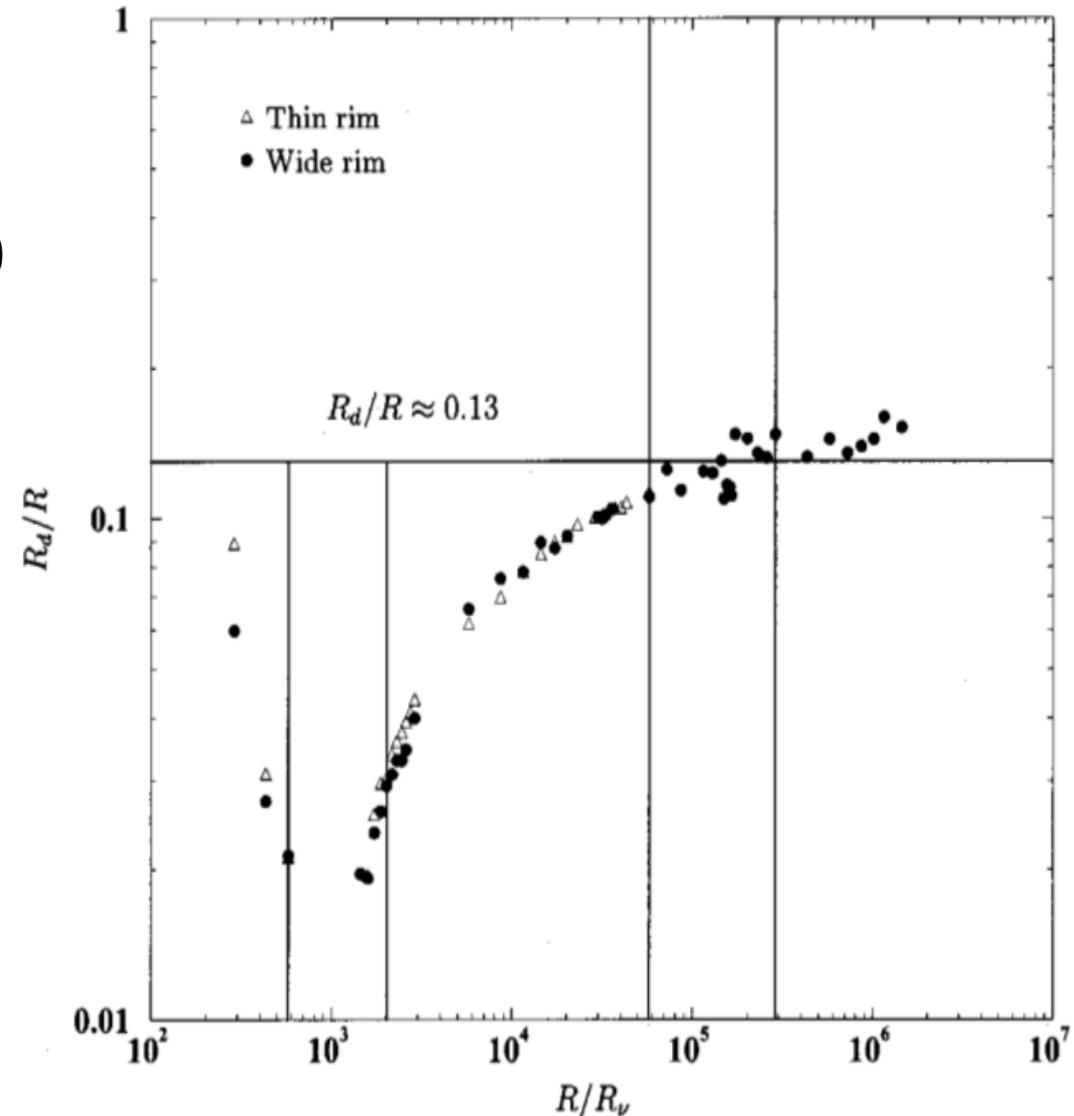
# Bubble bursting experiments



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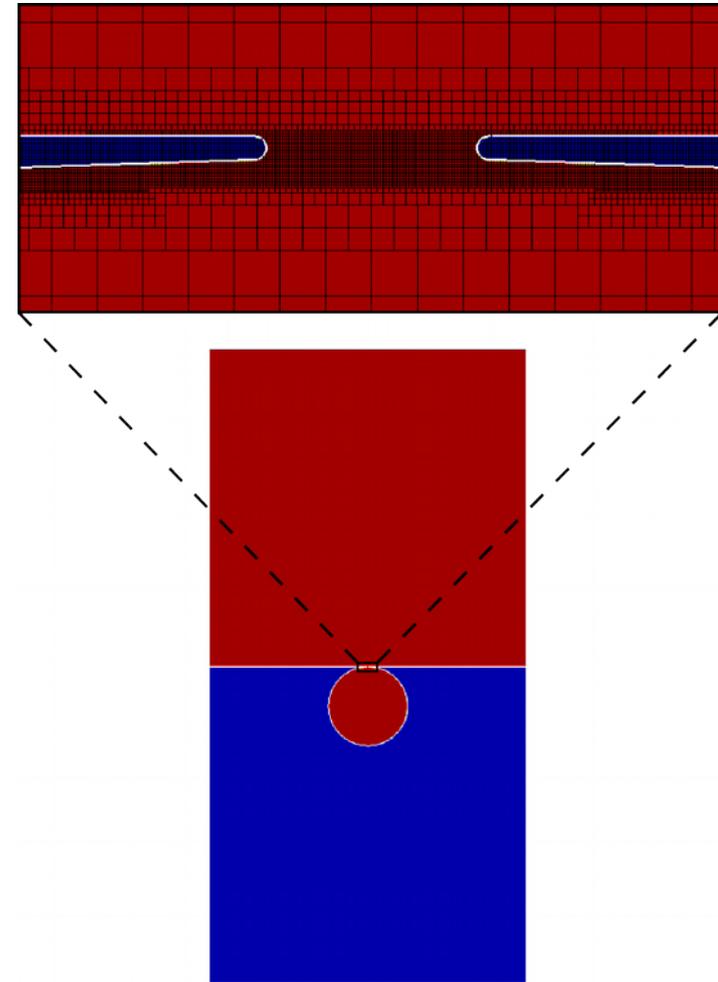
# Previous simulations

- Duchemin et al. 2002 also observed non-monotonic relationship in simulations
  - Limited resolution
  - Gap near minimum

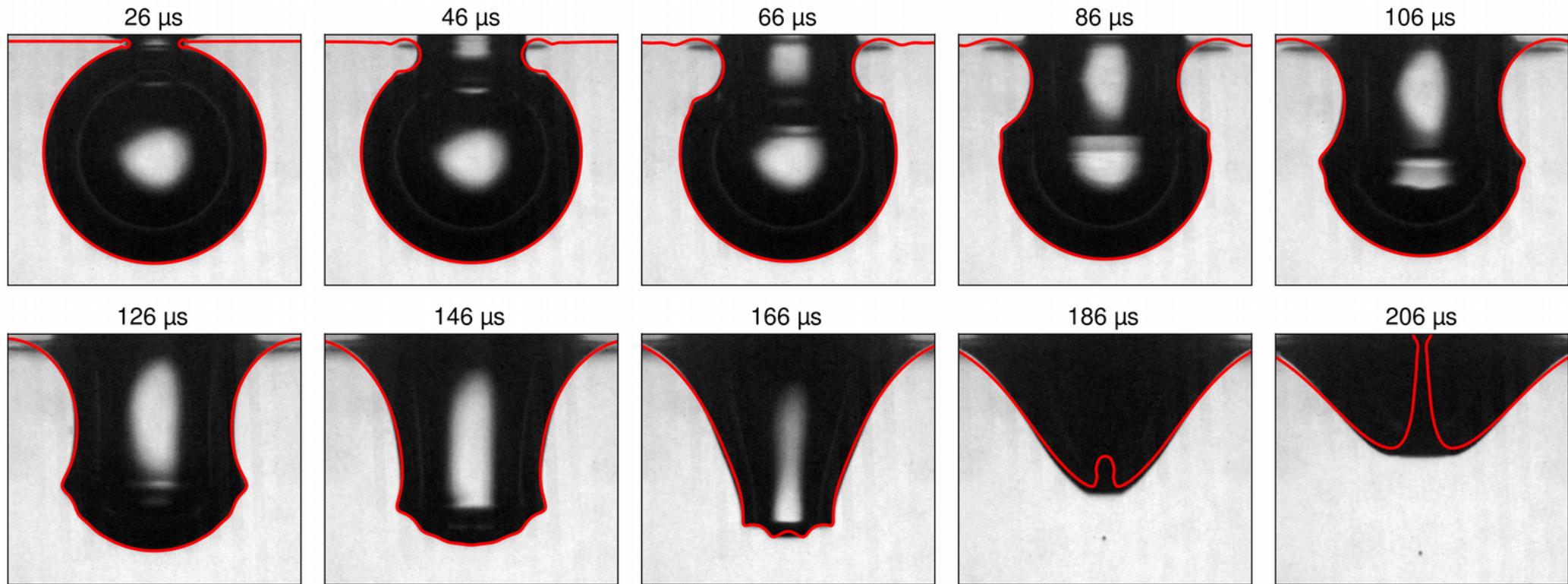


# Numerical simulations

- Axisymmetric simulations run in Gerris
  - Adaptive mesh refinement in regions of high curvature, vorticity: max level 14
  - Minimum cell size =  $2.4 \times 10^{-4} R$
- Initialized as spherical bubble with popped cap
  - Neglect gravity
- Vary  $La$ , fixing  $\rho_g/\rho = 1.2 \times 10^{-3}$   
 $\mu_g/\mu = 0.018$

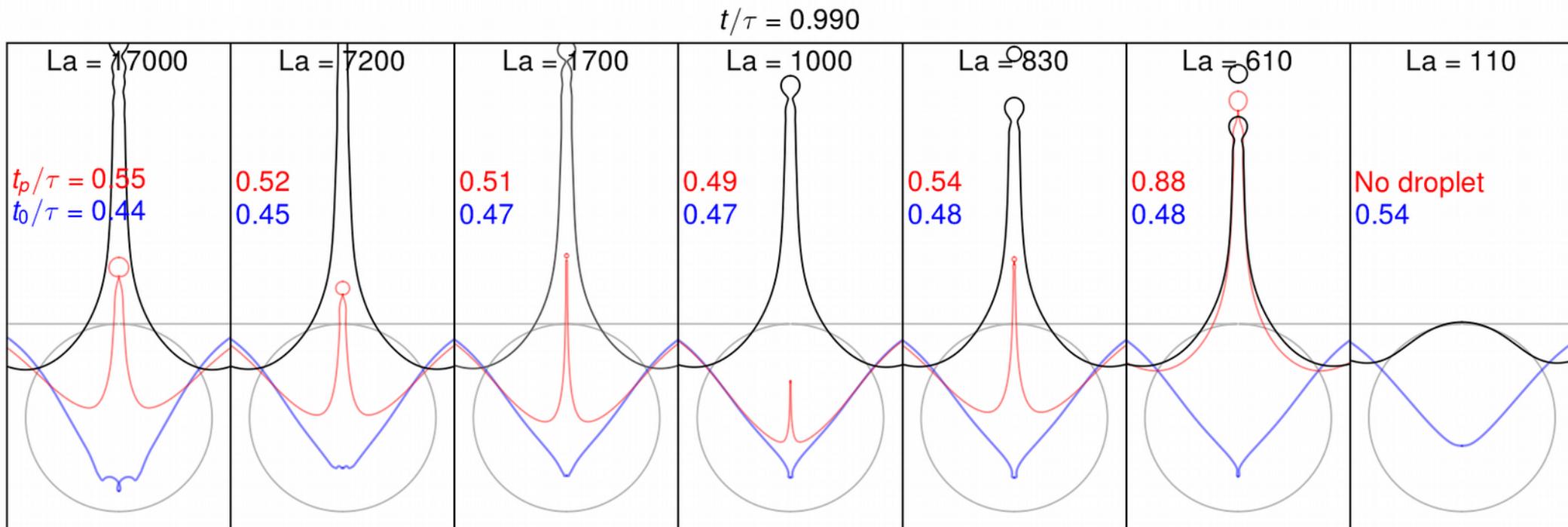


# Validation



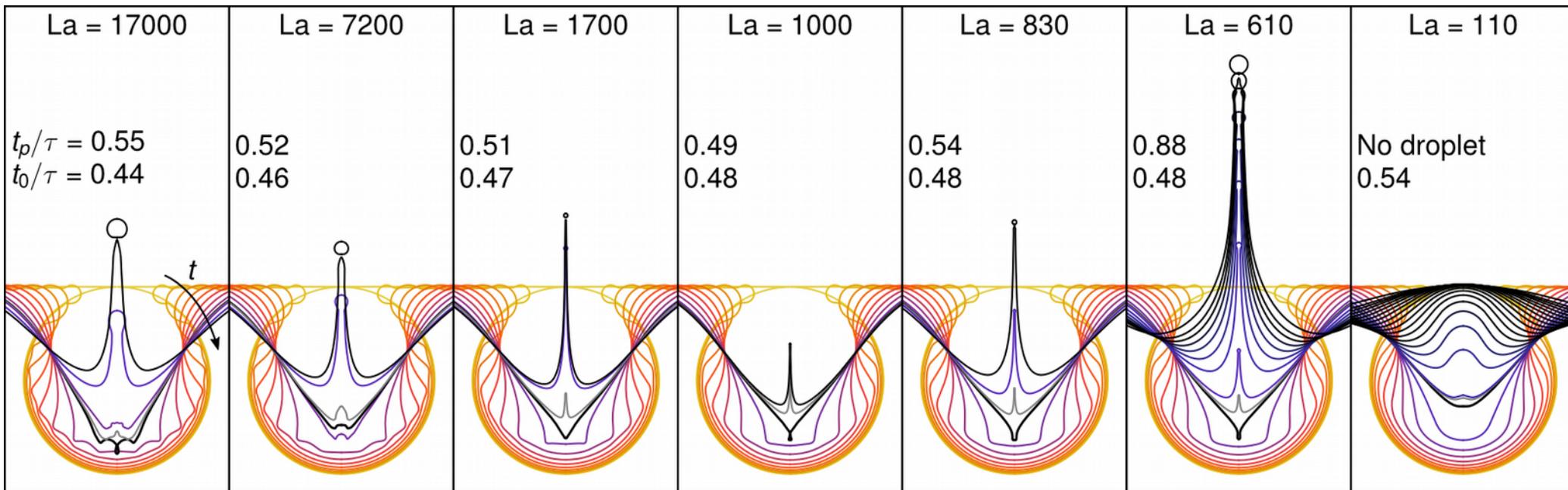
- Air bubble in water with  $R = 210 \mu\text{m}$   
( $\text{La} = 18000$ )

# Simulation results



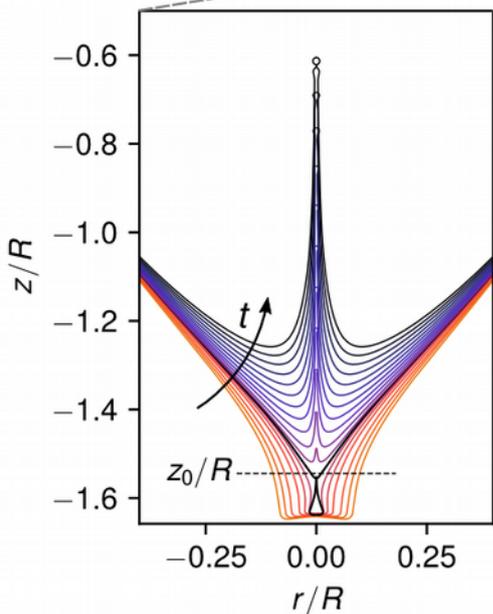
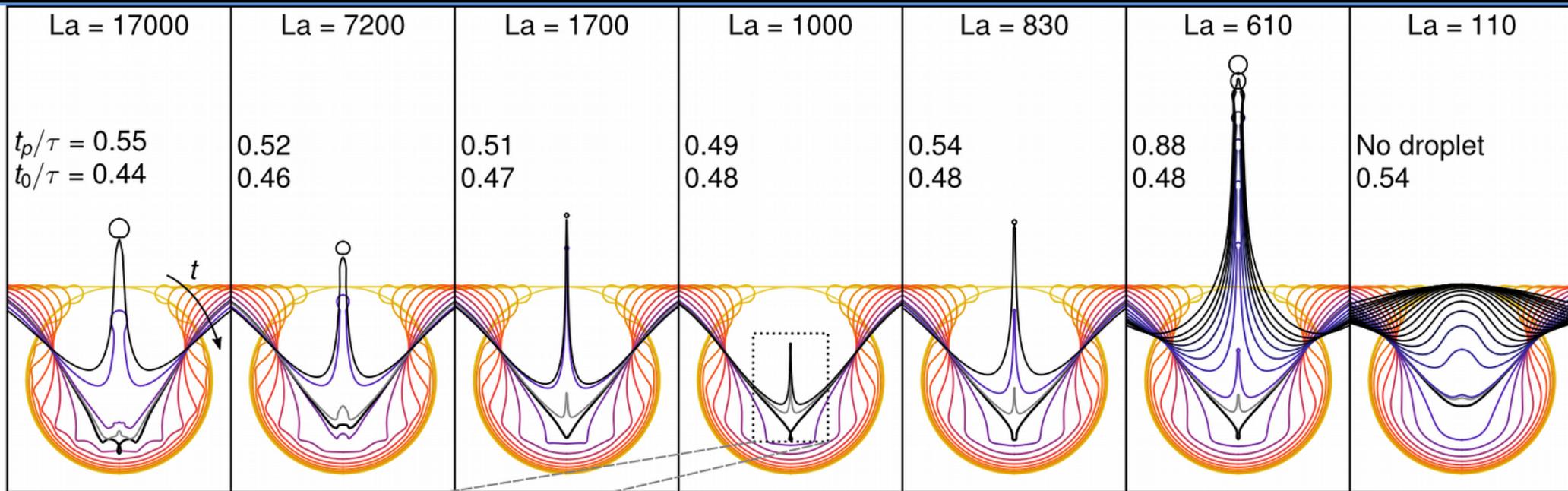
- Define **inversion time**  $t_0$  as time when velocity of interface at center is maximum
  - Time of **pinch-off**  $t_p$  also labelled
- Same non-monotonic relationship

# Simulation results

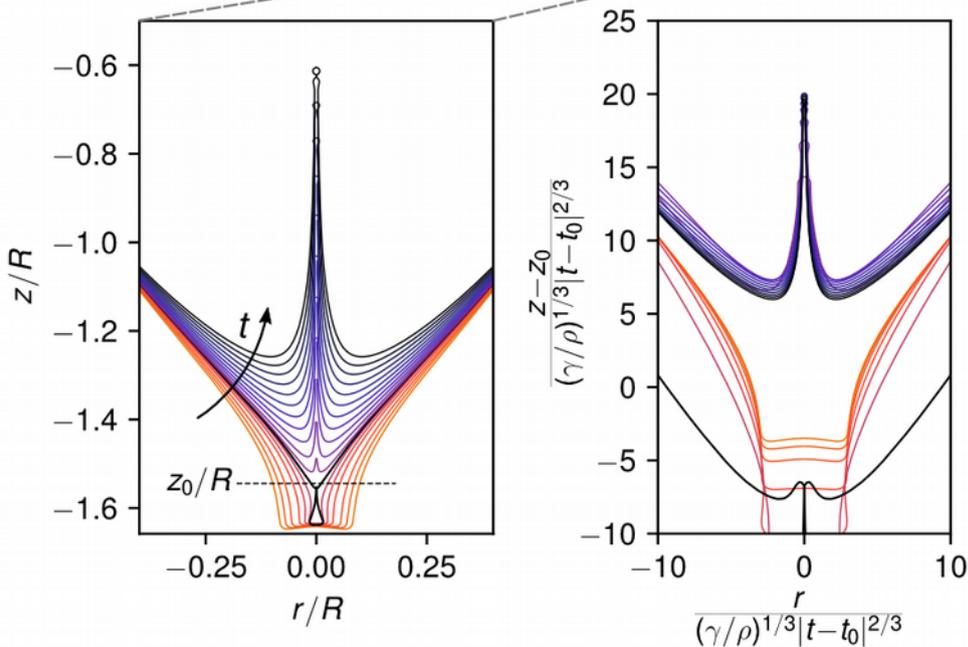
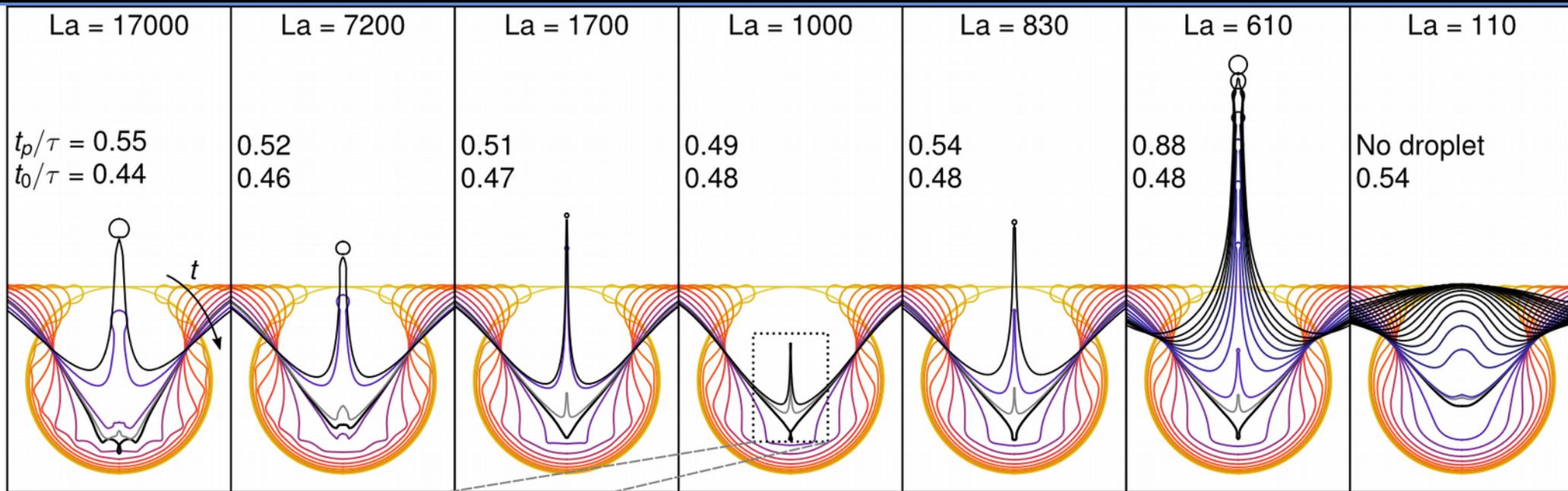


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# Self-similar jet growth

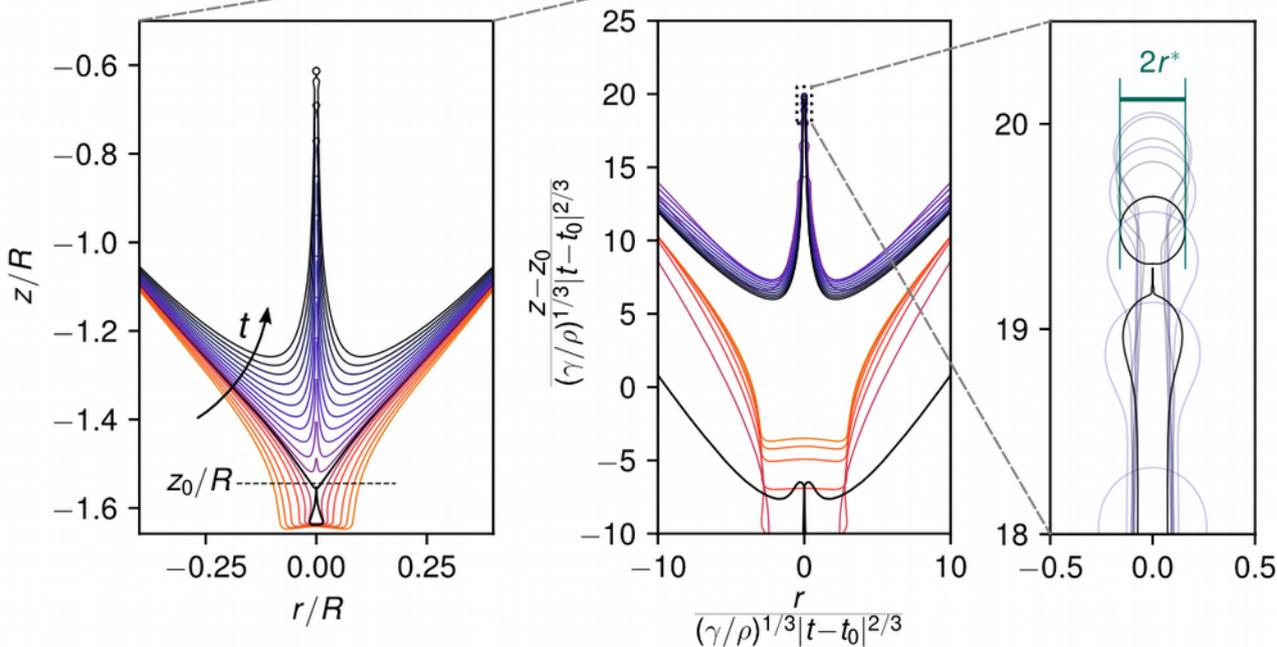
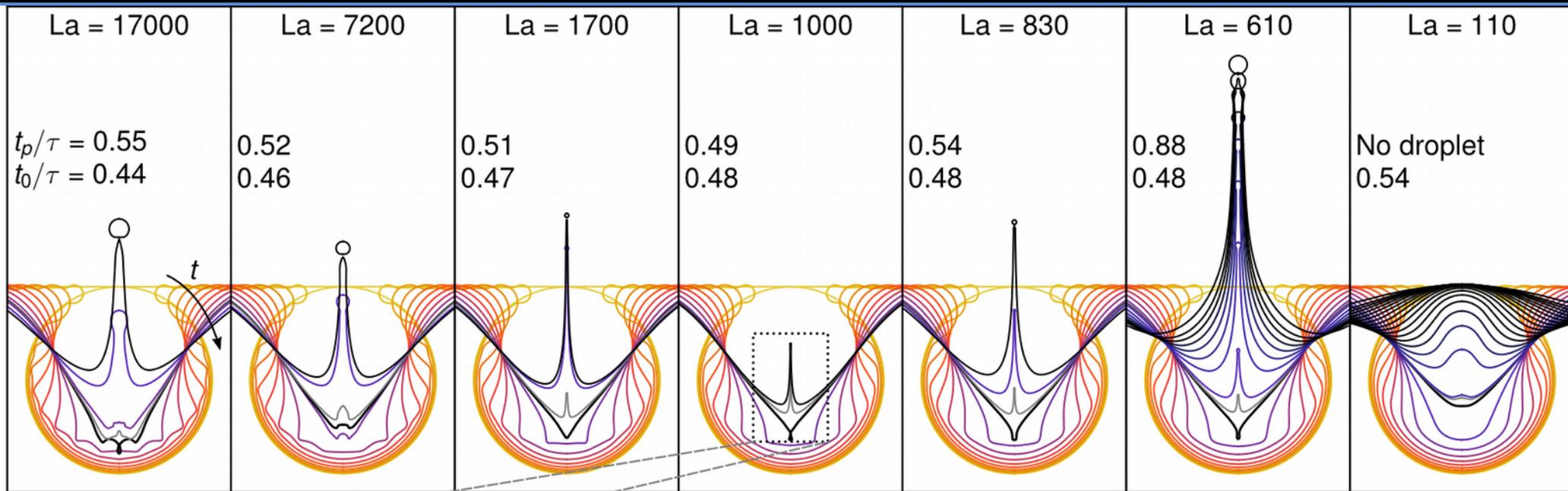


# Self-similar jet growth



- Self-similar scaling:
  - Lengths near inversion scale as  $(\gamma/\rho)^{1/3} |t - t_0|^{2/3}$

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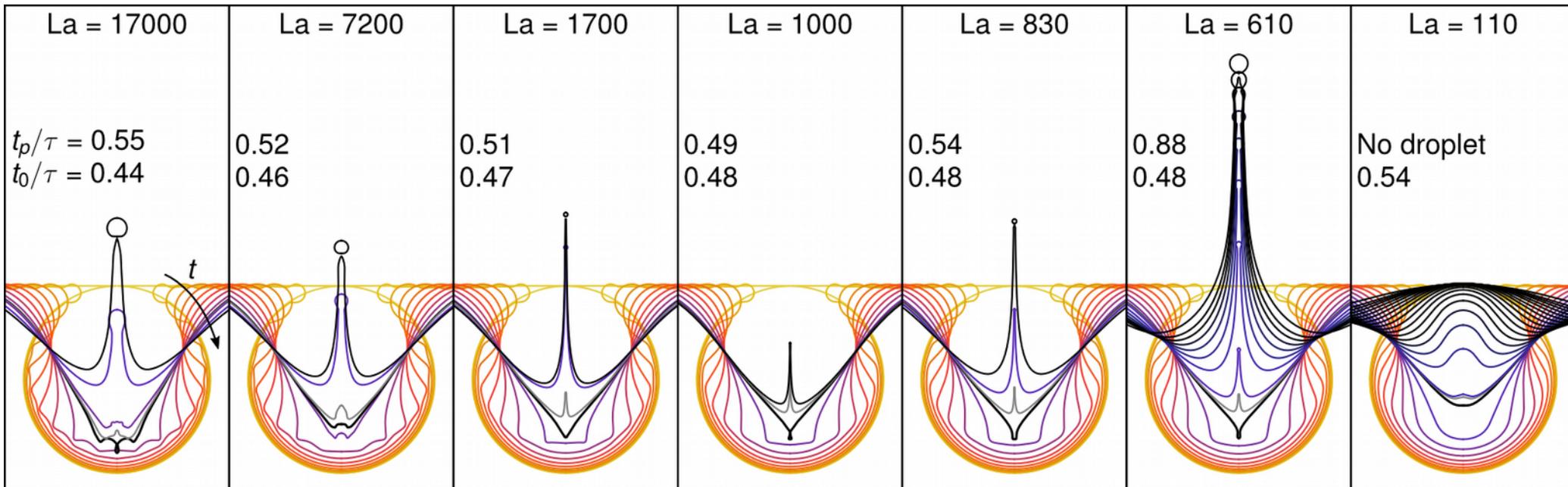
# Self-similar jet growth

- Decompose drop size into

- **Shape factor**  $r^* \equiv r_d(\rho/\gamma)^{1/3}(t_p - t_0)^{-2/3}$

- **Jet growth time**  $t^* \equiv (t_p - t_0)/\tau$

- Then  $r_d/R = r^*(\text{La})(t^*(\text{La}))^{2/3}$



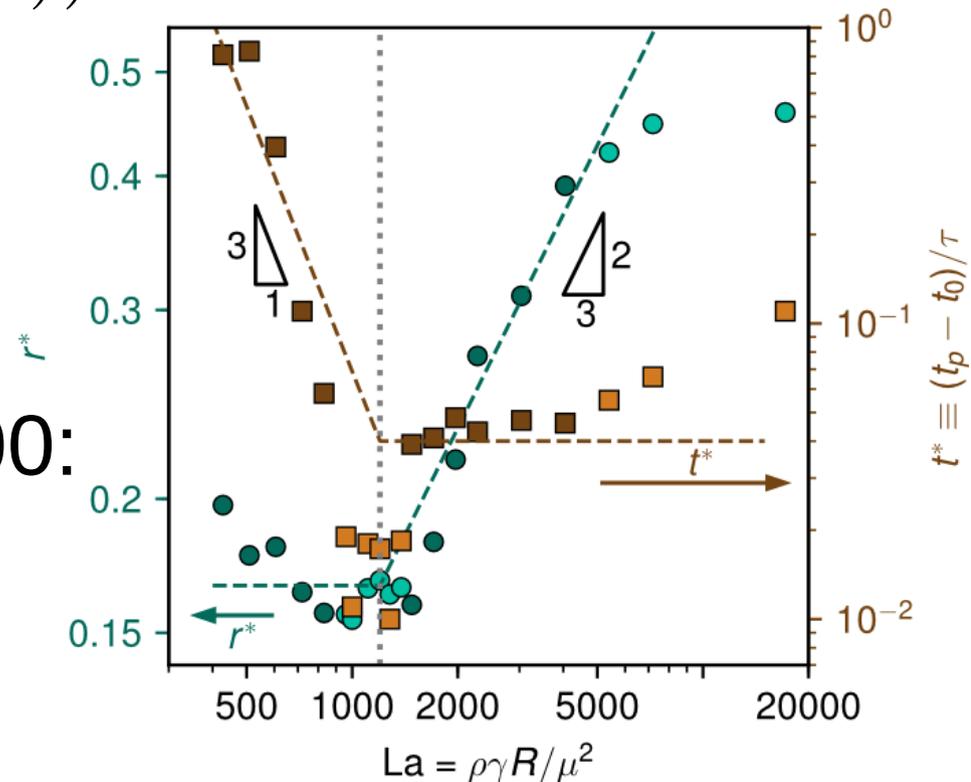
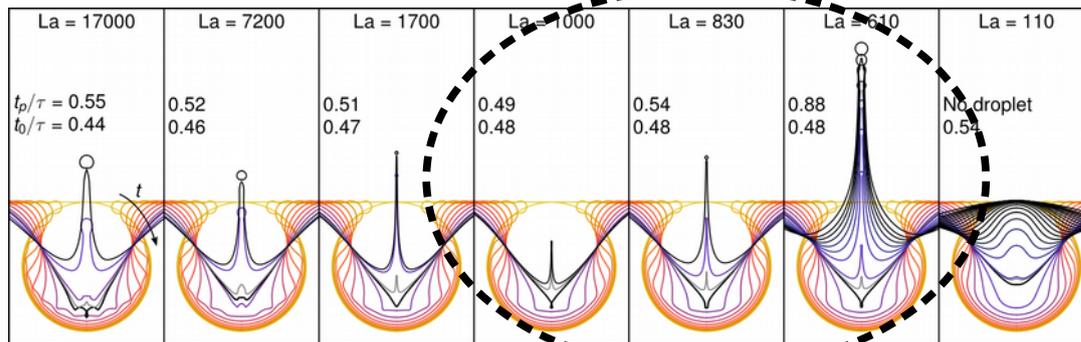
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- As  $La$  decreases below 1200:

- Viscosity delays pinch-off
- Drop size increases with  $t^*$

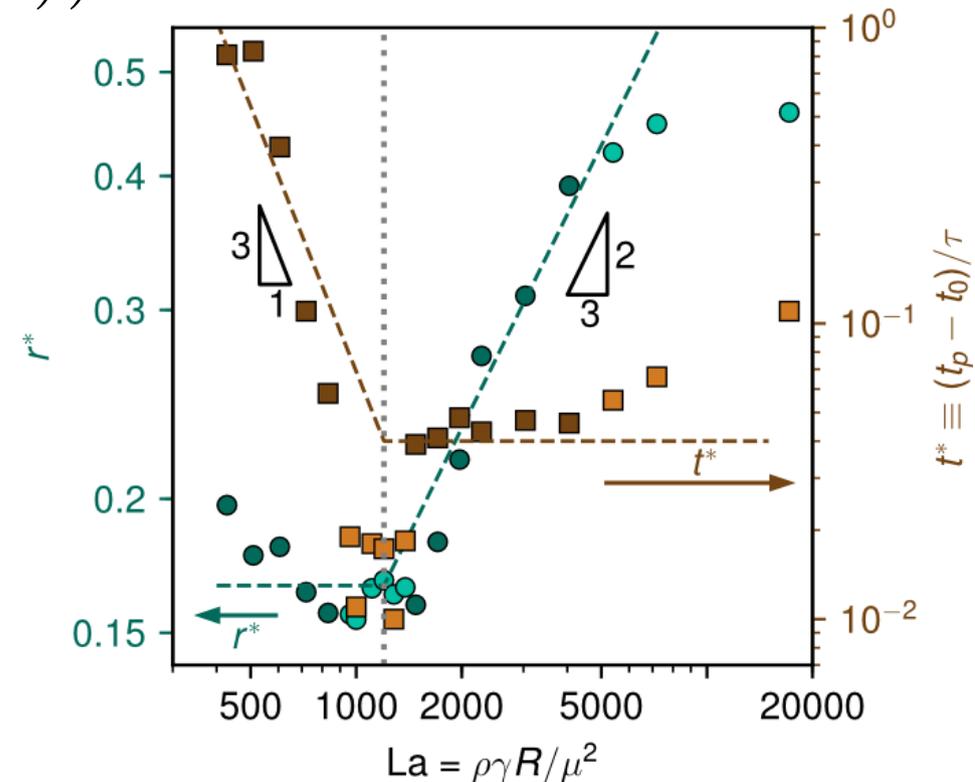
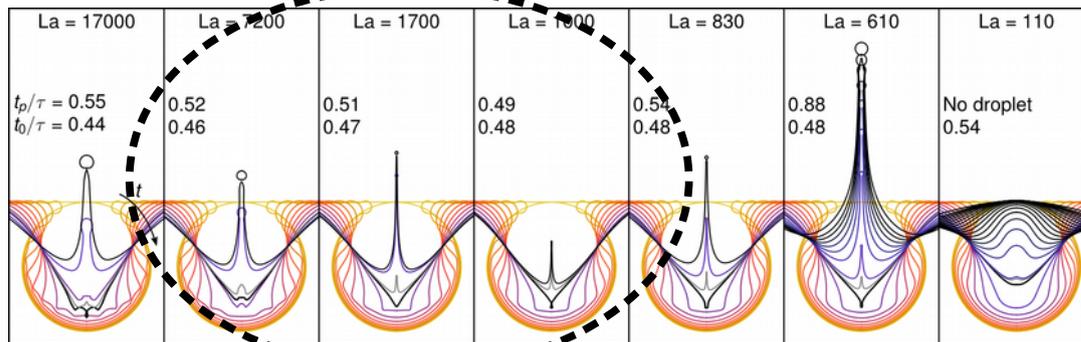
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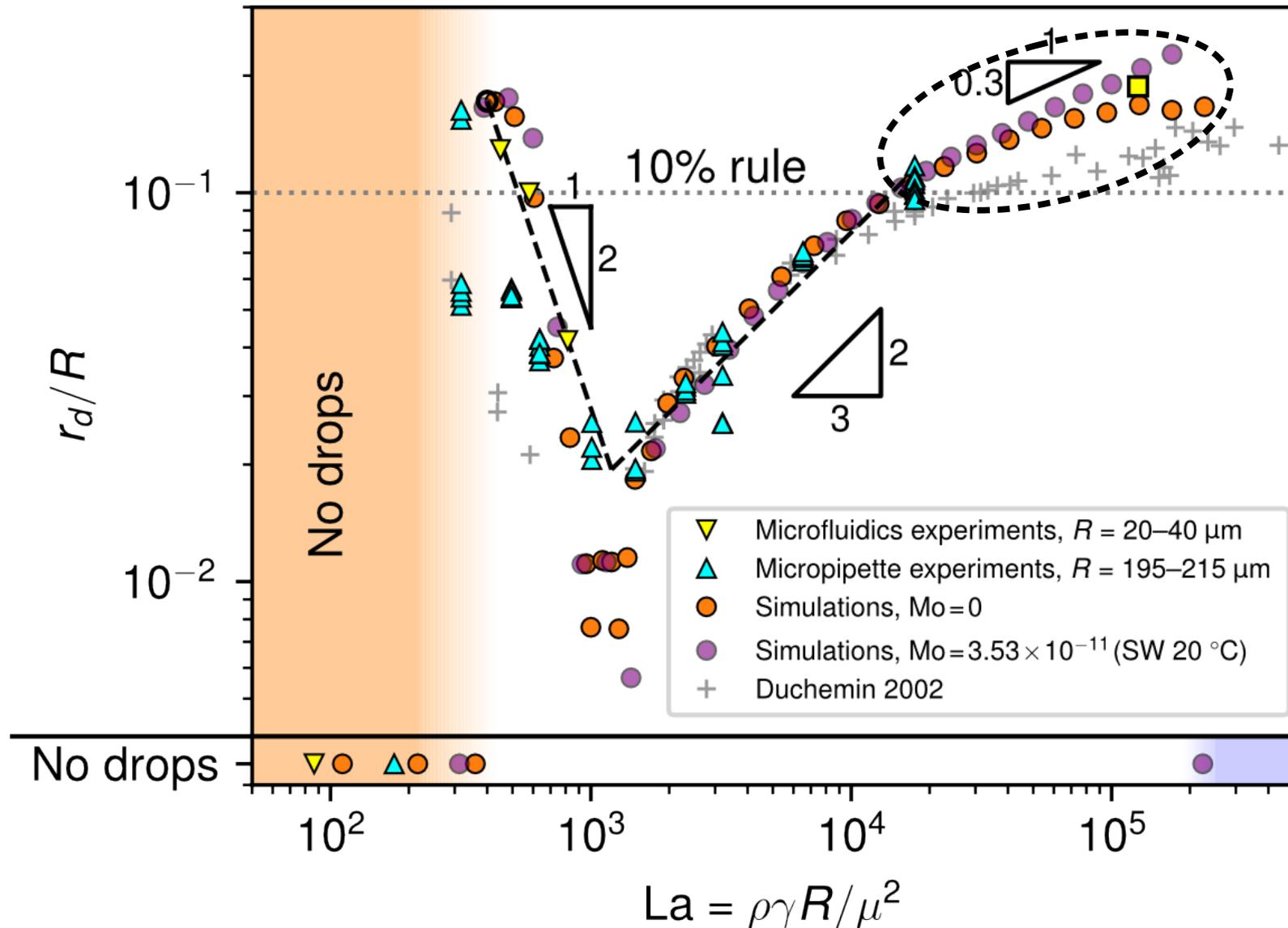
- Then  $r_d/R = r^*(La)(t^*(La))^{2/3}$



- As La increases above 1200:

- Less focusing of cavity with undamped capillary waves (Ghabache et al., 2014)
  - Drop size increases with  $r^*$

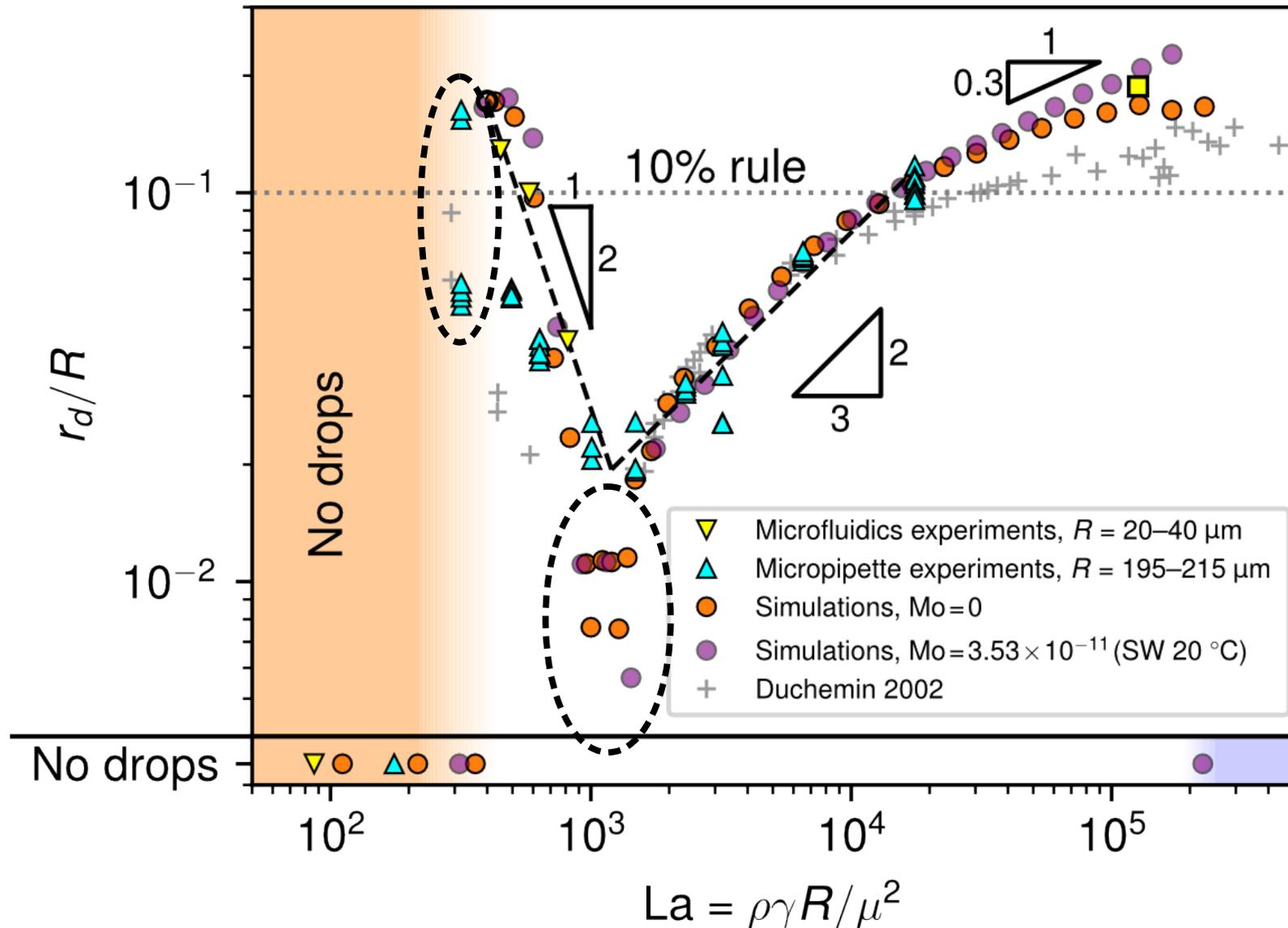
# Jet drop radius vs. bubble radius



- Dashed lines: model from fits to  $r^*$  and  $t^*$   

$$r_d/R = r^*(La)(t^*(La))^{2/3}$$

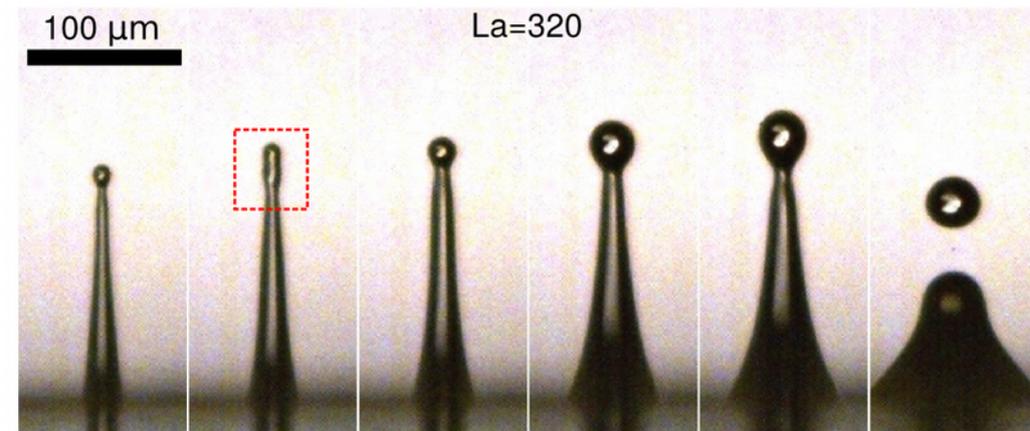
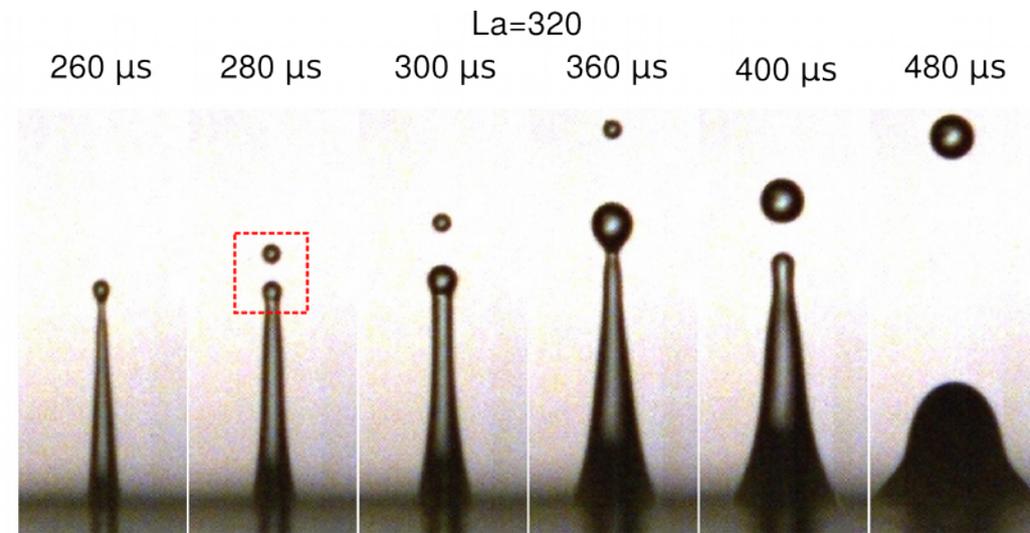
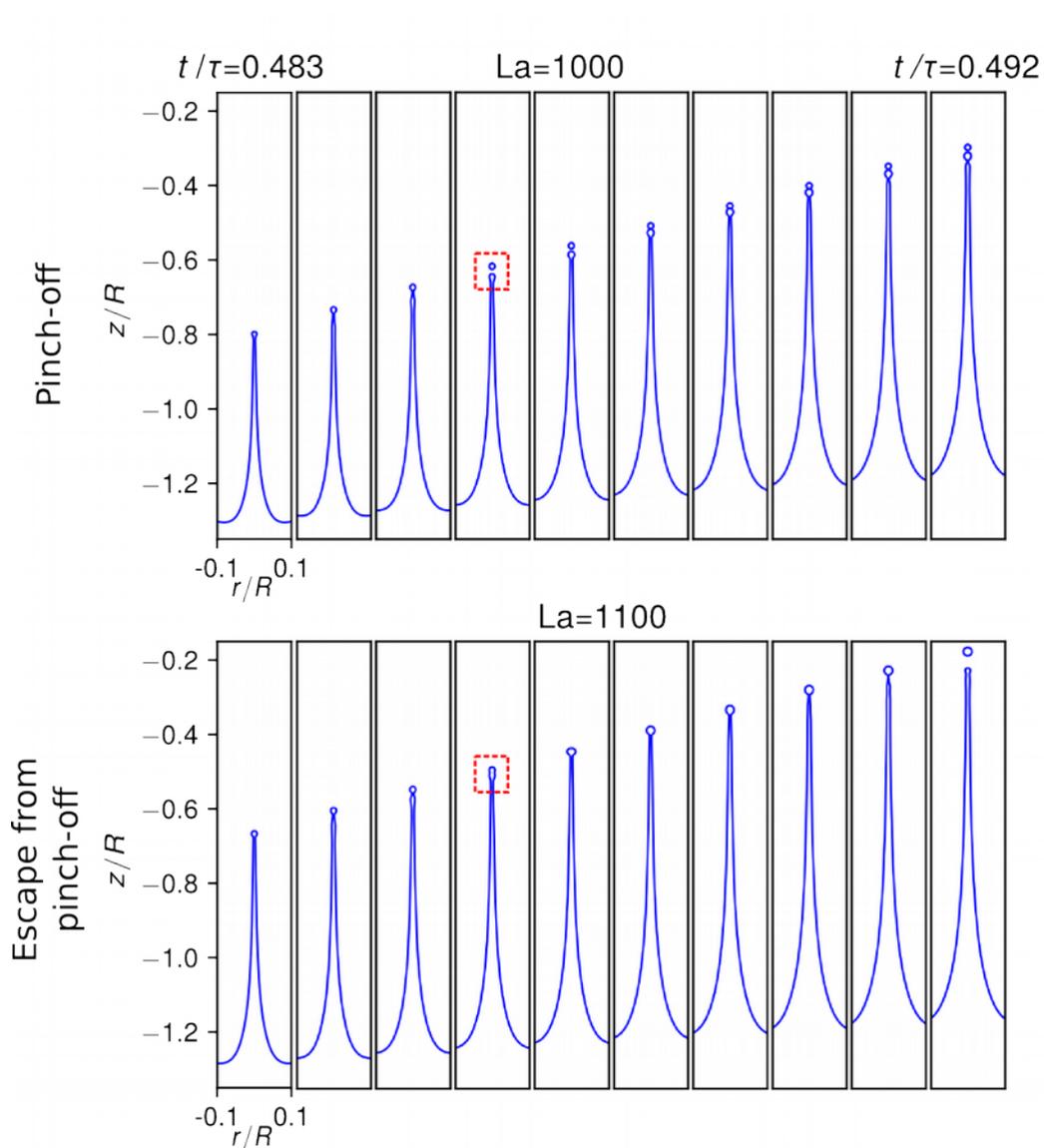
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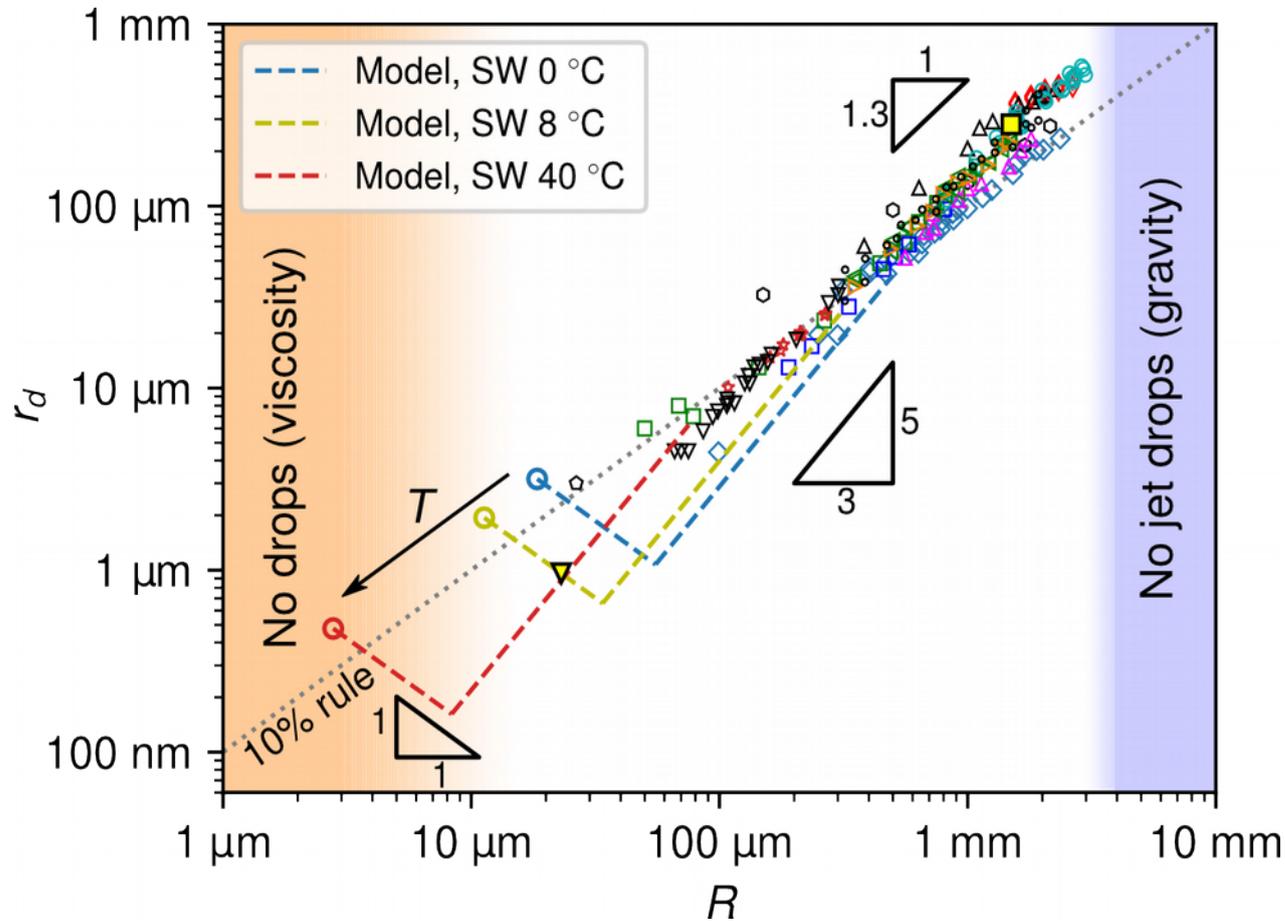
$$r_d/R = r^*(La)(t^*(La))^{2/3}$$

# Size variations due to escape from pinch-off



Hoepffner & Paré (2013), Recoil of a liquid filament: escape from pinch-off through creation of a vortex ring

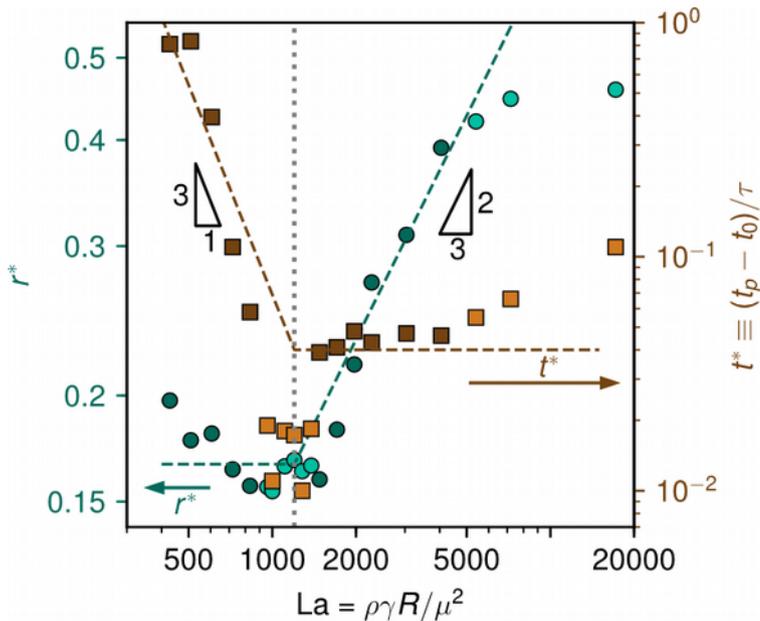
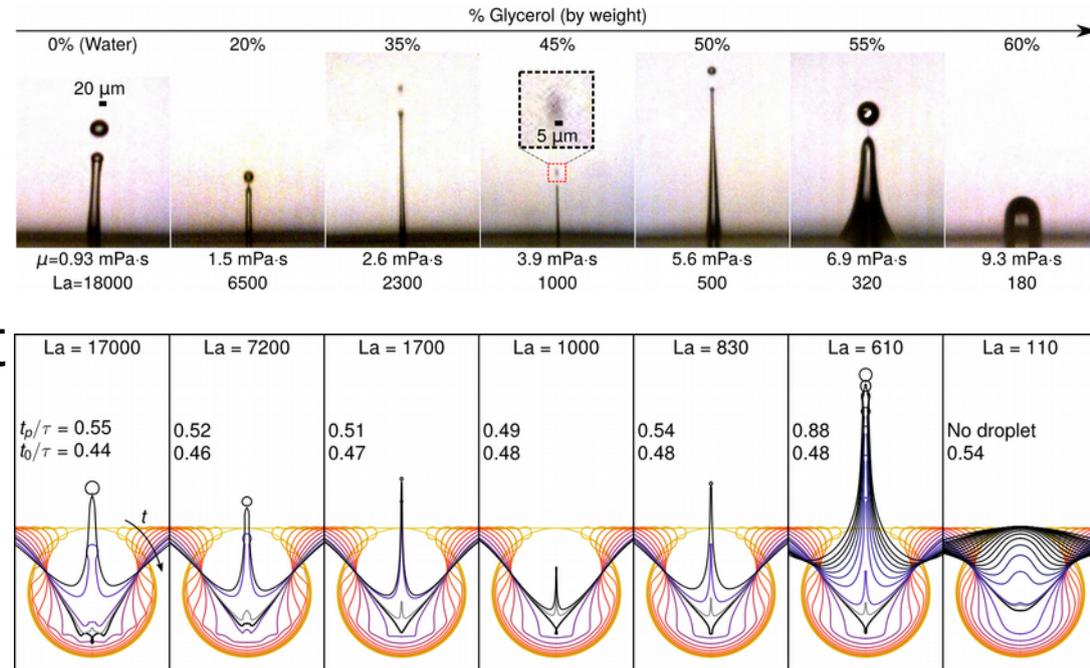
# Dimensional plot: Jet drop radius vs. bubble radius



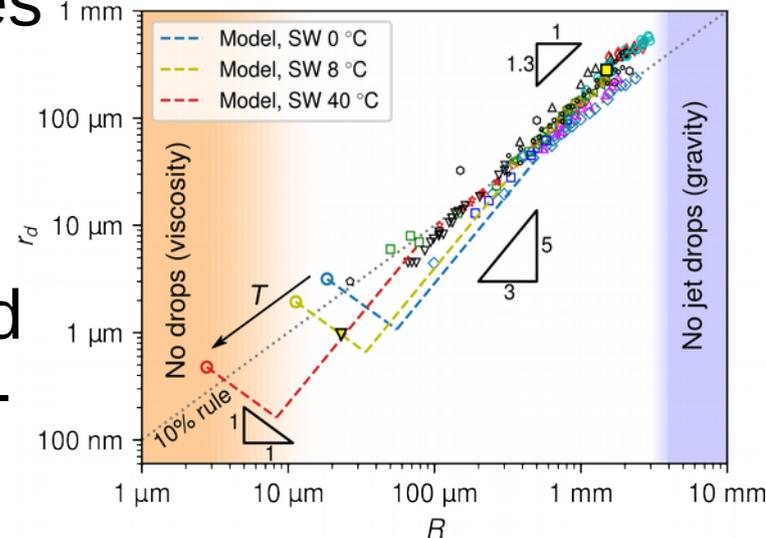
- Seawater viscosity varies by almost a factor of 3 from  $0^\circ\text{C}$  to  $40^\circ\text{C}$   $\rightarrow$  strong  $La$  dependence on temperature
  - Drop size increases with temperature for  $R \gtrsim 50 \mu\text{m}$
  - Jet drops as small as  $200 \text{nm}$  predicted in tropical waters

# Conclusions

- Non-monotonic size relationship between bubble and top jet drop observed
- Decomposing self-similar jet growth into shape and time components can capture non-monotonic behavior



- Jet drop sizes predicted significantly smaller than 10% rule and temperature-dependent

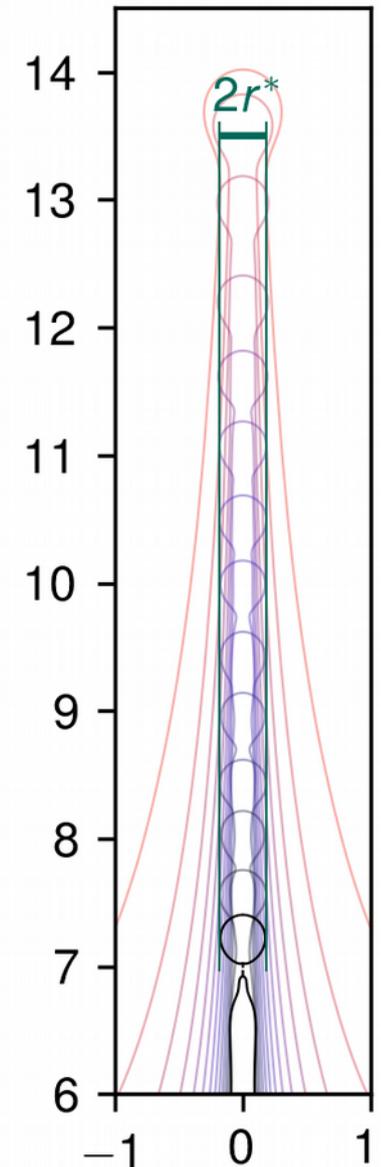
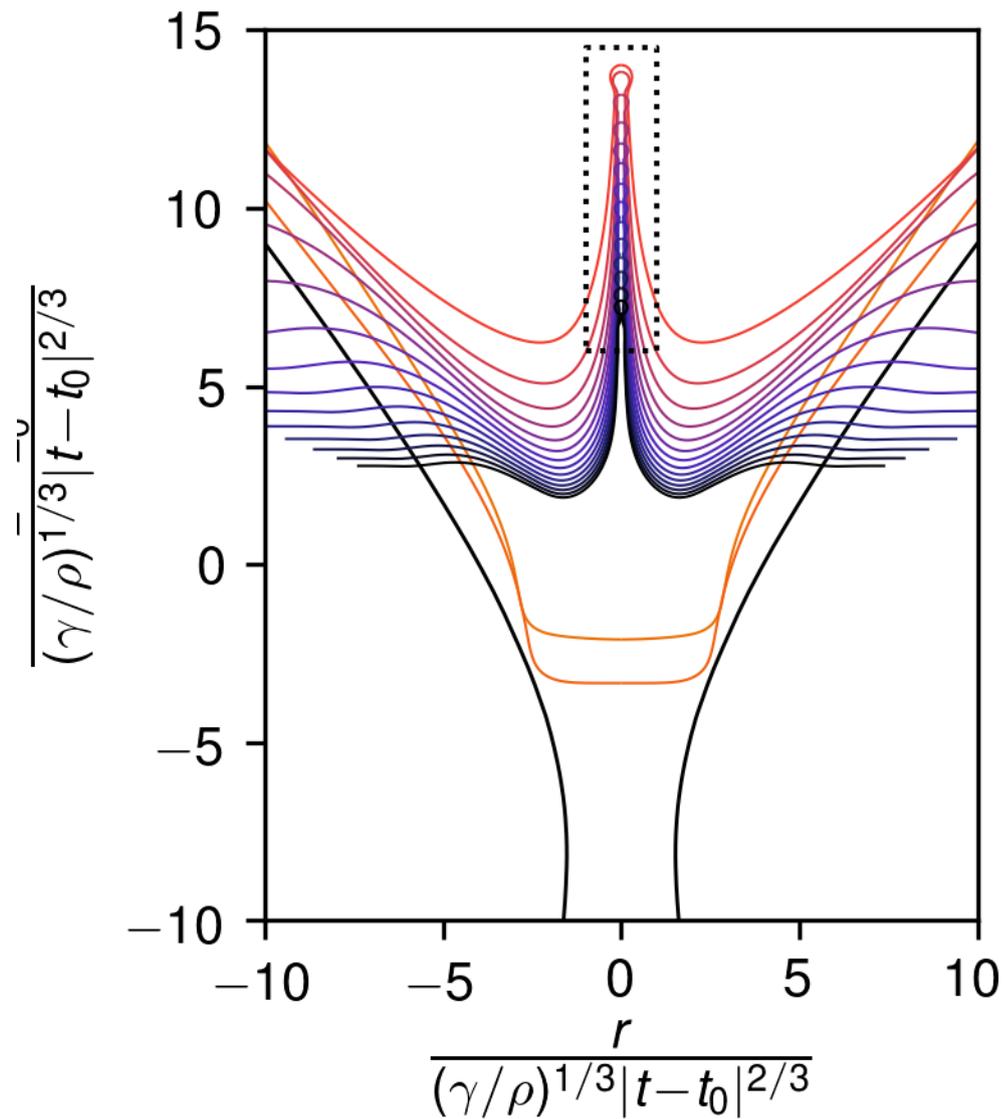
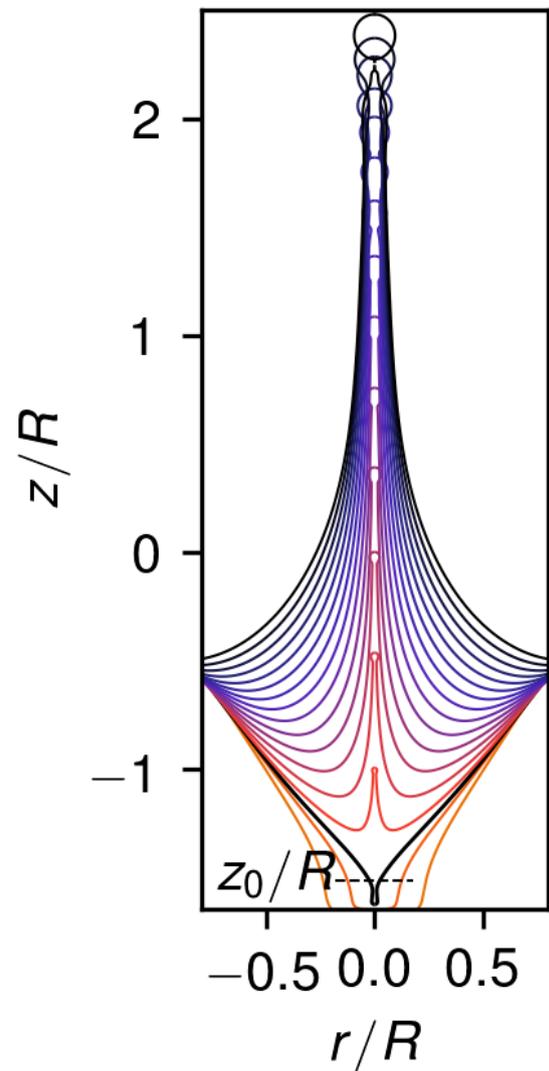


# Acknowledgments

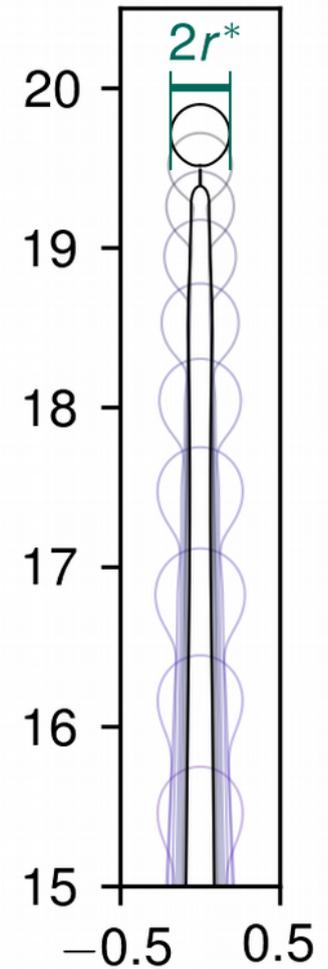
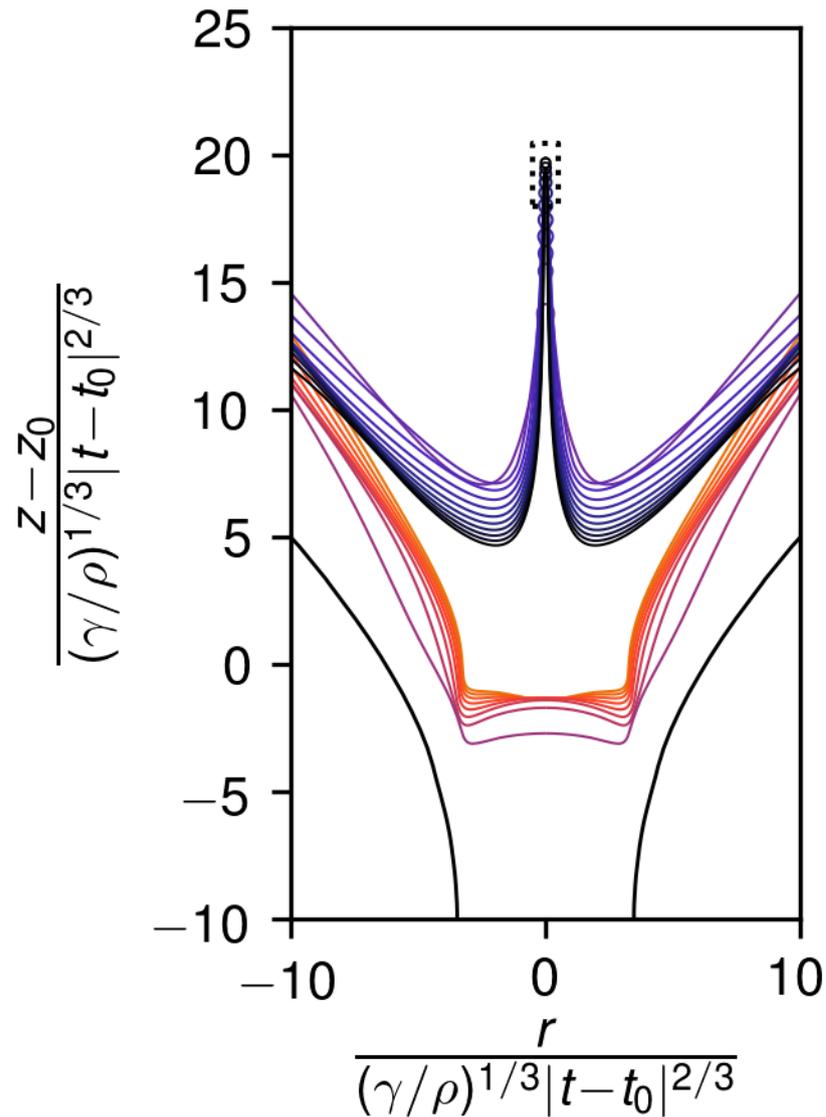
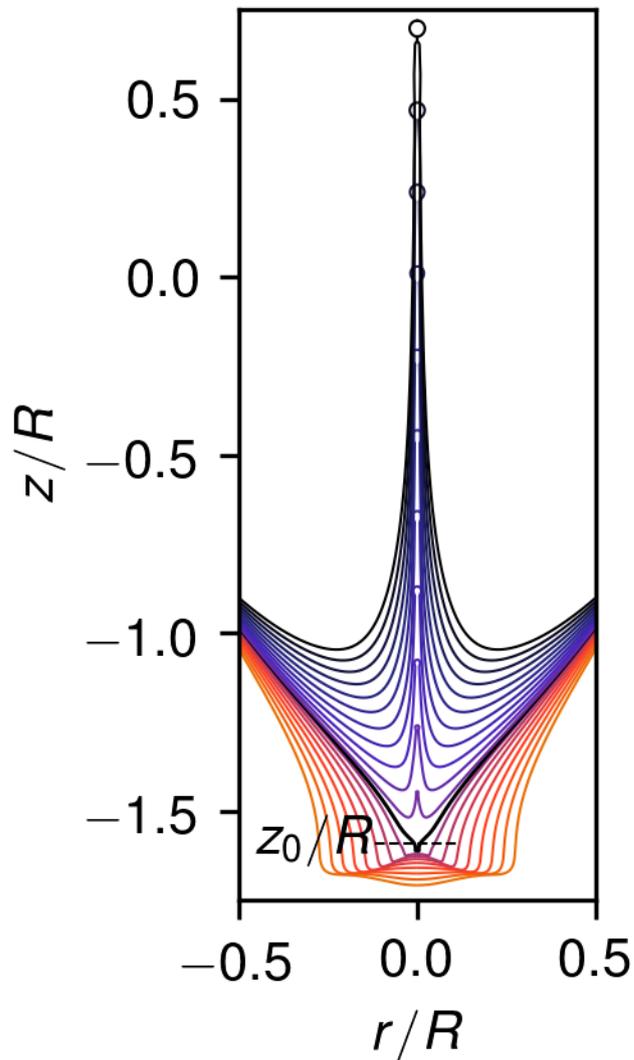
- National Science Foundation Grant No. 1351466
- Ernie Lewis



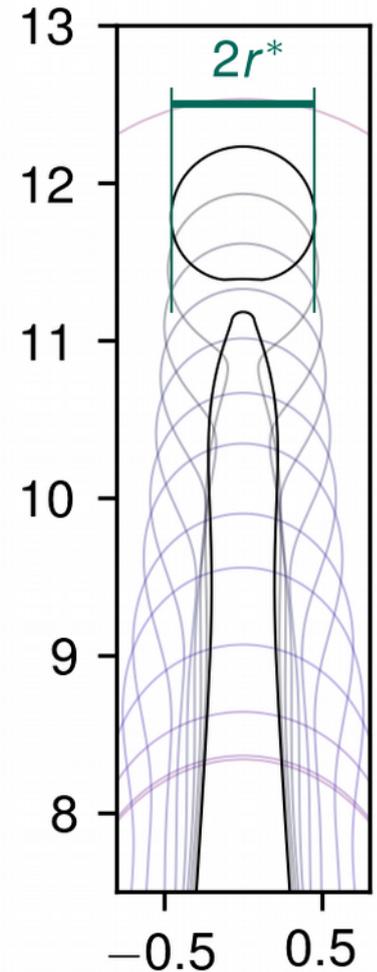
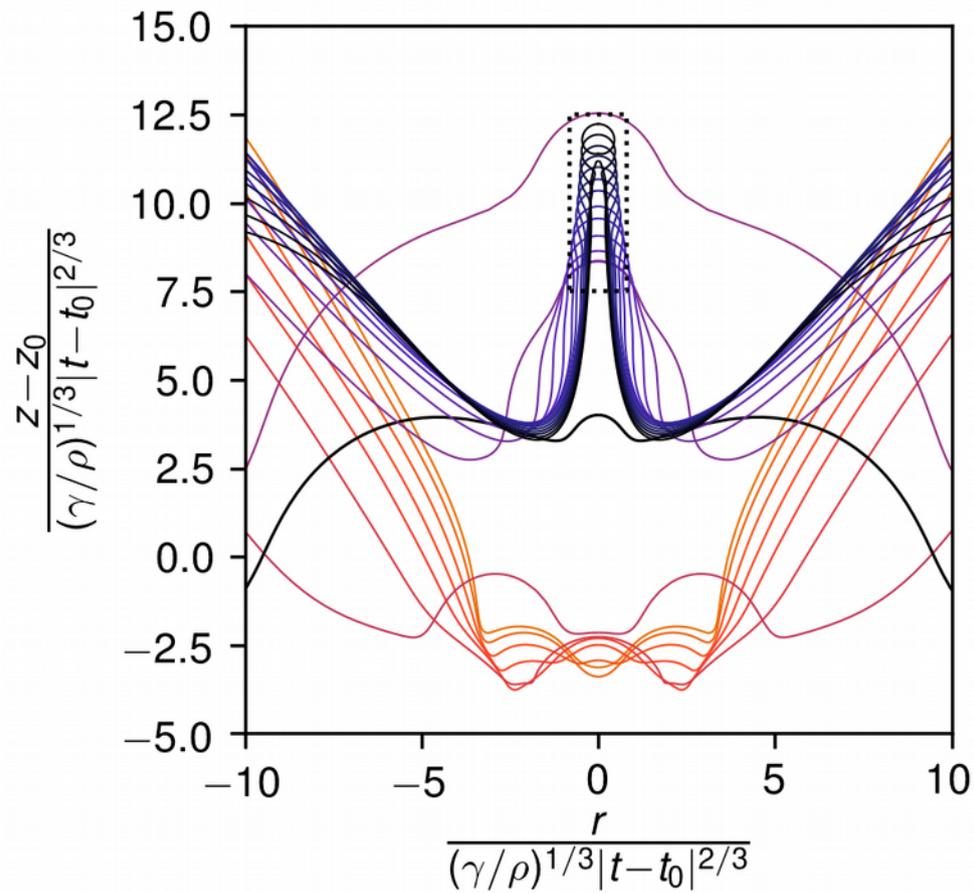
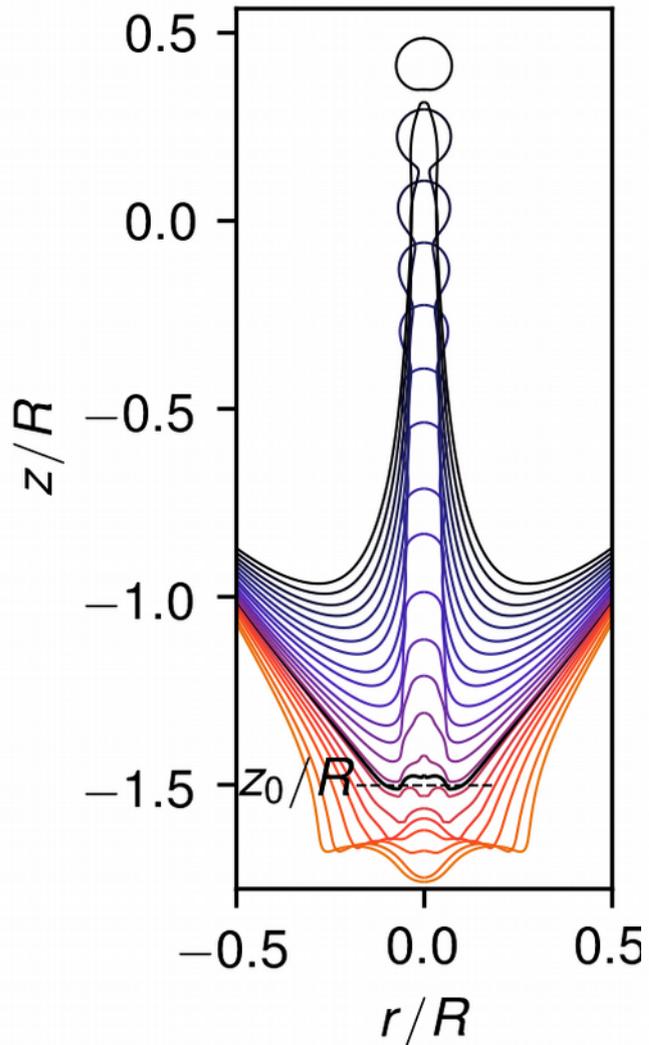
# Self-similar scaling: $La=610$



# Self-similar scaling: $La=1700$

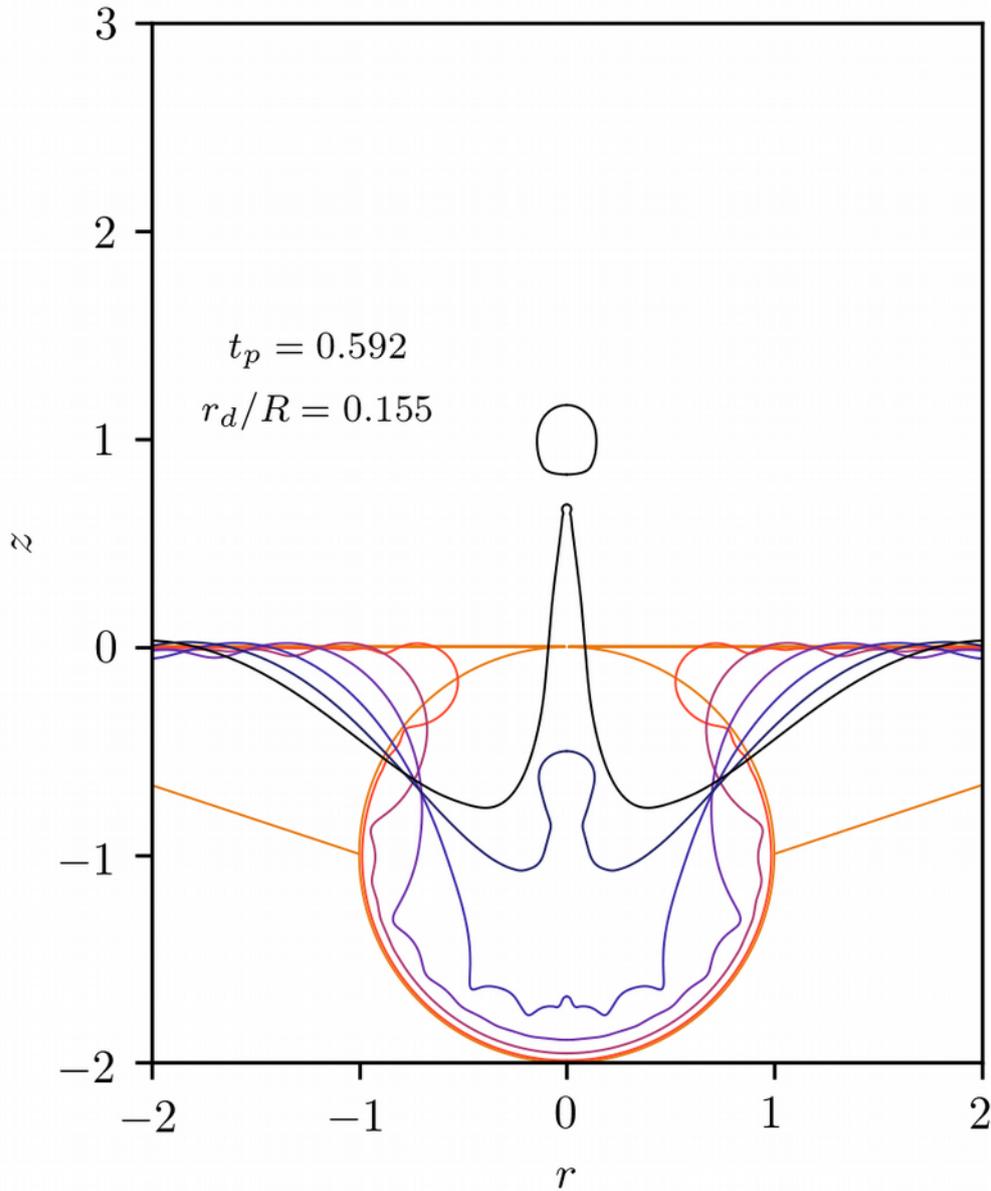


# Self-similar scaling: $La=7200$

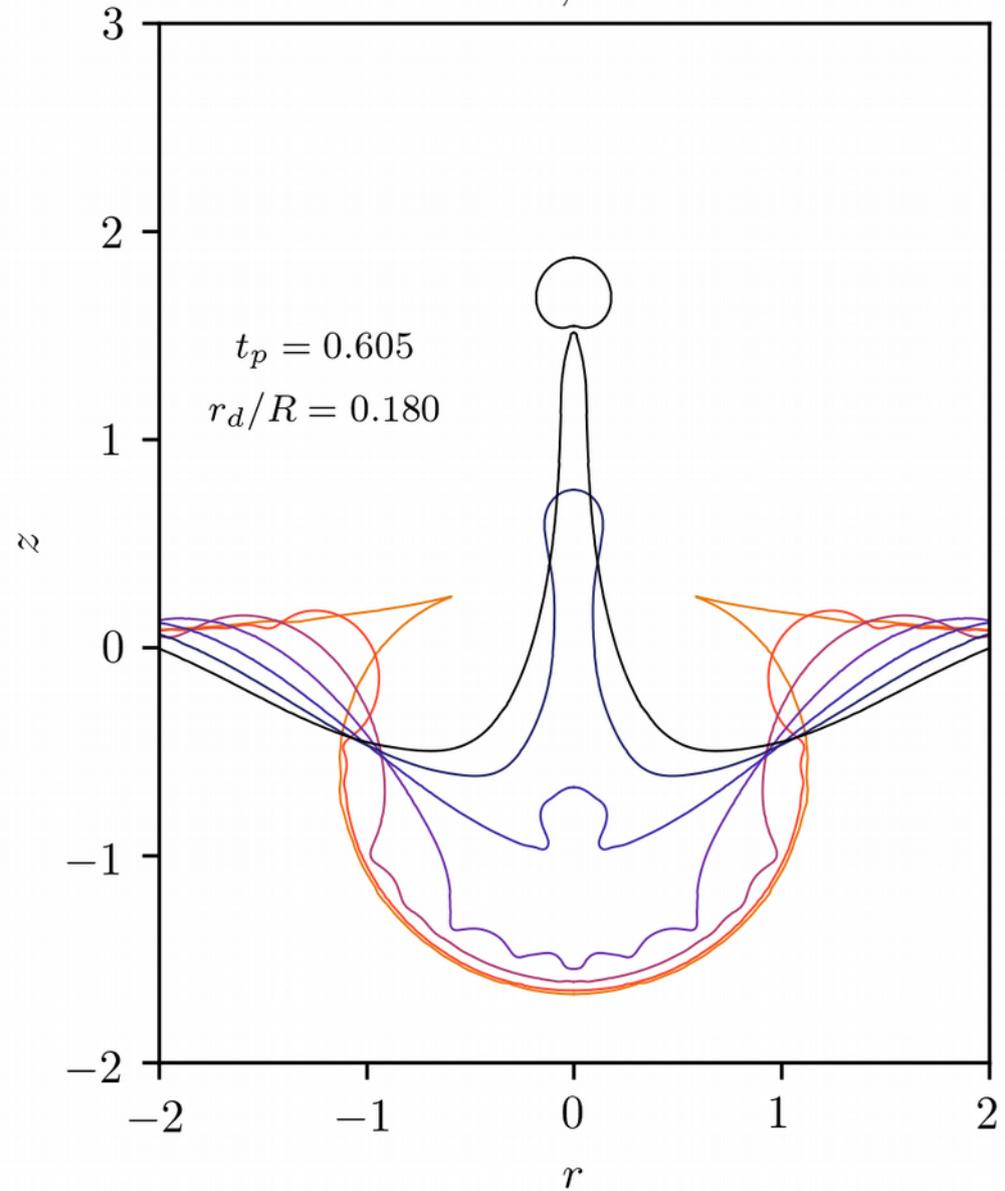


# Gravity effects: $Bo \sim 0.2$

$La = 71900$

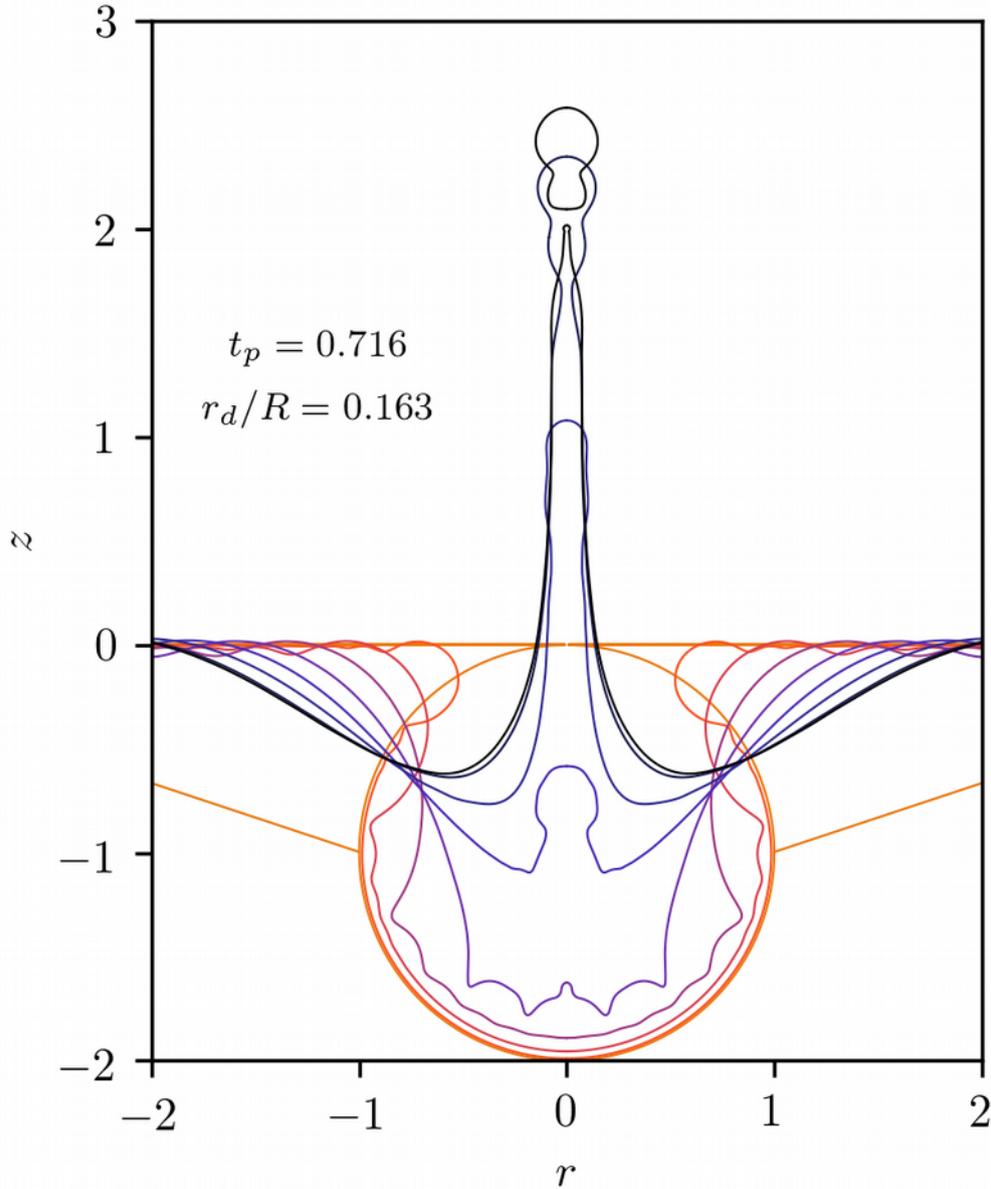


$La = 77600, Bo = 0.212$



# Gravity effects: $Bo \sim 1$

$La = 170000$



$La = 171000, Bo = 1.03$

