A decade of research with Gerris

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Basilisk/Gerris Users’ Meeting, Princeton 2017
A decade of research with Gerris

Most recent ones:

- **Liquid-solid interactions at nanoscale**: Kondic, Mahady (NJIT), Rack (UT, ORNL)
- **Variable surface tension**: Kondic, Seric (NJIT) *(her talk is this afternoon)*
- **Mixing of alcohol with water**: Seric (NJIT), Kim (KAIST), Stone (Princeton)
- **Ferrofluids**: Qiu, Feng (UBC)
- **Microfluidics**: Leshansky (Technion), Tabeling (ESPCI), Seemann (Saarbrucken)
- **Viscoelasticity**: Renardy (Virginia Tech)
- **Contact lines**: Zaleski, Popinet (UPMC)
- **APS/DFD Focus Session**: Modeling, Computations and Applications of Wetting/Dewetting
- **Breakup of liquid filament**: Nakrani, Dziedzic (NJIT)
- **Acknowledgement**: NSF Grants DMS-1320037 and CBET-1604351
Capillary self-focusing: Generation of monodisperse emulsions

Can we observe the capillary focusing?

Can we predict the transition between two different emulsification mechanisms?
Volume-of-fluid (VoF) based Hele-Shaw solver

\[
\frac{12\mu}{b^2} \mathbf{V} = -\nabla p^* + \gamma \kappa (x, y, t) \delta \mathbf{n},
\]

\[
p^* = p(x, y) + \frac{2\gamma}{b} f
\]

Advection of the VoF function

\[
\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{V} f) = 0
\]

At inlet:

\[
\frac{\partial p^*_i}{\partial x} = -\frac{12\mu_i Q_i}{b^3 w_i \infty}, \quad i = 1, 2,
\]

At outlet:

\[
p^* = p_0 + A \frac{2\gamma}{b} f
\]

[Hein et al., Microfluid. Nanofluid. 2015]
Generation of monodisperse emulsions

Step (Ca = 0.016)

Multi-Jet (IV)

Ca\textsubscript{ch} = 2.5 \cdot 10^{-2}
\text{w}_{1,\infty}/b = 16.4

Ca\textsubscript{ch} = 2.1 \cdot 10^{-2}
\text{w}_{1,\infty}/b = 23.0

Single-Jet (V)

Ca\textsubscript{ch} = 3.8 \cdot 10^{-2}
\text{w}_{1,\infty}/b = 7.9

Ca\textsubscript{ch} = 5.1 \cdot 10^{-2}
\text{w}_{1,\infty}/b = 19.6

Ca \approx 0.02, w_{1,\infty}/b \approx 8, A = 1
\[ \rho (\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot (2\mu D) + \sigma \kappa \delta_s \hat{n} + \nabla_s \sigma \delta_s \]
\[ \nabla \cdot \mathbf{u} = 0 \]

Spreading of alcohol on water-air surface


![Image of alcohol spreading on water-air surface with time stamps (t=0 ms to t=340 ms).](image)

![Graph showing the relationship between Δγ and U.](image)

![Equation for calculating the spreading velocity U.](image)
Marangoni-driven flow: liquid lens of diameter 30mm placed on water surface

\[ \Delta \sigma = 20 \text{ mN/m} \quad \Delta \sigma = 10 \text{ mN/m} \]

Afkhami et. al., in preparation (2018)
Dynamic surface tension: time evolution of surface tension, $\Delta \sigma = 20 \text{ mN/m}$

![Graph showing the evolution of surface tension over time](image)

- **$\sigma$**: Surface tension
- **$\sigma_{\text{water}}$**: Surface tension of water
- **$\sigma_{\text{drop}}$**: Surface tension of the drop

<table>
<thead>
<tr>
<th>Time (ms)</th>
<th>Surface Tension (mN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>72</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>68</td>
</tr>
<tr>
<td>5</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
</tr>
</tbody>
</table>

The graph shows the time evolution of surface tension from $t=2$ ms to $t=6$ ms.

- At $t=2$ ms, the surface tension is 72 mN/m.
- At $t=3$ ms, the surface tension is 70 mN/m.
- At $t=4$ ms, the surface tension is 68 mN/m.
- At $t=5$ ms, the surface tension is 66 mN/m.
- At $t=6$ ms, the surface tension is 64 mN/m.

These values are approximate due to the representation in the image.
Marangoni number $M = \frac{3\Delta \sigma}{(2\bar{\sigma} \theta^2)}$; $M_t \approx 2 \pm 0.225$
\[ \nabla \cdot \mathbf{u} = 0 \]
\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\nabla \cdot \mathbf{u}) \mathbf{u} \right) = \nabla \cdot \tilde{\tau} + \sigma \kappa \delta_s \mathbf{n} + \rho \mathbf{g} \]
\[ \tilde{\tau} = \rho \mathbf{l} + \mu \dot{\gamma} + \tau_m \]
\[ \dot{\gamma} = \nabla \mathbf{u} + \nabla \mathbf{u}^T \]
\[ \tau_m = -\frac{\mu_m}{2} \mathbf{H}^2 \mathbf{l} + \mu_m \mathbf{H} \mathbf{H}^T \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{H} = 0 \]
\[ \mathbf{B}(\mathbf{x}, t) = \begin{cases} \mu_d \mathbf{H} & \text{in the ferrofluid} \\ \mu_o \mathbf{H} & \text{in the matrix,} \end{cases} \]
\[ \mathbf{H} = \nabla \psi \]
\[ \nabla \cdot (\mu \nabla \psi) = 0, \]
\[ \mu_m = \mu_m(f) \]

Interaction of a Pair of Ferrofluid Drops in a Rotating Magnetic Field

[Qui et al., JFM, submitted, 2018]
Interaction of a Pair of Ferrofluid Drops in a Rotating Magnetic Field

[Qui et al., JFM, submitted, 2018]
Instability of nanoscale liquid metal filaments

\[ \text{Oh} = \frac{\mu}{\sqrt{\rho \sigma R}} \]

[Hartnett et al., Langmuir, 2015]

\[ 0.1 \leq \text{Oh} \leq 0.2 \]

\[ 10 \leq L_0 \leq 20 \]
Breakup of Finite Size Liquid Filaments Involving Substrate Effects

Oh = 0.15, $L_0 = 23$
